CLASS TEST

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# SIGNALS AND SYSTEMS

## EC + EE

**Date of Test : 17/01/2023**

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (d) | 7. | (c) | 13. | (b) | 19. | (c) | 25. | (b) |
| 2. | (c) | 8. | (b) | 14. | (b) | 20. | (a) | 26. | (d) |
| 3. | (d) | 9. | (c) | 15. | (a) | 21. | (b) | 27. | (b) |
| 4. | (a) | 10. | (d) | 16. | (c) | 22. | (a) | 28. | (c) |
| 5. | (a) | 11. | (c) | 17. | (c) | 23. | (d) | 29. | (b) |
| 6. | (c) | 12. | (b) | 18. | (b) | 24. | (a) | 30. | (d) |

## DETAILED EXPLANATIONS

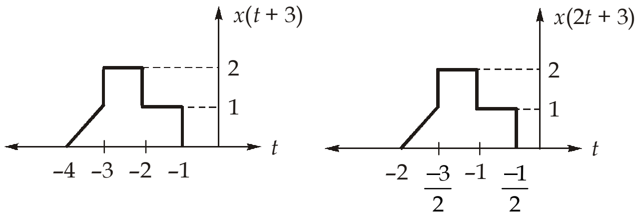
**1. (d)**

Energy of signal $x(n)$ = Energy in even part of signal $x(n)$ + Energy in odd part of signal $x(n)$

$$\text{Energy of signal } x(n) = 6 + 8 = 14$$

**2. (c)**

The signal $x(2t + 3)$ can be obtained by first shifting $x(t)$ to the left by 3 units and then scaling by 2 units.




**3. (d)**

$$u(t) \xrightarrow{\text{L.T.}} \frac{1}{s}$$

$$u(t-1) \xrightarrow{\text{L.T.}} \frac{e^{-s}}{s}$$

$$u(2t-1) \xrightarrow{\text{L.T.}} \frac{1}{2} \cdot \frac{e^{-s/2}}{s/2}$$

$$u(-2t-1) \xrightarrow{\text{L.T.}} -\frac{1}{2} \cdot \frac{e^{s/2}}{s/2} = \frac{-e^{s/2}}{s}$$

**4. (a)**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} = 1 + z^{-1} - z^{-2} - z^{-3}$$

$$X\left(\frac{1}{2}\right) = 1 + 2 - 4 - 8 = -9$$

**5. (a)**

System transfer function using laplace transform would be,

$$H(s) = \frac{Y(s)}{X(s)} = \frac{Ls}{R + Ls} = 1 - \frac{R/L}{(R/L) + s}$$

taking inverse laplace transform

$$h(t) = \delta(t) - \frac{R}{L} e^{-(R/L)t}$$

for stability of any system,

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{0}^{\infty}\left|\delta(t) - \frac{R}{L}e^{-(R/L)t}\right|dt = 1 - \left[\frac{R}{L}\frac{e^{-(R/L)t}}{(-R/L)}\right]_0^{\infty}$$

$$= 1 - (0 + 1) = 0$$

so $$\int_{-\infty}^{\infty} |h(t)dt| < \infty$$

so system is BIBO stable.

**6.** **(c)**

We first apply time-shifting operation to find,

$$y(n) = x(n-1) = \{3, \underset{\uparrow}{4}, 5, 6\}$$

$$y\left(\frac{n}{2}\right) = x(0.5n - 1) = \{3, 0, \underset{\uparrow}{4}, 0, 5, 0, 6, 0\}$$

**7.** **(c)**

$$x(t) = \underbrace{4\cos\left(\frac{2\pi}{3}t + 40^\circ\right)}_{x_1(t)} + \underbrace{3\sin\left(\frac{4\pi}{5}t + 20^\circ\right)}_{x_2(t)}$$

$$\omega_1 = \frac{2\pi}{3} \quad \Rightarrow \quad T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi/3} = 3$$

$$\omega_1 = \frac{4\pi}{5} \quad \Rightarrow \quad T_2 = \frac{2\pi}{4\pi/5} = \frac{5}{2}$$

$$T = \text{LCM of } (T_1, T_2)$$

$$\frac{T_1}{T_2} = \frac{3}{5/2} = \frac{6}{5}$$

$\Rightarrow$ $$T = 3 \times 5 \text{ or } 6 \times \frac{5}{2} = 15\,\text{sec}$$

**8.** **(b)**

$$e^{-(2t-2)}\,u(t-1) = e^{-2(t-1)}\,u(t-1)$$

Now, $$e^{-2t}\,u(t) \leftrightarrow \frac{1}{2 + j\omega}$$

$$e^{-2(t-1)}\,u(t-1) \leftrightarrow \frac{e^{-j\omega}}{2 + j\omega}$$

**9.** **(c)**

Overall impulse response is,

$$h_3(t) = h_1(t) * h_2(t)$$
$$= \delta(t+1) * e^{-t}\,u(t)$$
$$= e^{-(t+1)}\,u(t+1)$$

**10.** **(d)**

$$u[n] \leftrightarrow \frac{1}{1-z^{-1}}, \ |z|>1$$

$$u[n+2] \leftrightarrow \frac{z^2}{1-z^{-1}}, \ |z|>1$$

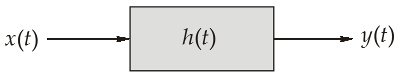
$$u[-n+2] \leftrightarrow \frac{z^{-2}}{1-z}, |z|<1 = \frac{z^{-1}}{z-z^2}, |z|<1$$

**11.** **(c)**

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output, $$y(t) = e^{-3t}\,u(t) - e^{-4t}\,u(t)$$




We know that, $$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

$\therefore$ $$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t}\,u(t)$$

**12.** **(b)**

Given, $$y(t) = e^{-t}\,u(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)$$

$$= e^{-t}\,u(t) * (.....+\delta(t+4)+\delta(t+2)+\delta(t)+\delta(t-2)+\delta(t-4)+.....)$$

Using convolution property of impulse response,

i.e., $$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$y(t) = ...+e^{-(t+4)}\,u(t+4)+e^{-(t+2)}\,u(t+2)+e^{-t}u(t)+e^{-(t-2)}\,u(t-2)+e^{-(t-4)}\,u(t-4)$$

+ ...

In the range $0 \le t < 2$, we may write $y(t)$ as,

$$y(t) = [...+e^{-(t+4)}\,u(t+4)+e^{-(t+2)}\,u(t+2)+e^{-t}u(t)+e^{-(t-2)}\,u(t-2)+e^{-(t-4)}\,u(t-4)+...]\;(u(t)-u(t-2))$$

$$= \left(e^{-t}+e^{-(t+2)}+e^{-(t+4)}+......\right) \ ; \ 0 \le t < 2$$

$$= e^{-t}\left(1+e^{-2}+e^{-4}+......\right) \ ; \ 0 \le t < 2$$

$$= e^{-t}\left[\frac{1}{1-e^{-2}}\right] \ ; \ 0 \le t < 2$$

$\because$ $$y(t) = Ae^{-t} \text{ for } 0 \le t < 2$$

$\therefore$ $$A = \frac{1}{1-e^{-2}}$$

**13. (b)**

**Linearity :**

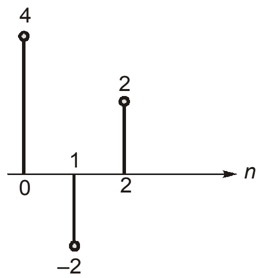
$$x_1(t) \rightarrow 3x_1(\sin t) = y_1(t)$$
$$x_2(t) \rightarrow 3x_2(\sin t) = y_2(t)$$
$$x_1(t) + x_2(t) \rightarrow 3[x_1(\sin t) + x_2(\sin t)] = y_1(t) + y_2(t) \Rightarrow \text{System is linear.}$$

**Causality :**

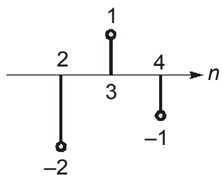
At $t = -\pi$, $\quad y(-\pi) = 3x(0) \Rightarrow$ Non-causal.

**14. (b)**

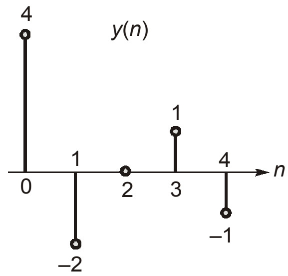
For $x_1[n] = 2\delta[n]$, $\quad y_1(n) = 2y(n) \Rightarrow$




For $x_2[n] = -\delta[n-2]$, $\quad y_2(n) = -y(n-2) \Rightarrow$




$y(n) = y_1(n) + y_2(n) \Rightarrow$




**15. (a)**

Given,
$$X(z) = \frac{10 - 8z^{-1}}{2 - 5z^{-1} + 2z^{-2}} = \frac{2}{(2 - z^{-1})} + \frac{4}{(1 - 2z^{-1})}$$
$$X(z) = \frac{2z}{2z-1} + \frac{4z}{z-2}$$
$$X(z) = \frac{z}{\left(z - \frac{1}{2}\right)} + \frac{4z}{(z-2)}$$

Since, ROC includes unit circle,

$\therefore$ ROC of $X(z)$ is $\frac{1}{2} < |z| < 2$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

$$\therefore \quad x(1) = \frac{1}{2} = 0.5$$

**16. (c)**

Integration is linear system

**Time-variant (or) time invariant system:**

Delay input by $t_o$ units

$$= \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau - t_0)\,d\tau \quad \text{...(i)}$$

Delay output by $t_o$ units (or) substitute $(t - t_0)$ in the place of $t$.

$$y(t - t_0) = \frac{1}{T}\int_{t-t_0-T/2}^{t-t_0+T/2} x(\tau)\,d\tau \quad \text{...(ii)}$$

From equation (i) and (ii), we can say equation (i) = equation (ii),

∴ The given system is time invariant.

**Causal (or) Non-causal system:**

$$y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)\,d\tau$$

Let, $T = 4$

$$y(0) = \frac{1}{4}\int_{-2}^{2} x(\tau)\,d\tau$$

here, $y(0)$ depends on future value $x(2)$.

∴ The given system is non-causal system.

**17. (c)**

$$x_1(t) \xrightarrow{\text{L.T.}} \frac{1}{s+2}$$

$$x_2(t) \xrightarrow{\text{L.T.}} \frac{1}{s+3}$$

$$x_1(t-2) \xrightarrow{\text{L.T.}} \frac{e^{-2s}}{s+2}$$

$$x_2(t+3) \xrightarrow{\text{L.T.}} \frac{e^{3s}}{s+3}$$

$$x_2(-t+3) \xrightarrow{\text{L.T.}} \frac{e^{-3s}}{3-s}$$

$$\therefore\ y(t) \xrightarrow{\text{L.T.}} \frac{e^{-2s}}{(s+2)} \cdot \frac{e^{-3s}}{(3-s)}$$

$$y(t) \xrightarrow{\text{L.T.}} \frac{e^{-5s}}{(s+2)(3-s)}$$

**18. (b)**

By applying Laplace transform on differential equation,

$$s^2Y(s) + 2sY(s) - 3\,Y(s) = X(s)$$

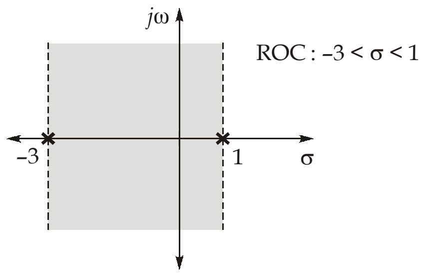
$$Y(s)\,[s^2 + 2s - 3] = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+2s-3} = \frac{1}{(s-1)(s+3)}$$

$$= \frac{A}{s-1} + \frac{B}{s+3}$$

$$= \frac{1/4}{s-1} - \frac{1/4}{s+3}$$

Given system is stable:




$$h(t) = -\frac{1}{4}e^{-3t}u(t) - \frac{1}{4}e^{t}u(-t)$$

**19. (c)**

$$y[n] = h[n] * x[n]$$
$$= h[n] * 3\delta[n-2]$$

$$h[n] = \frac{1}{3}y[n+2]$$

$$y[n] = \left[\frac{1}{2}\left(\frac{-1}{2}\right)^{n-2} + \frac{1}{2}\left(\frac{1}{4}\right)^{n-2}\right]u[n-2]$$

$$h[n] = \frac{1}{3}\left[\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)^{n-2+2} + \frac{1}{2}\left(\frac{1}{4}\right)^{n-2+2}\right]u[n+2-2]$$

$$= \frac{1}{6}\left(\left(-\frac{1}{2}\right)^{n} + \left(\frac{1}{4}\right)^{n}\right)u[n]$$

**20. (a)**

- $h[n] = 0$ for $n < 0 \Rightarrow$ causal
- $\sum_{n=-\infty}^{\infty} |h[n]|$ is finite $\Rightarrow$ stable

**21.** **(b)**

We have,

$$\frac{1}{s} \xleftrightarrow{L^{-1}} u(t)$$

$$\frac{e^{-3s}}{s} \xleftrightarrow{L^{-1}} u(t-3) \qquad \text{(Time shifting)}$$

$$\frac{d}{ds}\left(\frac{e^{-3s}}{s}\right) \longleftrightarrow -tu(t-3) \qquad \text{(Differentiation in s-domain)}$$

$$\frac{1}{s}\frac{d}{ds}\left(\frac{e^{-3s}}{s}\right) \xleftrightarrow{L^{-1}} \int_{-\infty}^{t} -\tau u(\tau-3)d\tau$$

$$x(t) = -\int_{3}^{t} \tau\, d\tau = -\left(\frac{t^2}{2}\right)_3^t \; ; \; t > 3$$

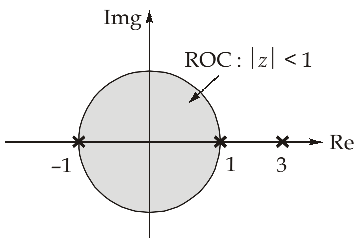
$$x(t) = -\frac{1}{2}(t^2-9)u(t-3)$$

**22.** **(a)**

$$X(z) = \frac{z^2+5z}{z^2-2z-3} = \frac{z(z+5)}{(z-3)(z+1)}$$

$$\frac{X(z)}{z} = \frac{z+5}{(z-3)(z+1)} = \frac{2}{z-3} - \frac{1}{z+1}$$

Thus, $$X(z) = \frac{2z}{z-3} - \frac{z}{z+1}$$




ROC : $|z| < 1$, which is not exterior of circle outside the outermost pole $z = 3$

So, $x[n]$ is anti-causal given as,

$$x[n] = [-2(3)^n + (-1)^n]\, u[-n-1]$$

**23.** **(d)**

$$\left(\frac{\sin 10^4 \pi t}{\pi t}\right) \Rightarrow f_{1\,max} = \frac{10^4 \pi}{2\pi} = 5\text{ kHz}$$

$$\left(\frac{\sin 2\times 10^4 \pi t}{\pi t}\right) \Rightarrow f_{2max} = 10\text{ kHz}$$

$$f_s = 2[\text{min of } (f_{1\,max}, f_{2\,max})]$$
$$= 2 \times 5 = 10\text{ kHz}$$

**24. (a)**

$$x\left(\frac{t-2}{3}\right) = x\left(\frac{t}{3}-\frac{2}{3}\right)$$

Now,

$$x\left(t-\frac{2}{3}\right) \leftrightarrow e^{-\frac{2}{3}s} X(s)$$

$$x\left(\frac{t}{3}-\frac{2}{3}\right) \leftrightarrow \frac{1}{\left|\frac{1}{3}\right|} e^{-\frac{2}{3}\frac{s}{1/3}} X\left(\frac{s}{1/3}\right)$$

$\Rightarrow$

$$x\left(\frac{t-2}{3}\right) \leftrightarrow 3e^{-2s} X(3s)$$

**25. (b)**

$$X(s)-\frac{3H(s)}{s^2} = H(s)$$

$\Rightarrow$

$$X(s) = \left(1+\frac{3}{s^2}\right)H(s)$$

•

$$2H(s)+\frac{H(s)}{s} = Y(s)$$

$\Rightarrow$

$$\left(2+\frac{1}{s}\right)H(s) = Y(s)$$

$\Rightarrow$

$$\left(2+\frac{1}{s}\right)\frac{X(s)}{\left(1+\frac{3}{s^2}\right)} = Y(s)$$

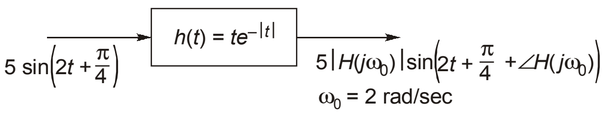
$\Rightarrow$

$$\frac{Y(s)}{X(s)} = \frac{2+\frac{1}{s}}{\frac{3}{s^2}+1} = \frac{s+2s^2}{3+s^2}$$

$\Rightarrow$

$$\frac{d^2y(t)}{dt^2}+3y(t) = \frac{dx(t)}{dt}+2\frac{d^2x(t)}{dt^2}$$

**26. (d)**




$$h(t) = te^{-|t|}$$

$$H(j\omega) = j\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right) = \frac{-4j\omega}{(1+\omega^2)^2}$$

$$|H(j\omega_0)| = \left|\frac{-4j(2)}{(1+4)^2}\right| = \frac{8}{25} \qquad (\omega_0 = 2 \text{ rad/sec})$$

$$\angle H(j\omega_0) = -90°$$

$$\text{output} = 5\times\frac{8}{25}\sin\left(2t+\frac{\pi}{4}-\frac{\pi}{2}\right) = \frac{8}{5}\sin\left(2t-\frac{\pi}{4}\right)$$

$$= \frac{8}{5}\left(\frac{\sin 2t}{\sqrt{2}}-\frac{\cos 2t}{\sqrt{2}}\right)$$

$$= \frac{8}{5\sqrt{2}}(\sin 2t-\cos 2t)$$

$$= 1.13\ (\sin 2t - \cos 2t)$$

**27. (b)**

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} [f(t)\cos\omega t - jf(t)\sin\omega t]\,dt$$

$$= \int_{-\infty}^{\infty} f(t)\cos\omega t\,dt - j\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt$$

$f(t) \Rightarrow$ even signal
$f(t)$ cos$\omega t \Rightarrow$ even signal
$f(t)$ sin$\omega t \Rightarrow$ odd signal

$$\int_{-\infty}^{\infty} f(t)\sin\omega t\,dt = 0$$

$$\int_{-\infty}^{\infty} f(t)\cos\omega t\,dt = 2\int_{0}^{\infty} f(t)\cos\omega t\,dt$$

$$\therefore \quad F(\omega) = 2\int_{0}^{\infty} f(t)\cos\omega t\,dt$$

**28. (c)**

We know that, unit impulse let $x(t)$,

$$x(t) = \delta(t)$$

for $\delta(t) \xleftrightarrow{LT} 1$

for $\frac{d}{dt}x(t) \xleftrightarrow{LT} sX(s)$

$$\frac{d}{dt}\delta(t) \xleftrightarrow{LT} s$$

$$\frac{d^2}{dt^2}\delta(t) \xleftrightarrow{LT} s^2$$

**29.** **(b)**

Given, $X(s) = \log(s+2) - \log(s+3)$

Differentiating both the sides with respect to $s$

$$\frac{d}{ds}X(s) = \frac{1}{s+2} - \frac{1}{s+3} \quad \text{...(i)}$$

From the properties of Laplace transform, we know that,

$$tx(t) \longleftrightarrow -\frac{d}{ds}X(s)$$

Thus equation (i) can be written as,

$$-tx(t) = [e^{-2t} - e^{-3t}]u(t)$$

or,

$$x(t) = \left[\frac{e^{-3t} - e^{-2t}}{t}\right]u(t)$$

**30.** **(d)**

$$C_k = j\delta(k+2) - j\delta(k-2) + 2\delta(k+3) + 2\delta(k-3)$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} C_k e^{jk\pi t}$$

$$= je^{-j2\pi t} - je^{j2\pi t} + 2e^{-j3\pi t} + 2e^{j3\pi t}$$

$$= 4\cos(3\pi t) + 2\sin(2\pi t)$$

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