S.No. : 01 PT_EE_A+C_300719
Electromagnetic Theory

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CLASS TEST 2019-2020

ELECTRICAL ENGINEERING

Electromagnetic Theory

Date of Test : 30/07/2019

Answer Key

1.	(b)	7.	(a)	13.	(d)	19.	(b)	25.	(b)
2.	(a)	8.	(d)	14.	(b)	20.	(c)	26.	(a)
3.	(a)	9.	(c)	15.	(b)	21.	(c)	27.	(d)
4.	(b)	10.	(b)	16.	(d)	22.	(c)	28.	(c)
5.	(c)	11.	(d)	17.	(a)	23.	(a)	29.	(a)
6.	(b)	12.	(a)	18.	(d)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (b)

$$\vec{A} = 3x^2yz\hat{a}_x + x^3 = \hat{a}_y + \left(x^3y - 2z\right)\hat{a}_z$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3x^2y & x^3z & x^3y - 2z \end{vmatrix}$$

$$= \hat{a}_x\left(x^3 - x^3\right) - \hat{a}_y\left(3x^2y - 3x^2y\right) + \hat{a}_z\left(3x^2z - 3x^2z\right)$$

$$\nabla \times \vec{A} = 0$$

Hence the given field is conservative (or) irrotational.

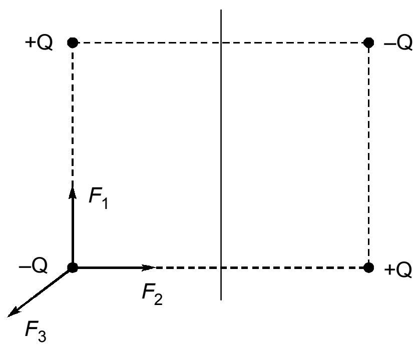
2. (a)

$$S = \text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.5}{1-0.5} = 3$$

Since minima is observed at load, $Z_L = \dfrac{Z_0}{s} = \dfrac{50}{3} = 16.67\ \Omega$

3. (a)

Because of image formation



With the resultant effect of all the forces when compared to F_1 alone the force is decreased.

4. (b)

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

Net force on sides parallel to the wire = $\dfrac{\mu_0 I^2 a}{2\pi a} - \dfrac{\mu_0 I^2 a}{2\pi 2a}$

1st term attractive, 2nd term repulsive and repulsion less than attraction.

$\therefore$ $F = \dfrac{\mu_0 I^2}{4\pi}$ towards the wire

5. (c)

$$F = \frac{Q_1 Q_2}{4\pi \in_0 \in_r d^2}$$

$$F \propto \frac{1}{\in_r}$$

$$\therefore \quad \frac{F_2}{F_1} = \frac{\in_r}{1}$$

$$\Rightarrow \quad F_2 = \in_r F_1 = 2.25\, F_1$$

6. (b)

$$\nabla \times \vec{A} = \vec{B}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{A}\right) = \nabla \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\Rightarrow \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

7. (a)

Total charge within the region is given by:

$$Q = \int_x \int_y \int_z \rho_V dx\, dy\, dz$$

$$Q = \int_0^2 \int_0^2 \int_0^2 40xyz\, dx\, dy\, dz = 40 \times \left[\frac{x^2}{2}\right]_0^2 \times \left[\frac{y^2}{2}\right]_0^2 \times \left[\frac{z^2}{2}\right]_0^2$$

$$= 40 \times 2 \times 2 \times 2 = 320 \text{ Coulomb}$$

8. (d)

From the given diagram, we can find that the distance of centre of loop from each side of square is equal to 2 cm.

$\therefore$ As we know analytically the magnetic field intensity $\vec{H}$ due to one side of loop is given by;

$$\vec{H} = -\frac{I}{\sqrt{2}\pi a}\hat{a}_z$$

Where direction of H is always $-\hat{a}_z$.

Therefore total magnetic field at centre of loop is obtained by adding field intensities due to 4 sides.

$$H_{total} = 4\, H_{each}$$

$$|\vec{H}| = \frac{4 \times I}{\sqrt{2} \times \pi a} = \frac{4 \times 2}{\sqrt{2} \times \pi \times 4 \times 10^{-2}}$$

$$|\vec{H}| = 45.016 \text{ A/m} \approx 45.02 \text{ A/m}$$

9. (c)

Since given function is a vector function and Stroke's theorem is valid for any vector function.

10. (b)

Work done = charge × potential difference

So, $\quad \text{W.D} = \Delta Q(V_2 - V_1) = -\Delta Q\,(V_1 - V_2)$

12. (a)

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A \cdot \varepsilon_0}$$

So,
$$\begin{aligned} q &= EA\,\varepsilon_0 \\ &= 10^3 \times 100 \times 10^{-4} \times 8.854 \times 10^{-12} \\ &= 8.854 \times 10^{-11}\ \text{C} \end{aligned}$$

13. (d)

(a) represents Maxwell's equation in integral from i.e.

$$\oint H.dl = \iint_s \left(\sigma E + \in \frac{\delta E}{\delta t}\right) ds$$

(b) represents Ampere's circuital law

(c) represents Maxwell's equation in point form

(d) is not true

14. (b)

An emf is not induced if the plane of the coil is parallel to the lines of the magnetic field, since the magnetic flux through the coil does not change when the coil shrinks.

15. (b)

The motional electric field is given by

$$\begin{aligned} |\vec{E}_m| &= |\vec{u}| \times |\vec{B}| \\ &= (0.01) \times (37.5) \\ &= 0.375\ \text{N/C} \approx 0.38\ \text{N/C} \end{aligned}$$

16. (d)

We can find the conversion from spherical to cylindrical cordinates, by first converting point P into cartesian coordinates and then to cylindrical coordinates.

$$x = r\sin\theta \cdot \cos\phi = 10 \times \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{3}\right) = 2.5\ \text{m}$$

$$y = r\sin\theta \cdot \sin\phi = 10 \times \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{3}\right) = 4.33\ \text{m}$$

$$z = r\cos\theta = 10 \times \cos\left(\frac{\pi}{6}\right) = 8.66\ \text{m}$$

Now,
$$\rho = \sqrt{x^2 + y^2} = \sqrt{(2.5)^2 + (4.33)^2} = 5\ \text{m}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4.33}{2.5}\right) \cong 60°$$

$$z = 8.66\ \text{m}$$

Then point P is,
$$P = (5, 60°, 8.66) \text{ in cylindrical coordinates}$$

17. (a)

The given vector is in cylindrical coordinates,

$$\bar{A} = \rho z \sin\phi\, \hat{a}_\rho + 3\rho z^2 \cos\phi\, \hat{a}_\phi$$

$$\bar{A} = A_\rho \cdot \hat{a}_\rho + A_\phi \cdot \hat{a}_\phi + A_z \cdot \hat{a}_z$$

$$div \cdot \vec{A} = \nabla \cdot \vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \cdot A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho \times \rho z \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(3\rho z^2 \cos\phi) + 0$$

$$= \frac{1}{\rho}(2\rho z \sin\phi) - \frac{1}{\rho}\cdot(3\rho z^2 \cdot \sin\phi)$$

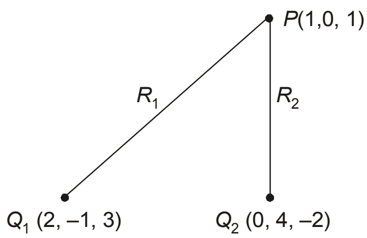
$$= 2\, z \sin\phi - 3z^2 \sin\phi = (2 - 3z)\, z \sin\phi$$

At point $\left(5, \frac{\pi}{2}, 1\right)$

$$div \cdot \vec{A} = (2 - (3 \times 1)) \times 1 \sin\frac{\pi}{2} = -1$$

18. (d)

Given,
$$Q_1 = -4\,\mu C$$
$$Q_2 = 5\,\mu C$$



From the figure, we can write,

$$\text{Distances, } R_1 = \sqrt{(2-1)^2 + (-1-0)^2 + (3-1)^2}$$
$$= \sqrt{6}\,\text{m}$$

and
$$R_2 = \sqrt{(0-1)^2 + (4-0)^2 + (-2-1)^2}$$
$$= \sqrt{26}\,\text{m}$$

Now potential at point P due to Q_1 is,

$$V_1 = \frac{Q_1}{4\pi\,\varepsilon_0 R_1} = \frac{-4 \times 10^{-6}}{4\pi \cdot \left(\frac{10^{-9}}{36\pi}\right) \times \sqrt{6}} = -14.70\text{ kV}$$

Potential at point P due to Q_2 is;

$$V_2 = \frac{Q_2}{4\pi\,\varepsilon_0 R_2} = \frac{5 \times 10^{-6}}{4\pi \cdot \left(\frac{10^{-9}}{36\pi}\right) \times \sqrt{26}} = 8.825\text{ kV}$$

Total potential at point P is obtained by superposition,

$$V = V_1 + V_2 = (-14.70 + 8.825)\text{ kV}$$
$$V = -5.875\text{ kV}$$

19. (b)

The incremental work done is

$$dW = -Q\vec{E} \cdot d\vec{l}$$

Where $\quad d\vec{l} = dl \cdot \hat{a}_l$

The work is done in the direction of $\hat{a}_x + \hat{a}_y + \hat{a}_z$

$$\hat{a}_l = \frac{(\hat{a}_x + \hat{a}_y + \hat{a}_z)}{\sqrt{3}}$$

$$d\vec{l} = \frac{1 \times 10^{-3}(\hat{a}_x + \hat{a}_y + \hat{a}_z)}{\sqrt{3}}$$

The work done is,

$$dW = -4 \times (400\hat{a}_x - 300\hat{a}_y + 500\hat{a}_z) \times \frac{10^{-3}(\hat{a}_x + \hat{a}_y + \hat{a}_z)}{\sqrt{3}}$$

$$= \frac{-4 \times 10^{-3}(400 - 300 + 500)}{\sqrt{3}} = \frac{-2.4}{\sqrt{3}}$$

$$= -1.39 \text{ Joules}$$

20. (c)

Electric potential is given by

$$V = \sum_{k=1}^{2}\left[\frac{p_k \cdot r_k}{4\pi\epsilon_0 r_k^3}\right] = \frac{1}{4\pi\,\epsilon_0}\left[\frac{P_1 \cdot r_1}{r_1^3} + \frac{P_2 \cdot r_2}{r_2^3}\right]$$

Where,

$$P_1 = -5a_z,$$

$$r_1 = (0, 0, 0) - (0, 0, -2)$$

$$= 2a_z,\ r_1 = |r_1| = 2$$

$$P_2 = 9a_z,$$

$$r_2 = (0, 0, 0) - (0, 0, 3)$$

$$= -3a_z,\ r_2 = |r_2| = 3$$

$$= \frac{1}{4\pi \times \frac{10^{-9}}{36\pi}}\left[\frac{-5\hat{a}_z . 2\hat{a}_z}{2^3} + \frac{9\hat{a}_z . (-3\hat{a}_z)}{3^3}\right] \times 10^{-9}$$

Hence,

$$V = \frac{1}{\left(4\pi \times \frac{10^{-9}}{36\pi}\right)}\left[-\frac{10}{2^3} - \frac{27}{3^3}\right] \times 10^{-9}$$

$$= -20.25\ V \qquad \left(\epsilon_0 = \frac{10^{-9}}{3\pi} = 8.84 \times 10^{-12}\ F/m\right)$$

21. (c)

In spherical co-ordinates

$$\nabla \cdot \vec{D} = \frac{1}{r^2} \cdot \frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$= \frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\theta}{\pi r^2}(1 - \cos 3r)\right) = \frac{3\theta}{\pi r^2}\sin 3r$$

22. (c)

$$V = \frac{kq_1}{L_1} + \frac{kq_2}{L_2} + \frac{kq_3}{L_3} + \frac{kq_4}{L_4}$$

$$q_1 = q \qquad L_1 = L$$
$$q_2 = q \qquad L_2 = L$$
$$q_3 = -q \qquad L_3 = \sqrt{5}L$$
$$q_4 = -q \qquad L_4 = \sqrt{5}L$$

$$V = \frac{1q}{4\pi\epsilon_0}\left(\frac{1}{L} + \frac{1}{L} - \frac{1}{\sqrt{5}L} - \frac{1}{\sqrt{5}L}\right) = \frac{1}{4\pi\epsilon_0}\cdot\frac{2q}{L}\left(1 - \frac{1}{\sqrt{5}}\right)$$

23. (a)

The magnetic field intensity produced due to a small current element $I\,dl$ is defined as

$$d\vec{H} = \frac{I\,dl \times \hat{a}_R}{4\pi R^2}$$

where dl is the differential line vector and $\hat{a}_R$ is the unit vector directed towards the point where field is to be determined. So for the circular current carrying loop, we have

$$dl = a\,d\phi\,\hat{a}_\phi$$

$$\hat{a}_R = -\hat{a}_\rho$$

Therefore the magnitude field intensity produced at the center of the circular loop is

$$\vec{H} = \int_{\phi=0}^{2\pi} \frac{I_a d\phi \hat{a}_\phi \times (-\hat{a}_\rho)}{4\pi a^2} = \frac{I_a}{4\pi a^2}[\phi]_0^{2\pi}\,\hat{a}_z = \frac{I}{2a}\hat{a}_z \text{ A/m}$$

24. (c)

Given, Magnetization, $M = 2.8$ A/m,

$\chi_m = 0.0025$

The magnetic field intensity in a material,

$$H = \frac{M}{\chi_m} = \frac{2.8}{0.0025} = 1120 \text{ A/m}$$

The flux density, $B = (1+\chi_m)\mu_0 H = (1+0.0025)\times 4\pi \times 10^{-7} \times 1120$

$= 1.411$ mT

25. (b)

Given, $\vec{A} = yz\hat{a}_x + xy\hat{a}_y + xz\hat{a}_z$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xy & xz \end{vmatrix}$$

$$\nabla \times \vec{A} = \hat{a}_x(0-0) - \hat{a}_y(z-y) + \hat{a}_z(y-z)$$
$$= -\hat{a}_y(z-y) + \hat{a}_z(y-z)$$

At point (0, 1, 2)

$$\nabla \times \vec{A} = -\hat{a}_y(2-1) + \hat{a}_z(1-2)$$
$$\nabla \times \vec{A} = -\hat{a}_y - \hat{a}_z$$
$$\left|\nabla \times \vec{A}\right| = \sqrt{1^2+1^2} = \sqrt{2} = 1.41$$

26. (a)

$$\frac{C}{L} = \frac{2\pi \in}{ln\frac{b}{a}} = \frac{2\pi \times 4 \times 8.854 \times 10^{-12}}{ln 5} = 138.3 \text{ pF}$$

27. (d)

According to Gauss Law, the volume charge density in a certain region is equal to the divergence of electric flux density in that region.

i.e.
$$\rho_v = \nabla.\vec{D}$$
$$= \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right).\left[(3y^2+4z)\hat{a}_x + 2xy\,\hat{a}_y + 4x\,\hat{a}_z\right]$$
$$\rho_v = 2x$$

So, total charge enclosed by the cube is:

$$Q = \int \rho_v dV = \int_0^2\int_0^2\int_{-1}^1 (2x)(dx\,dy\,dz) = \left[x^2\right]_0^2 \times 2 \times 2$$

Hence,
$$Q = 16 \text{ C}$$

28. (c)

According to Ampere's circuital law the contour integral of magnetic field intensity in a closed path is equal to the current enclosed by the path.

i.e.
$$\oint \vec{H}.d\vec{l} = I_{\text{enclosed}}$$

Now using right hand rule, we obtain the direction of the magnetic field intensity in the loop which will be opposite to the direction of L.

So,
$$\oint \vec{H}.d\vec{l} = -I_{\text{enclosed}} = -20 \text{ A}$$

(10 A is not inside the loop so it will not be considered)

29. (a)

Electric field at any point due to infinite surface charge distribution is defined as

$$\vec{E} = \frac{\rho_s}{2\in_0}\hat{a}_n$$

Where $\rho_s \rightarrow$ surface charge density,

$\hat{a}_n \rightarrow$ unit vector normal to the sheet directed towards the point where field is to be determined.

At origin electric field intensity due to sheet at $y = 1$ is

$$\vec{E}_1 = \frac{\rho_s}{2\in_0}(-\hat{a}_y) = -\frac{5}{2\in_0}\hat{a}_y$$

$$(\hat{a}_n = -\hat{a}_y)$$

and electric field intensity at origin due to sheet at $y = -1$ is

$$\vec{E}_{-1} = \frac{\rho_s}{2\in_0}(\hat{a}_y) = \frac{5}{2\in_0}\hat{a}_y$$

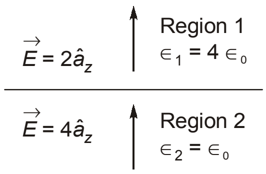
$$(\hat{a}_n = \hat{a}_y)$$

So, net field intensity at origin is

$$\vec{E} = \vec{E}_1 + \vec{E}_{-1} = -\frac{5}{2\in_0}\hat{a}_y + \frac{5}{2\in_0}\hat{a}_y = 0 \text{ V/m}$$

30. (a)

Consider the two dielectric as shown below,



Since the field is normal to the interface so, the fields are

$$E_{1n} = 2$$

and $$E_{2n} = 4$$

From boundary condition, we have

$$D_{1n} - D_{2n} = \rho_s$$

$$\in_1 E_{1n} - \in_2 E_{2n} = \rho_s$$

(where ρ_s is the surface charge density on the interface)

$$4(\in_0)(2) - (\in_0)(4) = \rho_s$$

$$8\in_0 - 4\in_0 = \rho_s$$

$$4\in_0 = \rho_s$$

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