S.No. : 01 BS_CS_S_190719

Discrete Mathematics

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Noida | Bhopal | Hyderabad | Jaipur | Lucknow | Indore | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

CLASS TEST 2019-2020

COMPUTER SCIENCE & IT

Date of Test : 19/07/2019

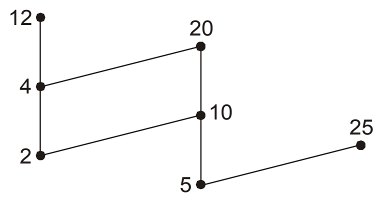
ANSWER KEY ➤ Discrete Mathematics

1.	(c)	7.	(a)	13.	(d)	19.	(b)	25.	(d)
2.	(b)	8.	(d)	14.	(b)	20.	(c)	26.	(b)
3.	(c)	9.	(b)	15.	(d)	21.	(d)	27.	(b)
4.	(d)	10.	(c)	16.	(c)	22.	(b)	28.	(c)
5.	(a)	11.	(b)	17.	(d)	23.	(c)	29.	(d)
6.	(a)	12.	(d)	18.	(d)	24.	(d)	30.	(a)

Detailed Explanations

1. (c)
The upper bounds of {1, 3, 4, 6} are 6, 8 and 9.
Hence there are only 3 upper bounds.

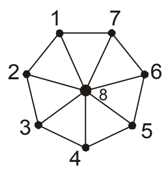
2. (b)



This is the hasse diagram for given poset.
Maximum elements are: 12, 20, 25
Minimal elements are: 2, 5
$\therefore$ The sum of all the maximal and all the minimal elements is 12 + 20 + 25 + 2 + 5 = 64.

3. (c)
According to Binomial expansion of $(x + y)^{25} = \sum_{r=0}^{25} {}^{25}C_r \cdot x^r \cdot y^{25-r}$.
Here $r = 2$
$\therefore$ The coefficient of $x^2 y^{23}$ is ${}^{25}C_2 = 300$.

4. (d)



Nullity for connected graph = $e - n + 1 = 14 - 8 + 1 = 7$
Chromatic number = 4
$\therefore$ 7 + 4 = 11 is the sum.

5. (a)
The total number of ways of choosing 6 squares out of 8 is ${}^8C_6 = 28$.
But out of these, 2 possibilities need to be removed.
One being the upper row being empty.
Second being the lower row being empty.
Both being empty at the same time is not a possibility.
$\therefore$ 28 – 2 = 26 ways are there.

6. (a)
$X \rightarrow Y$ is false only when X is True and Y is false. By substituting the truth values of X and Y in S_1 and S_2 we find that both S_1 and S_2 are False.
Note: $X \leftrightarrow Y$ is True only when both X and Y have same truth values.

7. (a)
The subset of a countable set is always countable.

8. (d)
Empty set ϕ satisfies all properties except reflexive property. Hence not an equivalence relation. A reflexive relation satisfies both symmetric and antisymmetric properties. Hence (b) is false.
The relation "divides" is not symmetric because 1 divides 2, but 2 does not divide 1.
Union of two transitive relations need not be transitive relation. Hence union need not be equivalence relation.

9. (b)
A partition of a set S is a collection of disjoint non-empty subsets of S that have S as their union. For partitionn in (b) {10} and {10, 20, 30, 41} are not disjoint and hence is not correct partition.

10. (c)
Let $|A| = n$, and $|B| = m$
In partial function every element in domain need not have a range in co-domain.
$\therefore$ Each element in A will have $(m + 1)$ choices.
For n elements in A

$$\underbrace{(m+1)(m+1)...(m+1)}_{n \text{ times}} = (m+1)^n.$$

In this question, $|A| = 4$, $|B| = 4$
The number of partial functions from A to B are $(4 + 1)^4$.
$\therefore$ $(4 + 1)^4 = 625$

11. (b)
Number of ways of distributing 5 blue pens to 6 children
where $n = 5$, $r = 6$

$$^{5+6-1}C_5 = {}^{10}C_5$$

Number of ways of distributing 6 black pens to 6 children

$$^{6+6-1}C_6 = {}^{11}C_6$$

$\therefore$ Total number of ways $= {}^{10}C_5 \times {}^{11}C_6 = 116424$

12. (d)
A graph G, having more than two vertices of odd degree, does not posses an Euler path.
Since the given graphs (I) and (II) both contains more than two nodes of odd degree, neither (I) nor (II) has any Euler path or Euler circuit.

13. (d)
The statement "not every P is Q" can be written as "there exist a P which is not Q".
i.e., $\exists x(P(x) \wedge \neg Q(x))$ which is same as option (a), (b) and (c).

14. (b)
S_1 is false because poset would be called as lattice iff any two elements must have both meet and join.
S_2 is true because if any element has complement then it must be unique.

15. (d)

$$\begin{aligned} A &= \{\{\}, \{x\}\} \\ A &= \{p, q\} \text{ [Assume } p = \{\}, q = \{x\}] \\ P(A) &= \{\{\}, \{p\}, \{q\}, \{p, q\}\} \\ &= \{\{\}, \{\{\}\}, \{\{x\}\}, \{\{\}, \{x\}\}\} \end{aligned}$$

16. (c)

Conjunction ($\wedge$) is commutative. Hence I is True.

Existential Quantifier ($\exists$) is distributive over disjunction ($\vee$) and not distributive over conjunction ($\wedge$). Hence II is false.

If we simplify III we get $\neg\forall x\ (\neg S\ (x) \vee \neg\ P\ (x))$ which is equal to $\exists x\ [S\ (x) \wedge P\ (x)]$ (same as given expression).

Hence only I and III are equivalent.

17. (d)

In complete graph of 'n' vertices all vertices will have ($n-1$) degree.

$\therefore$ Minimum degree = Maximum degree = 8 for K_9.

In complete bipartite graph with $K_{m,n}$, the size of $K_{m,n}$ is $m \times n$.

$\therefore$ In $K_{2,7}$ we will have $2 \times 7 = 14$ edges.

18. (d)

The operation is not commutative as $p * q \neq q * p$

$q * p = p$ and $p * q = r$

The operation is not associative as $p * (q * r) \neq (p * q) * r$

LHS $p * r = s$

RHS $r * r = p$

19. (b)

Consider choice (b) : $(\forall x(A(x) \Rightarrow B(x))) \Rightarrow ((\forall x A(x)) \Rightarrow (\forall x B(x)))$

Let the LHS of this implication be true

This means that

$$\begin{aligned} &A_1 \to B_1 \\ &A_2 \to B_2 \\ &\vdots \\ &A_n \to B_n \end{aligned}$$

Now we need to check if the RHS is also true. The RHS is $((\forall x A(x)) \Rightarrow (\forall x B(x)))$

To check this let us take the LHS of this as true i.e. take $\forall x\, A(x)$ to be true. This means that (A_1, A_2,...A_n) is taken to be true. Now A_1 along with $A_1 \to B_1$ will imply that B_1 is true. Similarly A_2 along with $A_2 \to B_2$ will imply that B_2 is true. And so on...

Therefore (B_1, B_2,...B_n) all true.

i.e. $\forall x\, B(x)$ is true. Therefore the statement (b) is a valid predicate statement.

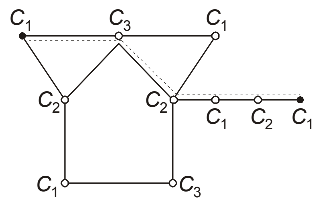
20. (c)

$$1 + x + x^2 + x^3 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x} \qquad [\because\ C \to 3]$$

$$1 + x + x^2 + x^3 + \ldots \infty = \frac{1}{1 - x} \qquad [\because\ B \to 1]$$

$$\sum_{r=0}^{\infty} {}^{n-1+r}C_r \cdot x^r = \frac{1}{(1 - x)^n} \qquad [\because\ A \to 2]$$

21. (d)
Only three colors (C_1, C_2 and C_3) are sufficient to color all vertices such that two adjacent vertices do not have same color.



The dotted lines shows that diameter is 5.

22. (b)

Total number of edges in complete graph of 6 vertices $\frac{6(6-1)}{2} = 15$.

$\therefore$ 15 – 7 = 8 edges are there in $\bar{G}$.

23. (c)
Euler formula says
Number of regions (r) = Number of edges (e) – Number of vertices (n) + 2

$$r = e - n + 2 \qquad \text{...(1)}$$

$$e = \frac{n \cdot k}{2} = \frac{8 \times 11}{2} = 44$$

$\therefore$ $r = 44 - 8 + 2 = 38$ regions.

24. (d)
(Z, +) is both group and Abelian group, as it satisfies commutative property and inverse element is $-a \; \forall a \in Z$.
(Z, –) is never semigroup, because subtraction operation is not associative and hence cannot be monoid too.
(Z, X) is monoid but not group, because inverse does not exist i.e. for any integer 'a' it's inverse is $1/a$ which is a rational number.
Hence it is monoid only.

25. (d)
Minimum degree of any graph can not exceed average degree.

$$\Delta_{min} = \left\lfloor \frac{2e}{n} \right\rfloor$$

$$= \left\lfloor \frac{2 \times 14}{12} \right\rfloor = 2$$

$\therefore$ 3 can never be the minimum degree for the given graph.

26. (b)
$A \oplus B$ is the symmetric difference i.e.

$$A \oplus B = (A \cup B) - (A \cap B)$$
$$C = \{1, 2, 3, 4, 5, 8, 12\} - \{1, 8\}$$
$$C = \{2, 3, 4, 5, 12\}$$
$$|C| = 5$$

27. (b)

Number of chits = $^{10}C_5 = 252$

Using Pigeon hole principle, $\left\lfloor \frac{252-1}{6} \right\rfloor + 1 = 42$

$\therefore$ Atleast 42 chits will be in same box.

28. (c)

R^1 is nothing but R itself.
Now, $R^2 = R \cdot R$ i.e. composite of R with R.
If $(a, b) \in R$, then $(a, c) \in R^2$ iff $(b, c) \in R$.
This composite of relations
$R^2 = \{(1, 1), (2, 1), (3, 1), (4, 2)\}$
$R^3 = \{(1, 1), (2, 1), (3, 1) (4, 1)\}$
$P = \{(1, 1) (2, 1), (3, 2), (4, 1), (4, 2), (4, 3), (3, 1)\}$
$\therefore$ Cardinality of $P = 7$.

29. (d)

This assignment is similar to onto function from set of tasks to set of employees.
The number of onto functions are

$$= 3^5 - {}^3C_1(2)^5 + {}^3C_2(1)^5$$
$$= 243 - 96 + 3 = 150$$

30. (a)

$$\begin{aligned} x'_1 &= x_1 - 1 \,;\, x_1 \ge 0 \\ x'_2 &= x_2 - 2 \,;\, x_2 \ge 0 \\ x'_3 &= x_3 - 2 \,;\, x_3 \ge 0 \\ x'_4 &= x_4 - 4 \,;\, x_4 \ge 0 \\ x'_5 &= x_5 - 6 \,;\, x_5 \ge 0 \\ x'_6 &= x_6 - 5 \,;\, x_6 \ge 0 \end{aligned}$$

The solution for an equation of type $x'_1 + x'_2 + \ldots + x'_n = r$
where, $x'_1, x'_2, \ldots, x'_n \ge 0$ is $^{n+r-1}C_r$
Number of solutions = $^{n+r-1}C_r$
Here $r = 31 - (1 + 2 + 2 + 4 + 6 + 5) = 11$
$x'_1 + x'_2 + x'_3 + x'_4 + x'_5 + x'_6 = 11$
$n = 6,\ r = 11$

$\therefore$ Number of solutions $= {}^{6+11-1}C_{11}$
$= {}^{16}C_{11} = 4368$

■■■■