CLASS TEST

S.No. : 01 IG_CE_A_170123

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 17/01/2023

ANSWER KEY

1.	(d)	7.	(a)	13.	(c)	19.	(c)	25.	(b)
2.	(b)	8.	(a)	14.	(d)	20.	(b)	26.	(a)
3.	(c)	9.	(c)	15.	(a)	21.	(b)	27.	(a)
4.	(a)	10.	(a)	16.	(c)	22.	(a)	28.	(c)
5.	(d)	11.	(d)	17.	(b)	23.	(c)	29.	(a)
6.	(a)	12.	(d)	18.	(d)	24.	(a)	30.	(b)

DETAILED EXPLANATIONS

1. (d)

$$\frac{e^x}{(1-e^x)}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

Integrating on both sides, we get,

$$-\ln(1-e^x) + \ln(\tan y) = C_1$$

$$\ln\left(\frac{\tan y}{(1-e^x)}\right) = C_1$$

$$\frac{\tan y}{(1-e^x)} = e^{C_1} = C$$

$$\tan y = C(1-e^x)$$

2. (b)

$$y = cx^k$$

$$\Rightarrow \quad \frac{dy}{dx} = ckx^{k-1}$$

$$\Rightarrow \quad c = \frac{1}{k}x^{1-k}\frac{dy}{dx}$$

$$\therefore \quad y = \frac{1}{k}x\frac{dy}{dx}$$

To get orthogonal trajectories, $\frac{dy}{dx}$ is replaced by $-\frac{dx}{dy}$

$$\Rightarrow \quad y = -\frac{1}{k}x\frac{dx}{dy}$$

$$\therefore \quad ky^2 + x^2 = \text{constant}$$

3. (c)

As per Euler's equation

Since $u(x, y)$ is homogenous function of degree 4.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4u$$

4. (a)

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

5. (d)

For function to be differentiable i.e. continuous $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)}$$

$$= \lim_{x\to 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3}$$

$$f(0^+) = \lim_{x\to 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)}$$

$$= \lim_{x\to 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\frac{5}{3}$

6. (a)

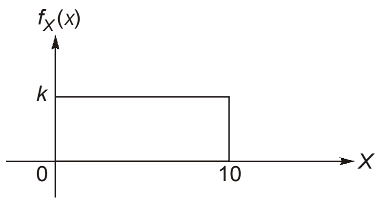
$$h = 0.2$$

$$\int_{4.0}^{5.2} f(x)dx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 .....y_5)]$$

$$= \frac{0.2}{2}\left[\begin{matrix}(2.3863 + 2.6484) + 2(2.4351 + 2.4816 \\ + 2.5261 + 2.5686 + 2.6094)\end{matrix}\right]$$

$$= 3.0276$$

7. (a)



$$\int_0^{10} k dx = 1$$

$$kx\Big|_0^{10} = 1$$

$$10k = 1$$

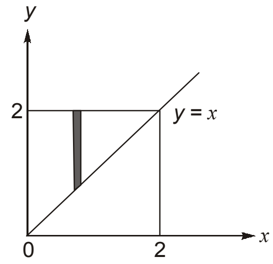
$$k = \frac{1}{10}$$

$$P(2.5 \le X \le 7.5) = \int_{2.5}^{7.5} \frac{1}{10} dx = \frac{1}{10} x\Big|_{2.5}^{7.5} = \frac{1}{10}(7.5 - 2.5) = \frac{1}{2}$$

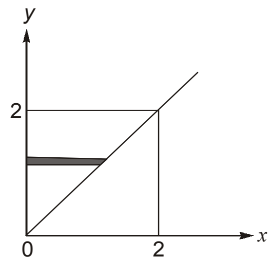
Mean square value,

$$\int \frac{1}{10} x^2 dx = \frac{1}{10} \frac{x^3}{3}\Bigg|_0^{10} = \frac{10^3 - 0^3}{30} = \frac{1000}{30} = \frac{100}{3}$$

8. (a)



$x < y < 2$
$0 < x < 2$



$0 < x < y$
$0 < y < 2$

$$I = \int_0^2 \int_0^y f(x,y)\,dx\,dy$$

$$r = p = 0$$
$$q = y$$
$$s = 2$$

9. (c)

Case-I: White ball is transferred from urn A to urn B

Probability of drawing white ball from $B = \frac{2}{2+4} \times \frac{6}{13} = \frac{2}{13}$

Case-II: Black ball is transferred from A to B

Probability of drawing black ball from $B = \frac{4}{2+4} \times \frac{5}{13} = \frac{10}{39}$

Required probability $= \frac{2}{13} + \frac{10}{39} = \frac{16}{39}$

10. (a)

$$\phi = 4x^2y + z^3$$

$$\nabla\phi = \frac{\partial}{\partial x}(4x^2y + z^3)\hat{i} + \frac{\partial}{\partial y}(4x^2y + z^3)\hat{j} + \frac{\partial}{\partial z}(4x^2y + z^3)\hat{k}$$

$$= 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla\phi_{(1,2,1)} = 16\hat{i} + 4\hat{j} + 3\hat{k}$$

The desired directional derivative $= (16\hat{i} + 4\hat{j} + 3\hat{k}) \cdot \frac{(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{16 + 8 + 6}{3} = 10$

11. (d)

$$AX = B$$

Augmented matrix, $[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$

$R_3 \to R_3 + R_2 + R_1$:

$$|A : B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $l + m + n = 0$

Rank of $[A : B] = 2$

Rank of $[A]$ = Rank of $[A : B] = 2 < 3$ (Number of variables)

$\Rightarrow$ Infinitely many solutions are possible.

12. (d)

Eigen values are real, so the matrix should be symmetric.

i.e. $\alpha = \beta$ ...(i)

If all the leading minors of a symmetric matrix are positive, then all its eigen values are positive.

So, $\begin{vmatrix} 1 & \alpha \\ \beta & 2 \end{vmatrix} = 2 - \alpha\beta > 0$ ...(ii)

Conditions (i) and (ii) should be satisfied.

13. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, the given equation is not exact.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2+1)} = \frac{1}{y}$$

$$IF = e^{\int \frac{1}{y}dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

14. (d)

The equation can be re-written as

$$\frac{dy}{dx} + y\tan x = \cos^2 x$$

$$\text{IF} = e^{\int P dx}$$

$$P = \tan x$$

$$\text{IF} = e^{\int \tan x \, dx} = e^{ln|\sec x|}$$

$$\text{IF} = |\sec x|$$

15. (a)

$$\int f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3...) + 2(y_2 + y_4...)\right]$$

Here $h = \frac{1}{2}$. So, $$\int_0^2 f(x)dx = \frac{1}{2\times 3}\left[(0+4) + 4(0.25 + 2.25) + 2(1)\right]$$

$$= \frac{1}{6}\left[4 + 10 + 2\right] = 2.667$$

16. (c)

From Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$1.5 = 1 - \frac{x_0^3 + 3x + A}{3x_0^2 + 3}$$

$$-\frac{1}{2} = \frac{1+3+A}{6}$$

$$-3 = 4 + A$$

$$A = -7$$

17. (b)

$$y = \frac{1}{x} ln x$$

$$\frac{dy}{dx} = \frac{1}{x}\cdot\frac{1}{x} + ln\, x\left(\frac{-1}{x^2}\right)$$

$$= \frac{1}{x^2}(1 - ln x)$$

for maxima $\frac{dy}{dx} = 0$

$ln\, x = 1$ $\Rightarrow$ e is a stationary point

$$\frac{d^2y}{dx^2} = -\frac{1}{x^3}(3 - 2\, ln x)$$

at $x = e$

$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = \frac{-1}{e^3}$$

hence maxima at $x = e$

18. (d)

Given function is $y = \frac{1}{x}$ [hyperbolic function]

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

hence, option (d) is correct.

19. (c)

$$\text{volume of the solid} = \int_a^b \pi y^2 dx$$

given as $y = \dfrac{1}{\sqrt{x}}$

$$\text{volume of the solid} = \int_2^3 \frac{\pi}{x} \cdot dx = (\pi \, l\text{n}\, x)_2^3$$

$$= \pi \, l\text{n}\frac{3}{2} = \pi \, l\text{n}(1.5)$$

20. (b)

Let $\tan^{-1}(x) = \theta$, $\quad x = \tan\theta$

$$g(x) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \frac{\theta}{2}$$

$$\frac{df(x)}{dg(x)} = \frac{d\theta}{d(\theta/2)} = 2$$

21. (b)

Let $\tan^{-1}(x) = \theta$, $\quad x = \tan\theta$

$$g(x) = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right) = \frac{\theta}{2}$$

$$\frac{df(x)}{dg(x)} = \frac{d\theta}{d(\theta/2)} = 2$$

22. (a)

$[A : B]$

$$\begin{bmatrix} 1 & 2 & -2 & 0 & : & 0 \\ 2 & -1 & -1 & 0 & : & 0 \\ 1 & 2 & -1 & 0 & : & 0 \\ 4 & -1 & -1 & 3 & : & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_2, \quad R_3 \to (R_3 - R_1)$

$$= \begin{bmatrix} 1 & 2 & -2 & 0 & : & 0 \\ 2 & -1 & -1 & 0 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 \\ 2 & 0 & 0 & 3 & : & 0 \end{bmatrix}$$

$R_1 \rightarrow \frac{1}{3}(R_1 + 2R_2 + 4R_3)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 \\ 2 & -1 & -1 & 0 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 \\ 2 & 0 & 0 & 3 & : & 0 \end{bmatrix}$$

$R_2 \rightarrow (2R_1 - R_2 - R_3), \quad R_4 \rightarrow \frac{1}{3}(R_4 - 2R_1)$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & : & 0 \\ 0 & 1 & 0 & 0 & : & 0 \\ 0 & 0 & 1 & 0 & : & 0 \\ 0 & 0 & 0 & 1 & : & 0 \end{bmatrix}$$

$\rho(A : B) = \rho(A) = 4$ = number of variables

$\Rightarrow$ system is consistent with trivial solution.

23. (c)

$$(A - \lambda I)\hat{X} = 0$$

$$\begin{bmatrix} 3-\lambda & -2 & 2 \\ 0 & p-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$6 - 2\lambda - 10 = 0 \quad \Rightarrow \lambda = -2$$

$$5p - 5\lambda = 0 \quad \Rightarrow p = \lambda$$

$$p = \lambda = -2$$

$$p = -2$$

24. (a)

The given curve is

$$\sqrt{x} + \sqrt{y} = \sqrt{6}$$

$$\sqrt{y} = \sqrt{6} - \sqrt{x}$$

$$y = \left(\sqrt{6} - \sqrt{x}\right)^2$$

for $x = 0, \; y = 6$

for $y = 0, \; x = 6$

Now, the area bounded is $= \int_0^6 y\,dx = \int_0^6 \left(\sqrt{6} - \sqrt{x}\right)^2 dx$

$$= \int_0^6 \left(6 + x - 2\sqrt{6}\sqrt{x}\right) dx = \left[6x + \frac{x^2}{2} - 2\sqrt{6}\,\frac{x^{3/2}}{3/2}\right]_0^6$$

$$= \left[36 + 18 - \frac{2\sqrt{6} \times 6 \times \sqrt{6} \times 2}{3}\right]$$

$$= 54 - 48 = 6 \text{ unit}^2$$

25. (b)

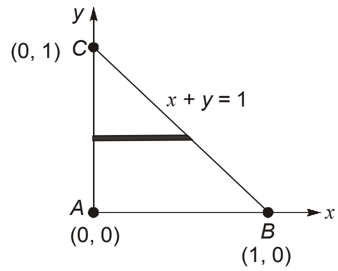
$$\lim_{x\to 0}\frac{e^{ax}-e^{-ax}}{\log(1+bx)} = \lim_{x\to 0}\frac{(e^{ax}-e^{-ax})\times 2ax\times b}{2ax\times bx\log(1+bx)}$$

$$= \lim_{x\to 0}\left(\frac{e^{ax}-e^{-ax}}{2ax}\right)\times\lim_{x\to 0}\frac{bx}{\log(1+bx)}\left(\frac{2a}{b}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sinh ax}{ax}\right)\lim_{x\to 0}\frac{bx}{\log(1+bx)}\left(\frac{2a}{b}\right) = 1\times 1\times\frac{2a}{b} = \frac{2a}{b}$$

26. (a)

$$I = \int_0^1\int_0^{1-x} 3y\,dy\,dx$$



$$= \int_0^1\left[\frac{3y^2}{2}\right]_0^{1-x} dx$$

$$= \int_0^1\frac{3}{2}(1-x)^2 dx$$

$$= -\frac{3}{2}\frac{(1-x)^3}{3}\bigg|_0^1 = -\frac{1}{2}\frac{(1-x)^3}{1}\bigg|_0^1 = \frac{1}{2}$$

27. (a)

Since the probability of occurrence is very small, this follows Poisson distribution

$$\text{mean} = m = np$$
$$= 2000\times 0.001 = 2$$

Probability that more than 2 will get a bad reaction

$$= 1-p(0)-p(1)-p(2) = 1-\left[e^{-m}+\frac{e^{-m}\cdot m^1}{1!}+\frac{e^{-m}\cdot m^2}{2!}\right]$$

$$= 1-\left[e^{-2}+\frac{e^{-2}\cdot 2}{1}+\frac{2^2\cdot e^{-2}}{2}\right] = 1-\left[\frac{1}{e^2}+\frac{2}{e^2}+\frac{2}{e^2}\right]$$

$$= 1-\frac{5}{e^2}$$

28. (c)

$$\text{Work done} = \int_c \vec{F}\cdot\vec{dr}$$

$$\vec{r} = x\hat{i}+y\hat{j}$$
$$d\vec{r} = dx\hat{i}+dy\hat{j}$$

$$= \int_c\left(2x^2y\hat{i}+3xy\hat{j}\right)\cdot\left(dx\hat{i}+dy\hat{j}\right)$$

$$= \int_c\left(2x^2y\,dx+3xy\,dy\right)$$

$$y = 4x^2$$
$$dy = 8x\,dx$$

$$\text{Work done} = \int_c \left(2x^2 y dx + 3xy \cdot 8x dx\right) = \int_0^1 2x^2 \cdot 4x^2 dx + 24x^2 \cdot 4x^2 dx$$

$$= \int_0^1 104x^4 dx = 104\frac{x^5}{5}\bigg|_0^1 = \frac{104}{5} = 20.8 \text{ J}$$

29. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)\,dx = 1$$

$$\frac{kx^2}{2}\bigg|_0^2 + 2kx\big|_2^4 + \left(\frac{-kx^2}{2} + 6kx\right)\bigg|_4^6 = 1$$

$$\frac{k}{2}(2^2 - 0) + 2k(4-2) - \frac{k}{2}(6^2 - 4^2) + 6k(6-4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2 dx + \int_2^4 \frac{1}{4}x dx + \int_4^6 \left(-\frac{1}{8}x^2 + \frac{3}{4}x\right)dx$$

$$= \frac{1}{8}\frac{x^3}{3}\bigg|_0^2 + \frac{1}{4}\frac{x^2}{2}\bigg|_2^4 - \frac{1}{8}\frac{x^3}{3}\bigg|_4^6 + \frac{3}{4}\frac{x^2}{2}\bigg|_4^6 = \frac{1}{3} + \frac{3}{2} - \frac{19}{3} + \frac{15}{2} = 3$$

30. (b)

$$\phi_1 = ax^2 - byz - (a+2)x$$

$$\nabla\phi_1 = [2ax - (a+2)]\hat{i} - bz\hat{j} - by\hat{k}$$

$$\nabla\phi_1(1, -1, 2) = (a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$

$$\phi_2 = 4x^2 y + z^3 - 4$$

$$\nabla\phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla\phi_2(1, -1, 2) = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

Since surfaces are orthogonal to each other at (1, –1, 2)

$$\nabla\phi_1 \cdot \nabla\phi_2 = 0$$

$$[(a-2)\hat{i} - 2b\hat{j} + b\hat{k}] \cdot [-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$$

$$-8(a-2) - 8b + 12b = 0 \qquad \text{... (i)}$$

Also point (1, –1, 2) lies on the surface.

$$\Rightarrow \quad a \times 1 + 2b = (a+2)1$$

$$b = 1$$

Putting this in equation 1, we get,

$$-8(a-2) - 8 + 12 = 0$$

$$a - 2 = -\frac{1}{8} \times (-4) = 0.5$$

$$a = 2.5$$

■■■■