CLASS TEST

S.No. : 01 SP_ME_A_05012023

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ENGINEERING MATHEMATICS

MECHANICAL ENGINEERING

Date of Test : 05/01/2023

ANSWER KEY

1.	(b)	7.	(a)	13.	(a)	19.	(b)	25.	(b)
2.	(b)	8.	(c)	14.	(c)	20.	(c)	26.	(c)
3.	(a)	9.	(d)	15.	(d)	21.	(a)	27.	(d)
4.	(b)	10.	(c)	16.	(c)	22.	(b)	28.	(a)
5.	(d)	11.	(a)	17.	(d)	23.	(b)	29.	(b)
6.	(b)	12.	(a)	18.	(a)	24.	(b)	30.	(c)

DETAILED EXPLANATIONS

1. (b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}_{3\times 3}$$

$$\text{order of matrix} = 3$$

$$\text{Rank} = 2$$

$\therefore$ dimension of null space of $A = 3 - 2 = 1$.

2. (b)

$$f(x) = -2 + 6x - 4x^2 + 0.5x^3$$

$$f'(x) = 6 - 8x + 1.5x^2$$

$$x_{ini} = 0$$

By Newton Raphson Method, $x_1 = x_{ini} - \dfrac{f(x_{ini})}{f'(x_{ini})} = 0 - \dfrac{-2}{6}$

$\Rightarrow$ $x_1 = \dfrac{1}{3}$

$\therefore$ $\Delta x = x_1 - x_{ini} = \dfrac{1}{3}$

3. (a)

$$L(\cos \omega t) = \frac{s}{s^2 + \omega^2}$$

4. (b)

eigen values of $(A + 5I)$ are $\alpha + 5$ and $\beta + 5$

eigen values of $(A + 5I)^{-1} = \dfrac{1}{\alpha + 5}$ and $\dfrac{1}{\beta + 5}$

5. (d)

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{\pi}{4}$$

6. (b)

$$2x + y + 2z = 0$$
$$x + y + 3z = 0$$
$$4x + 3y + z = 0$$

$$[A : B] = \begin{bmatrix} 2 & 1 & 2 & 0 \\ 1 & 1 & 3 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 4 & 3 & 1 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & -4 & 0 \\ 0 & -1 & -11 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$,

$$= \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

Rank of $[A : B] = 3$

Rank of $[A] = 3 =$ Rank of $[A : B] =$ number of unknowns

So, unique soluton exists

7. (a)

$$I = \int_0^{\pi/2} \log\left(\frac{\sin x}{\cos x}\right) dx$$

$$= \int_0^{\pi/2} [\log(\sin x)dx - \log(\cos x)dx] \qquad \left[\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$= \int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right) dx - \int_0^{\pi/2} \log(\cos x)dx$$

$$= \int_0^{\pi/2} \log(\cos x)dx - \int_0^{\pi/2} \log(\cos x)dx$$

$$I = 0$$

8. (c)

$$p = 0.1$$
$$q = 0.9$$
$$n = 400$$

$$\text{Standard deviation} = \sqrt{npq} = \sqrt{400 \times 0.1 \times 0.9} = 6$$

9. (d)

The roots of auxiliary equation are 2, $\pm 2i$

$$a = -(2 + 2i - 2i) = -2$$
$$b = 2 \times (2i) + 2 \times (-2i) + 2i \times (-2i) = 4$$
$$c = -(2 \times 2i \times (-2i) = -8$$
$$a + b + c = -2 + 4 - 8 = -6$$

10. (c)

$$x\,dy - y\,dx + 2x^3 dx = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = -2x^2$$

$$\Rightarrow \quad \text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

11. (a)

Putting

$$f'(x) = 6x^2 - 6x - 36 = 0$$

$$\Rightarrow \quad x^2 - x - 6 = 0$$

$$\Rightarrow \quad x = 3 \text{ or } -2$$

Now $\quad f''(x) = 12x - 6$

and $\quad f''(3) = 30 > 0$ (minima)

and $\quad f''(-2) = -30 < 0$ (maxima)

Hence maxima is at $x = -2$ only.

12. (a)

$$f(t) = L^{-1}\left[\frac{1}{s^2(s+1)}\right]$$

$$\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1}$$

$$\frac{1}{s^2(s+1)} = \frac{As(s+1) + B(s+1) + C(s^2)}{s^2(s+1)}$$

Matching coefficient of s^2, s and constant in numerator we get,

$$A + C = 0 \quad \text{... (i)}$$

$$A + B = 0 \quad \text{... (ii)}$$

$$B = 1 \quad \text{... (iii)}$$

Solving we get $A = -1$, $B = 1$, $C = 1$

So, $$f(t) = L^{-1}\left[\frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}\right]$$

$$= -1 + t + e^{-t} = t - 1 + e^{-t}$$

13. (a)

A. $$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{dy}{y} = \int \frac{dx}{x}$$

$\log y = \log x + \log c = \log cx$

$y = cx$ ... Equation of straight line.

B. $\dfrac{dy}{dx} = \dfrac{-y}{x}$

$$\frac{dy}{y} = \frac{-dx}{x} \Rightarrow \int\frac{dy}{y} = -\int\frac{dx}{x}$$

$$\log y = -\log x + \log c$$
$$\log y + \log x = \log c$$
$$\log yx = \log c$$
$$yx = c$$
$$y = c/x \quad \text{... Equation of hyperbola.}$$

C. $\dfrac{dy}{dx} = \dfrac{x}{y},\ y\,dy = x\,dx$

$$\Rightarrow \int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} - \frac{x^2}{2} = \frac{c^2}{2} \rightarrow \text{const}$$

$$y^2 - x^2 = c^2$$

$$\frac{y^2}{c^2} - \frac{x^2}{c^2} = 1 \quad \text{... Equation of hyperbola.}$$

D. $\dfrac{dy}{dx} = \dfrac{-x}{y} \Rightarrow \int y\,dy = -\int x\,dx$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{c^2}{2}$$

$$\frac{y^2}{2} + \frac{x^2}{2} = \frac{c^2}{2}$$

$$x^2 + y^2 = c^2 \quad \text{... Equation of a circle}$$

14. (c)

$$\frac{dy}{dx} - y\cos x = \sin x\cos x$$

$$\text{IF} = e^{-\int\cos x\,dx} = e^{-\sin x}$$

$$ye^{-\sin x} = \int \sin x\cos x\, e^{-\sin x}\,dx$$

$$ye^{-\sin x} = -(1+\sin x)e^{-\sin x} + C_0$$

$$y + 1 + \sin x = C_0 e^{\sin x}$$

15. (d)

$$A^{-1} = \frac{(adj\,A)}{|A|}$$

$$|A| = -6 \times 3 = -18$$

$$|A| \cdot (A^{-1}) = (\text{adj}\,A)$$

$$\lambda \text{ of adj}\,A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-18}{-6}, \frac{-18}{3}$$

$$= 3, -6$$

16. (c)

$$(D^2 + 1)y = \sin x$$

$$PI = \frac{\sin x}{D^2 + 1}$$

putting $D^2 = -1$

$$PI = \frac{\sin x}{-1+1} \qquad \text{[Makes denominator zero]}$$

∴ Differentiating numerator and denominator

$$PI = x \cdot \frac{\sin x}{2D}$$

$$PI = \frac{1}{2}x\int \sin x\, dx$$

$$PI = -\frac{1}{2}x\cos x$$

17. (d)

$$D^2 + 7D + 12 = 0$$

$$(D + 3)(D + 4) = 0$$

$$D = -3, -4$$

$$y = C_1 e^{-3x} + C_2 e^{-4x}$$

$$y(0) = C_1 + C_2 = 1$$

$$y'(0) = -3C_1 - 4C_2 = 0$$

$$\Rightarrow \quad -3C_1 - 4C_2 = 0$$

$$3C_1 + 3C_2 = 3$$

$$C_2 = -3$$

$$C_1 = 4$$

$$y(x) = 4e^{-3x} - 3e^{-4x}$$

18. (a)

$$P(X = k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!} \qquad [\lambda \text{ is mean}]$$

$$P(X = 2) = \frac{e^{-\lambda} \cdot \lambda^2}{2!}$$

$$P(X = 4) = \frac{e^{-\lambda} \cdot \lambda^4}{4!}$$

$$P(X=6) = \frac{e^{-\lambda}\cdot\lambda^6}{6!}$$

Given that, $P(X=2) = 9P(X=4) + 90P(X=6)$

$$\frac{e^{-\lambda}\cdot\lambda^2}{2} = \left(\frac{9\cdot\lambda^4}{24} + \frac{90\cdot\lambda^6}{30\cdot 24}\right)e^{-\lambda}$$

$\Rightarrow$ $12\lambda^2 = 9\lambda^4 + 3\lambda^6$

$\Rightarrow$ $4\lambda^2 = 3\lambda^4 + \lambda^6$

$\Rightarrow$ $\lambda \neq 0$

$4 = 3\lambda^2 + \lambda^4$

$\lambda^2 = 1$ or $\lambda^2 = -4$ which is not possible

So, $\lambda = \pm 1$

Thus option (a) is correct.

19. (b)

$$\nabla\times\vec{F} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix}$$

$$= -a_x(1) - a_y - a_z$$

$$= -a_x - a_y - a_z$$

$$\vec{ds} = (dx\,dz)\hat{a}_y$$

$$\int_s (\nabla\times\vec{F})\cdot\vec{ds} = -\iint dx\,dz$$

$$= -\pi r^2\Big|_{r=2}$$

$$= -\pi(4) = -4\pi \approx -12.57$$

20. (c)

$f'(x) = 2x - 1$

solve, $f'(x) = 0$

$\Rightarrow$ $x = \frac{1}{2}$ point of inflection

$f''(x) = 2 > 0$

so, at $x = \frac{1}{2}$, $f'(x)$ has minima

$$f\left(\frac{1}{2}\right) = 0.25 - 0.5 - 2 = -2.25$$

$$f(-4) = 16 + 4 - 2 = 18$$

$$f(4) = 16 - 4 - 2 = 10$$

21. (a)

$$\int_0^{0.4} f(x)dx = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$$

$$= \frac{0.1}{2}[(0+160) + 2(10+40+90)] = 22$$

22. (b)

According to question $A \times B = C$

Matrix C is a unit matrix. So matrix B will be inverse of A.

$$B = A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

23. (b)

Number of ways of throwing 6 is five ⇒ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)

Number of ways of throwing 7 is six ⇒ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \frac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \frac{5}{36} = \frac{31}{36}$

Probability of throwing 7, $q_1 = \frac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \frac{6}{36} = \frac{30}{36}$

$$\text{Probability of } B \text{ winning} = p_2q_1 + p_2q_2\,p_2q_1 + p_2q_2p_2q_2p_2q_1 + \ldots.$$

$$= p_2q_1[1 + p_2q_2 + (p_2q_2)^2 + (p_2q_2)^3 + \ldots..]$$

$$= \frac{p_2q_1}{(1 - p_2q_2)} = \frac{\frac{31}{36} \times \frac{6}{36}}{1 - \frac{31}{36} \times \frac{30}{36}} = \frac{31 \times 6}{366} = \frac{31}{61}$$

24. (b)

$$\frac{d^2y}{dx^2} = 0$$

Let, $y = C_1x + C_2$

$$C_1 = \frac{dy}{dx} = 3$$

At $x = 0$, $y = 7 = C_2$

∴ $y = C_1x + C_2 = 3x + 7$

At $x = 18$, $f(18) = 3 \times 18 + 7 = 54 + 7 = 61$

25. (b)

$$\frac{dx}{dt} = 3x$$

$$\int \frac{dx}{x} = \int 3dt$$

$$\ln x = 3t + C$$

at $t = 0$, $x = 5$

$$\ln 5 = C$$

So, $\ln x = 3t + \ln 5$

$$\ln \frac{x}{5} = 3t$$

$$\frac{x}{5} = e^{3t}$$

$$x = 5e^{3t}$$

At $t = 4$, $\quad x = 5e^{12}$

26. (c)

$$\log\sqrt{\frac{1+x}{1-x}} = \log\left(\frac{1+x}{1-x}\right)^{1/2}$$

$$= \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

$$= \frac{1}{2}\{\log(1+x) - \log(1-x)\}$$

$$= \frac{1}{2}\left\{\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\ldots\right) - \left[-\left(x + \frac{x^2}{2} + \frac{x^3}{3}\ldots\right)\right]\right\}$$

$$= x + \frac{x^3}{3} + \frac{x^5}{5}\ldots..$$

27. (d)

$$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{x}$$

$$= \lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{3-x}}{x}\times\frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}+\sqrt{3-x}}$$

$$= \lim_{x\to 0}\frac{(3+x)-(3-x)}{x(\sqrt{3+x}+\sqrt{3-x})}$$

$$= \lim_{x\to 0}\frac{2x}{x(\sqrt{3+x}+\sqrt{3-x})}$$

$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

28. (a)

$$\left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 3 & -2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -3/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -8 & 1 & -3/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -3/2 & -8 \\ 0 & 1 & 0 & 0 & 1/2 & 2 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{array}\right]$$

$$\therefore\ A^{-1} = \begin{bmatrix} 1 & -3/2 & -8 \\ 0 & 1/2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

29. (b)

$$h = 10$$

$$\text{Area} = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \ldots\ldots + y_{n-1}) + y_n\right]$$

$$= \frac{10}{2}\left[0 + 2(4 + 7 + 9 + 12 + 15 + 14 + 8) + 3\right]$$

$$= 705\ \text{m}^2$$

30. (c)

The equation $x^2 + bx + c = 0$ has roots α and β.

So $x^2 + bx + c = (x - \alpha)(x - \beta)$

$$\lim_{x\to\alpha}\frac{1-\cos(x^2+bx+c)}{(x-\alpha)^2} = \lim_{x\to\alpha}\frac{2\sin^2\left(\dfrac{x^2+bx+c}{2}\right)}{(x-\alpha)^2}$$

$$= \lim_{x\to\alpha}\frac{2\sin^2[(x-\alpha)(x-\beta)/2]}{(x-\alpha)^2}$$

$$= 2\lim_{x\to\alpha}\left[\frac{\sin(x-\alpha)(x-\beta)/2]}{\frac{1}{2}(x-\alpha)\cdot(x-\beta)}\right]^2 \frac{1}{4}(x-\beta)^2$$

$$= \frac{2}{4}(\alpha-\beta)^2$$

$$= \frac{2}{4}\left[(\alpha+\beta)^2 - 4\alpha\beta\right] \qquad \left[\lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= \frac{2}{4}\left[b^2 - 4c\right]$$

$$= \frac{1}{2}\left[b^2 - 4c\right]$$

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