CLASS TEST

S.No. : 01 SK_EC+EE_A_271222

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ENGINEERING MATHEMATICS

EC & EE

Date of Test : 27/12/2022

ANSWER KEY

1. (c)	7. (d)	13. (a)	19. (a)	25. (a)
2. (a)	8. (a)	14. (b)	20. (c)	26. (d)
3 . (a)	9. (b)	15. (a)	21. (d)	27. (c)
4. (c)	10. (b)	16. (c)	22. (c)	28. (d)
5. (b)	11. (b)	17. (c)	23. (d)	29. (b)
6. (d)	12. (c)	18. (b)	24. (c)	30. (c)

DETAILED EXPLANATIONS

1. (c)

For the system to be consistent,

$$\begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$$

$$1 + (-abc) + (-abc) - b^2 - a^2 - c^2 = 0$$

$$a^2 + b^2 + c^2 + 2abc = 1$$

2. (a)

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

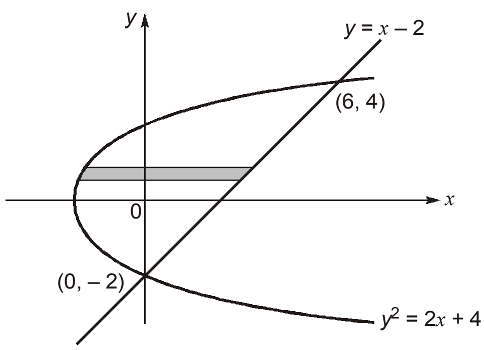
3 . (a)

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x^2\,dx + \int_1^2 (2-x)x\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^1 + \left.\left(x^2 - \frac{x^3}{3}\right)\right|_1^2 = \frac{1}{3} + 4 - 1 - \frac{8-1}{3} = 1$$

4. (c)

The point of intersection of line and parabolic are (0, –2) and (6, 4).



$$\text{Area} = \int_{-2}^{4}\int_{\left(\frac{y^2-4}{2}\right)}^{y+2} dx\,dy = \int_{-2}^{4} x\Big|_{\frac{y^2-4}{2}}^{y+2} dy$$

$$= \int_{-2}^{4}\left(y+2-\frac{y^2}{2}+2\right)dy = \left(\frac{y^2}{2}+4y-\frac{y^3}{6}\right)\Bigg|_{-2}^{4} = 18$$

5. (b)

$$\frac{\partial z}{\partial x} = f(x^2-y^2)\,2x$$

$$\frac{\partial z}{\partial y} = f(x^2-y^2)\,(-2y)$$

$$y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y} = 0$$

6. (d)

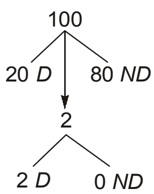
Given function is $y = \dfrac{1}{x}$ [hyperbolic function]

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

hence, option (d) is correct.

7. (d)

Problem can be solved by hypergeometric distribution



$$p(X=2) = \frac{20C_2 \times 80C_0}{100C_2} = \frac{19}{495}$$

8. (a)

Eigen value of A are, $\lambda_1,\ \lambda_2, \lambda_3$

$$|A| = \lambda_1\cdot\lambda_2\cdot\lambda_3$$

Eigen value of A^{-1} is $\dfrac{1}{\lambda_1},\dfrac{1}{\lambda_2},\dfrac{1}{\lambda_3}$

$$\frac{1}{\lambda_1} = 1 \Rightarrow \qquad \lambda_1 = 1$$

$$\frac{1}{\lambda_2} = 2 \Rightarrow \qquad \lambda_2 = \frac{1}{2}$$

$$\frac{1}{\lambda_3} = 5 \Rightarrow \qquad \lambda_3 = \frac{1}{5}$$

$$\lambda_1\lambda_2\lambda_3 = (1)\left(\frac{1}{2}\right)\left(\frac{1}{5}\right) = \frac{1}{10} = 0.1$$

$$|A| = 0.1$$

9. (b)

$$(D^2+4)y = 10\sin x$$

to get PI, put $D^2 = -1$

$$PI = \left.\frac{10\sin x}{D^2+4}\right|_{D^2=-1} = \frac{10\sin x}{3}$$

$$A\sin x = 3.33\sin x$$

$$A = 3.33$$

10. (b)

Given that the partial differential equation is parabolic.

Here $A = 3$, $C = 3$

$\therefore$ $$B^2 - 4AC = 0$$

$\therefore$ $$B^2 - 4(3)(3) = 0$$

$$B^2 - 36 = 0$$

$$B^2 = 36$$

11. (b)

$$\begin{bmatrix}(4-\lambda) & 1\\ 0 & (7-\lambda)\end{bmatrix} = 0$$

$\Rightarrow$ $$(4-\lambda)(7-\lambda) = 0$$

$\therefore$ $$\lambda = 4, 7$$

Putting the value of $\lambda = 4$

$\Rightarrow$ $$\begin{bmatrix}0 & 1\\ 0 & 3\end{bmatrix}\begin{bmatrix}1\\ p\end{bmatrix} = 0$$

$\Rightarrow$ $$p = 0$$

Putting the value of $\lambda = 7$

$\Rightarrow$ $$\begin{bmatrix}-3 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}1\\ q\end{bmatrix} = 0$$

$\Rightarrow$ $$q = 3$$

$\therefore$ $$p + q = 3$$

12. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$IF = e^{\int \frac{1}{y}dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

13. (a)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$$

$$\vec{v}\Big|_{t=1} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

Component of velocity in direction $\hat{i} - 3\hat{j} + 2\hat{k}$ will be,

$$\frac{\vec{v}\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{(4\hat{i} - 2\hat{j} + 3\hat{k})\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}} = \frac{4+6+6}{\sqrt{14}} = \frac{16}{\sqrt{14}} = 4.276$$

14. (b)

$$p = \frac{d\,f(x,y)}{dx} = 2x + 6$$

$\Rightarrow$ $p = 0$ at $x = -3$

$$q = \frac{df(x,y)}{dy} = 2y$$

$\Rightarrow$ $q = 0$ at $y = 0$

$\therefore$ (–3, 0) is a stationary point

$$r = \frac{d^2f(x,y)}{dx^2} = 2$$

$$s = \frac{d^2f(x,y)}{dx\,dy} = 0$$

$$t = \frac{d^2f(x,y)}{dy^2} = 2$$

At (–3, 0), $rt - s^2 = 4 > 0$ and $r = 2 > 0$

$\therefore$ $f(x, y)$ has a minimum value at (–3, 0)

$\therefore$ $f(-3, 0) = 1$

15. (a)

Let $$f(x) = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}\,dx \ (a > 0) \quad \text{...(i)}$$

We know $$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$$

$\therefore$ $$f(x) = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1 + a^x}\,dx \quad \text{...(ii)}$$

from (i) and (ii)

$\Rightarrow$ $$2f(x) = \int_{-\pi}^{\pi} \cos^2 x\,dx = 2\int_0^{\pi} \cos^2 x\,dx$$

$\Rightarrow$ $$2f(x) = 2 \times 2\int_0^{\pi/2} \cos^2 x\,dx$$

$\Rightarrow$ $$2f(x) = 4 \times \frac{1}{2} \times \frac{\pi}{2}$$

$$\left[\text{By using } \int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{1}{2}\times\frac{\pi}{2} \text{ if } n \text{ is even}\right]$$

$$f(x) = \frac{\pi}{2}$$

16. (c)

$$np = 3$$

$$npq = \sigma^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

from here

$$q = \frac{3}{4}$$

$$p = \left(1-\frac{3}{4}\right) = \frac{1}{4}$$

$$n\times\frac{3}{4}\times\frac{1}{4} = \frac{9}{4}$$

$$n = 12$$

17. (c)

Given equation are

$$x + 2y + z = 6$$
$$2x + y + 2z = 6$$
$$x + y + z = 5$$

Given system can be written as

$$\begin{bmatrix}1 & 2 & 1\\2 & 1 & 2\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\6\\5\end{bmatrix}$$

Augmented matrix is $\left[\begin{array}{ccc|c}1 & 2 & 1 & 6\\2 & 1 & 2 & 6\\1 & 1 & 1 & 5\end{array}\right]$

By gauss elimination

$$\left[\begin{array}{ccc|c}1 & 2 & 1 & 6\\2 & 1 & 2 & 6\\1 & 1 & 1 & 5\end{array}\right]\xrightarrow[R_3-R_1]{R_2-2R_1}\left[\begin{array}{ccc|c}1 & 2 & 1 & 6\\0 & -3 & 0 & -6\\0 & -1 & 0 & -1\end{array}\right]$$

$$\xrightarrow{R_3-\frac{1}{3}R_2}\left[\begin{array}{ccc|c}1 & 2 & 1 & 6\\0 & -3 & 0 & -6\\0 & 0 & 0 & 1\end{array}\right]$$

$$r(A) = 2$$
$$r(A\,|\,B) = 3$$

Since the rank of coefficient matrix is 2 and rank of argument matrix is 3, which is not equal. Hence system has no solution i.e. system is inconsistent.

18. (b)

Let $$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \quad \text{...(i)}$$

Since $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \quad \text{...(ii)}$$

(i) + (ii) ⇒ $$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

⇒ $$2I = \int_0^a dx$$

⇒ $$2I = a$$

⇒ $$I = a/2$$

19. (a)

$$AB^T = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} 3 & 8 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$$

20. (c)

$$\frac{dy}{dt} = -5y$$

$$\int \frac{dy}{y} = -\int 5dt$$

$$\ln y = -5t + C$$

at $$t = 0$$

$$y = 2$$

$$\ln 2 = C$$

So, $$\ln y = -5t + \ln 2$$

$$\ln\frac{y}{2} = -5t$$

$$\frac{y}{2} = e^{-5t}$$

$$y = 2e^{-5t}$$

at $$t = 3$$

$$y = 2e^{-15}$$

21. (d)

$P(\text{A wins}) = p(6 \text{ in first throw by } A) + p(A \text{ not } 6, B \text{ not } 6, A\, 6) + ...$

$$= \frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \cdots$$

$$= \frac{1}{6}\left(1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \cdots\right) = \frac{1}{6}\cdot\frac{1}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

22. (c)

For $f(x)$ to be probability density function $\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\frac{1}{A}\int_2^4 (2x+3)\,dx = 1$$

$$\frac{1}{A}\left[2\frac{x^2}{2} + 3x\right]_2^4 = 1$$

$$A = (4^2 - 2^2) + 3(4-2)$$
$$= 16 - 4 + 3 \times 2 = 18$$

23. (d)

$$\vec{A}\times\vec{B}\times\vec{C} = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

$$\therefore \quad \nabla\times\nabla\times\vec{A} = (\nabla\cdot\vec{A})\nabla - (\nabla\cdot\nabla)\vec{A} = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$$

24. (c)

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r}\cdot\hat{i} = x, \qquad \vec{r}\cdot\hat{j} = y, \qquad \vec{r}\cdot\hat{k} = z$

$$A = x(\vec{r}\times\hat{i}) + y(\vec{r}\times\hat{j}) + z(\vec{r}\times\hat{k})$$

$$= (\vec{r}\times x\hat{i}) + (\vec{r}\times y\hat{j}) + (\vec{r}\times z\hat{k}) = \vec{r}\times(x\hat{i} + y\hat{j} + z\hat{k}) = \vec{r}\times\vec{r}$$

$$A = 0 \qquad \text{(always)}$$

25. (a)

Since the probability of occurrence is very small, this follows Poisson distribution

$$\text{mean} = m = np$$
$$= 2000 \times 0.001 = 2$$

Probability that more than 2 will get a bad reaction

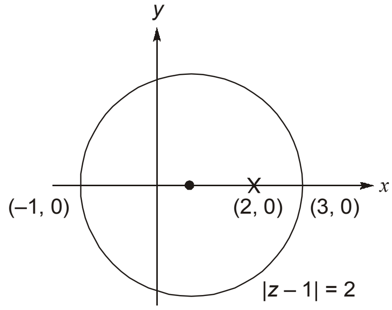
$$= 1 - p(0) - p(1) - p(2)$$

$$= 1 - \left[e^{-m} + \frac{e^{-m}\cdot m^1}{1!} + \frac{e^{-m}\cdot m^2}{2!}\right]$$

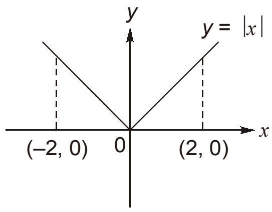
$$= 1 - \left[e^{-2} + \frac{e^{-2}\cdot 2}{1} + \frac{2^2\cdot e^{-2}}{2}\right] = 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2}\right] = 1 - \frac{5}{e^2}$$

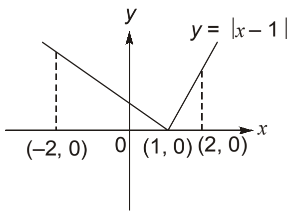
26. (d)

$$\frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz = f(a)$$
$$= 2 \times x\Big|_{z=2+i0}$$
$$= 2\,x\Big|_{x+iy=2}$$
$$= 2 \times 2$$
$$= 4$$



27. (c)





$$\int_{-2}^{2}\left(|x|\,dx\right) + \int_{-2}^{2}\left(|x-1|\,dx\right) = \text{Area under the curves}$$

$$= 2\times\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 3\times 3+\frac{1}{2}\times 1\times 1 = 4+\frac{9}{2}+\frac{1}{2} = 9 \text{ unit}^2$$

28. (d)

Let $d \rightarrow$ defective

$y \rightarrow$ supplied by y

$$P\left(\frac{y}{d}\right) = \frac{P(y\cap d)}{P(d)}$$

$$P(y\cap d) = 0.3\times 0.02 = 0.006$$
$$P(d) = 0.6\times 0.01 + 0.3\times 0.02 + 0.1\times 0.03$$
$$= 0.015$$

$$P\left(\frac{y}{d}\right) = \frac{0.006}{0.015} = 0.4$$

29. (b)

The maximum variation is in direction of grad T.

$$T = x^2 + 4xy + y^2$$

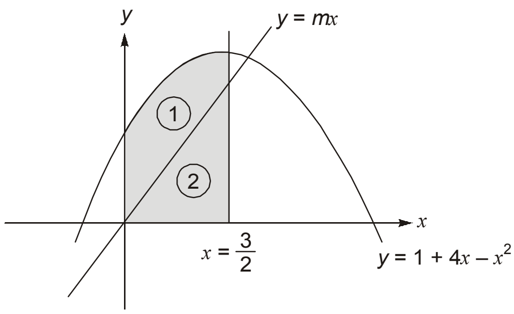
$$\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} = (2x+4y)\hat{i} + (4x+2y)\hat{j}$$

$$\nabla T\Big|_{(2,2)} = (4+8)\hat{i} + (8+4)\hat{j}$$
$$= 12\hat{i} + 12\hat{j}$$

The direction in which rate is slowest is perpendicular the direction in which variation is maximum.

$$\nabla T\Big|_{\min} = 12\hat{i} - 12\hat{j} \text{ or } \hat{i} - \hat{j}$$

30. (c)



$$\text{Area of (1)} = \text{Area of (2)} = \int_0^{3/2} mx\,dx = \frac{1}{2}[\text{Area of (1)} + \text{Area of (2)}]$$

$$\frac{1}{2}\times\int_0^{3/2}\left(1+4x-x^2\right)dx = \frac{1}{2}\left[x+\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^{3/2}$$

$$\int_0^{3/2} mxdx = \frac{1}{2}\left[\left(\frac{3}{2}-0\right)+2\left(\frac{9}{4}-0\right)-\left(\frac{27}{8\times 3}-0\right)\right]$$

$$\left[m\frac{x^2}{2}\right]_0^{3/2} = \frac{1}{2}\left[\frac{3}{2}+\frac{9}{2}-\frac{9}{8}\right]$$

$$m\times\frac{9}{4\times 2} = \frac{1}{2}\times\frac{39}{8}$$

$$m = \frac{13}{6} = 2.17$$

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