RIB RANK IMPROVEMENT BATCH
ELECTRONICS ENGINEERING

RIB-R | T4

Session 2019-20 | S.No. : 140719_LS1A

ANSWER KEY ➤ Signal & System

1. (d)	7. (a)	13. (d)	19. (b)	25. (a)
2. (d)	8. (b)	14. (c)	20. (a)	26. (a)
3. (b)	9. (a)	15. (a)	21. (b)	27. (a)
4. (a)	10. (a)	16. (a)	22. (a)	28. (a)
5. (a)	11. (b)	17. (a)	23. (c)	29. (d)
6. (a)	12. (a)	18. (b)	24. (c)	30. (d)

DETAILED EXPLANATIONS

1. (d)

$$2^n u[n] \xleftrightarrow{z} \frac{1}{1-2z^{-1}}$$

$\therefore$ ROC $\Rightarrow |z| > 2$

and $$(4)^n u[-n-1] \xleftrightarrow{z} \frac{1}{1-4z^{-1}}$$

$\therefore$ ROC $\Rightarrow |z| < 4$

$\therefore$ for the signal to converge

ROC $\Rightarrow 2 < |z| < 4$

2. (d)

Since the transfer function has one pole on the R.H.S thus the system is unstable, so the final value theorem is not applicable in this case.

3. (b)

$$y[n] = x[n] * \delta[n-2] = x[n-2]$$

$$\therefore \quad y[2] = x[0] = 1$$

4. (a)

$$\because \quad j\frac{d}{dt}[F(j\omega)] \longleftrightarrow t[f(t)]$$

thus at $t = 0$ the answer will be zero.

6. (a)

$$\because \quad X[k] = W_N^{n_0 k}$$

for $N = 4$, $\quad X[k] = W_N^{n_0 k}$ for $k = 0, 1, 2, 3$

taking IDFT, we get, $\quad x[n] = \delta[n-n_0]$

$$\therefore \quad \text{energy} = \Sigma|x[n]|^2 = 1$$

7. (a)

$$X(e^{j\omega}) = e^{j\omega} + e^{-j\omega} + 2(e^{2j\omega} - e^{-2j\omega}) + 3(e^{3j\omega} + e^{-3j\omega})$$

$$= 2\cos\omega + 4j\sin(2\omega) + 2\cos 3\omega = 2\cos(\pi) + 4j\sin(2\pi) + 6\cos(3\pi)$$

$$= -2 + 0 - 6 = -8$$

$$|Xe^{j\pi}| = 8$$

8. (b)

Hilbert transform will shift the signal by $\frac{-\pi}{2}$ thus $x(t) = e^{j(2\pi ft - \pi/2)} = -je^{j2\pi ft}$

9. (a)

$\because$ For causal system degree of numerator < degree of denominator. (degree is highest power of z)

10. (a)

$$Re\{x(t)\} = \frac{x(t) + x^*(t)}{2}$$

$\therefore$ The Fourier coefficient of $x^*(t)$ are

$$b_K = \frac{1}{T}\int_T x^*(t)e^{-jK\frac{2\pi}{T}t}\,dt$$

Taking conjugate on both sides

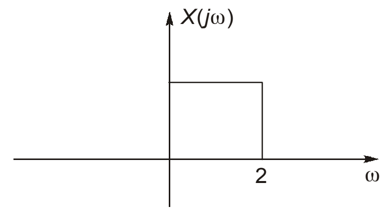
$$b_K^* = \frac{1}{T}\int_T x(t)e^{-j(-K)\frac{2\pi}{T}t}\,dt$$

$$\therefore \quad a_{-K} = b_K^*$$

$\therefore$ Fourier series Coefficient of $Re\{x(t)\} = \frac{a_K + a_{-K}^*}{2}$

11. (b)

The signal can be represented as



Since the signal is neither even nor odd symmetric in frequency domain thus it will be a complex signal in time domain.

12. (a)

$$Y(z) = X(z)\cdot H(z) = 2[z^3 + z^2 - 3z^{-1} + 5z^{-3}]\cdot[2(z^{-2})]$$
$$= 2[z + 2z^{-1} - 3z^{-3} + 5z^{-5}]$$

$\because$ $z^{-2} = 0$

thus $Y[2] = 0$

13. (d)

1. $x(t) = 0$ for $t < 0$ thus the signal is causal

$\because$ $x(t)$ is real thus $X(j\omega)$ is an even symmetric signal

now $$\text{Re}\{X(j\omega)\} \xleftrightarrow{F.T} \text{even}\{x(t)\}$$

$\therefore$ $$|t|e^{-|t|} = \text{even}\{x(t)\}$$

$\therefore$ $$\frac{x(t)+x(-t)}{2} = |t|e^{-|t|}$$

$\therefore$ $$x(t) = 2te^{-|t|}$$

$\{\because$ right sided $x(-t) = 0\}$

$$x(t) = 2te^{-t}u(t)$$

14. (c)

$$x(t) \xleftrightarrow{L.T} X(s) = \frac{s+3}{(s+3)^2+4}$$

$$y(t) \Rightarrow \int x(\tau)d\tau \Rightarrow \frac{s+3}{s[(s+3)^2+4]}$$

15. (a)

$$\frac{\sin\left(\frac{3}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)} \xleftrightarrow{F.T} x_1[n] = \begin{cases}1 & -1 < n < 1\\ 0 & \text{otherwise}\end{cases}$$

now $$\sum_{k=-\infty}^{n} x[n] \xleftrightarrow{F.T} \frac{1}{1-e^{-j\omega}}X_1(e^{j\omega}) + 3\pi\delta(\omega)$$

$$2\pi\delta(\omega) \xleftrightarrow{F.T} 1$$

$\therefore$
$$x[n] = 1 + \sum_{k=-\infty}^{n} x_1[k]$$

$$= \begin{cases} 1 & n < -4 \\ n+3 & -1 < n < 1 \\ 4 & n \geq 2 \end{cases}$$

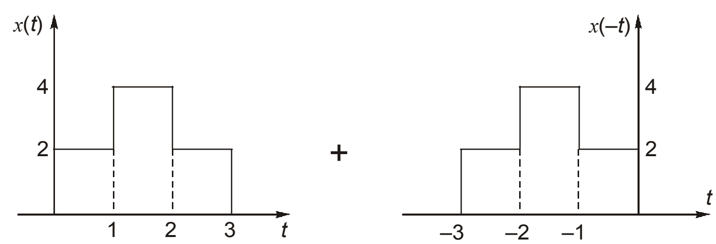
$\therefore$ at $n \to \infty$

$$x[n] = 4$$

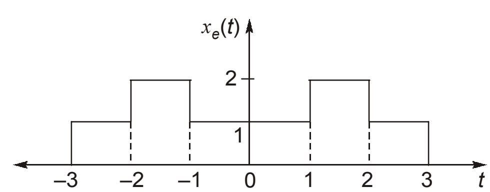
16. (a)

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$\therefore$ $x(t) =$



$\therefore$ $x_e(t) =$



17. (a)

Let
$$x'(t) = e^{-2t}u(t)$$

$$X'(j\omega) = \frac{1}{2 + j\omega}$$

now applying time reversal property

$$x(-t) \xrightarrow{FT} X(-j\omega)$$

$$\therefore \quad e^{2t}u(-t) \xrightarrow{FT} \frac{1}{2 - j\omega}$$

18. (b)

$$Y(e^{j\omega}) = X(e^{j\omega}) * X(e^{j\omega - \pi/2})$$

$\therefore$ In time domain

$$Y(e^{j\omega}) = 2\pi x_1[n] \cdot x_2[n]$$

now, If $$X(e^{j\omega}) \longleftrightarrow n\left(\frac{3}{4}\right)^{|n|}$$

Then $$X(e^{j\omega-\pi/2}) \longleftrightarrow n\cdot e^{j\pi/2}\left(\frac{3}{4}\right)^{|n|}$$

$\therefore$ $$y[n] \longleftrightarrow 2\pi n^2 e^{j\pi n/2}\left(\frac{3}{4}\right)^{2|n|}$$

19. (b)

$$X(z) = \frac{z}{z-1} \qquad |z| > 1$$

$$Y(z) = \frac{2z}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3}$$

$$H(z) = \frac{2(z-1)}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3}$$

$$X'(z) = \frac{z}{z-\frac{1}{2}} \qquad |z| > \frac{1}{2}$$

$$Y'(z) = \text{H}(z)\cdot X'(z)$$

$$= \frac{2z(z-1)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \qquad |z| > \frac{1}{2}$$

Taking inverse z transform

$$y[n] = \left[-6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{3}\right)^n\right]u[n]$$

$k_1 = -6, \quad k_2 = 8$

so, $$k_1 + k_2 = 2$$

20. (a)

$$x(t) = \cos\left(2\left(t-\frac{1}{2}\right)\right)$$

$$= \frac{1}{2}\left[e^{j2(t-1/2)} + e^{-j2(t-1/2)}\right]$$

$$= \frac{1}{2}\left[e^{j2t}\cdot e^{-j} + e^{-j2t}\cdot e^{j}\right]$$

According to the given condition the output is

$$= \frac{1}{2}\left[e^{-j}e^{j3t} + e^{j}e^{-j3t}\right] = \frac{1}{2}\left[e^{j3(t-1/3)} + e^{-j3(t-1/3)}\right]$$

$$y(t) = \cos\left[3\left(t - \frac{1}{3}\right)\right]$$

$$\therefore \quad y\left(\frac{1}{3}\right) = \cos\left[3\left(\frac{1}{3} - \frac{1}{3}\right)\right]$$

$$= \cos 0 = 1$$

21. (b)

If $x[n]$ is real

$$\text{odd}[x[n]] \xrightarrow{FT} jIm[X(e^{j\omega})]$$

$$\therefore \quad \text{odd}[x[n]] = F^{-1}\left[\frac{1}{2}\left(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}\right)\right]$$

$$= \frac{1}{2}\left[\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right]$$

$$\because \quad \text{odd}\,[x[n]] = \frac{x[n] - x[-n]}{2}$$

Since,

$$x[n] = 0 \text{ for } n > 0$$
$$x[n] = 2\,\text{odd}[x[n]]$$
$$= \delta[n+1] - \delta[n+2] \text{ for } n < 0$$

using parshevel's theorem

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|X\left(e^{j\omega}\right)\right|^2 d\omega = \sum_{n=-\infty}^{\infty}|x[n]|^2$$

$$(x[0])^2 - 2 = 3$$
$$x[0] = \pm 1$$
$$\because \quad x[0] > 0$$
$$\therefore \quad x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

22. (a)

$$|H(j\omega)| = \left|3\cos\left(\frac{\omega}{2}\right)\right|; \qquad \angle H(j\omega) = -\omega/2$$

So,

$$H(\omega) = 3\cos\left(\frac{\omega}{2}\right)\cdot e^{-j\omega/2} = \frac{3}{2}\left[e^{j\omega/2} + e^{-j\omega/2}\right]\cdot e^{-j\omega/2}$$

$$= \frac{3}{2}\left[1 + e^{-j\omega}\right]$$

So,

$$H(z) = \frac{3}{2}\left[1 + z^{-1}\right]$$

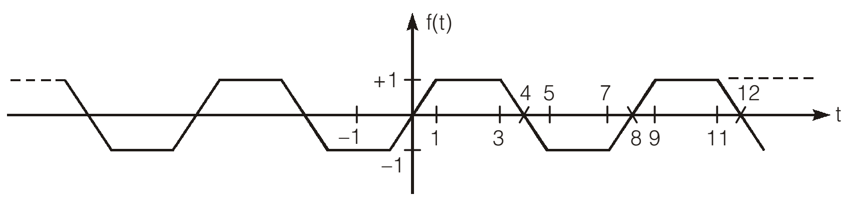
23. (c)

Taking Fourier transform of the signal, we obtain

$$H(e^{j\omega}) = \frac{\frac{3}{2}-\frac{1}{2}e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-2j\omega}}$$

$$\therefore\ y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=\frac{3}{2}x[n]-\frac{1}{2}x[n-1]$$

24. (c)



The signal f(t) has hidden, odd and half-wave symmetry.

So,
$$a_0 \neq 0$$
$$a_n = 0;\ \forall n$$
$$b_n \neq 0;\ n = 1, 3, 5$$

Therefore, non zero Fourier series coefficients are

a_0 and b_n, $n = 1, 3, 5, \ldots\ldots\ldots\ldots$

25. (a)

As, $$x(t)\cos^2 t = x(t)\left[\frac{1}{2}+\frac{1}{2}\cos 2t\right] = \frac{1}{2}x(t)+\frac{1}{2}\cos 2t$$

Now, $$g(t) = x(t).\cos^2 t * \frac{\sin t}{\pi t}$$

$$= x(t)\left[\frac{1+\cos 2t}{2}\right] * \frac{\sin t}{\pi t}$$

$$\therefore\ G(j\omega) = \left[\frac{1}{2}X(j\omega)+\frac{1}{4}X(\omega-2)+\frac{1}{4}X(\omega+2)\right]\times \text{rect}\left(\frac{\omega}{2}\right)$$

Thus the given solution will be

$$\therefore\ G(j\omega) = \frac{1}{2}X(j\omega)$$

or $$g(t) = \frac{1}{2}x(t)$$

Thus to get the desired result

$$h(t) = \frac{1}{2}\delta(t)$$

26. (a)

$$x[n] = \delta[n]$$
$$X(e^{j\omega}) = 1$$
$$\frac{dX(e^{j\omega})}{d\omega} = 0$$
$$\therefore \quad Y(e^{j\omega}) = e^{-j\omega} X(e^{j\omega})$$
$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega}.e^{j\omega n}d\omega = \frac{\sin\pi(n-1)}{\pi(n-1)}$$

27. (a)

$$\because \quad x^*(t) \xleftrightarrow{F} X^*(-j\omega)$$

and
$$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{F} \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

28. (a)

$$X(j0) = \int_{-\infty}^{\infty} x(t)dt = 7$$

29. (d)

$$H(j\omega) = \frac{1+2e^{-j\omega}}{1+\frac{1}{2e^{-j\omega}}} = \frac{1+2e^{-j\omega}}{2e^{-j\omega}+1}\cdot 2e^{-j\omega}$$
$$|H(j\omega)| = 2$$

30. (d)
The impulse response convolves with the input signal and the output can be broken up into shifted versions of input.

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