ANSWER KEY ➤ Signal & System

1.	(d)	7.	(b)	13.	(a)	19.	(d)	25.	(a)
2.	(c)	8.	(c)	14.	(c)	20.	(b)	26.	(a)
3.	(b)	9.	(d)	15.	(a)	21.	(b)	27.	(d)
4.	(a)	10.	(c)	16.	(d)	22.	(d)	28.	(c)
5.	(a)	11.	(b)	17.	(d)	23.	(b)	29.	(a)
6.	(c)	12.	(d)	18.	(b)	24.	(c)	30.	(d)

DETAILED EXPLANATIONS

1. (d)

The input $x[n]$ is non zero for range of $n \Rightarrow -3$ to 4

and $h(n)$ is non zero for range of $n \Rightarrow -1$ to 2.

Then output will be non zero for –4 to 6.

2. (c)

we know that the Laplace transform of

$$\sin(at)\,u(t) = \frac{a}{s^2 + a^2}$$

$$\therefore \quad \sin(\pi t)\,u(t) = \frac{\pi}{s^2 + \pi^2}$$

now, the above function can be written as

$$x(t) = \sin(\pi t)\,u(t) - \sin[\pi(t-2)]\,u(t-2)$$

Taking Laplace transform

$$X(s) = \frac{\pi}{s^2 + \pi^2}(1 - e^{-2s}) \qquad (\because x(t-t_0) = X(s)\cdot e^{-st_0}) \text{ shifting property}$$

3. (b)

Since, $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

thus the

$$\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega}\, d\omega + \int_{-\infty}^{\infty} X(\omega)\, d\omega + \int_{-\infty}^{\infty} X(\omega)\, e^{-j\omega}\, d\omega$$

is $2\pi[x(+1) + x(0) + x(-1)] \Rightarrow 10\pi$

4. (a)

$$N_1 = \frac{2\pi}{\Omega}\cdot k$$

$$= \frac{2\pi}{\pi/9}\cdot k = 18 \qquad (k = 1)$$

$$N_2 = \frac{2\pi}{\pi/7}k = 14 \qquad (k = 1)$$

$\therefore$ $$\frac{N_1}{N_2} = \frac{18}{14} = \text{Rational}$$

$$N = \text{LCM}(18, 14) = 126$$

5. (a)

Taking Laplace transform

$$H(s) = \frac{1/s}{1+\frac{1}{s}} = \frac{1}{s+1}$$

$\therefore$ $$h(t) = e^{-t}\, u(t)$$

thus $$h(t) = 0 \text{ for } t < 0 \qquad \Rightarrow \text{causal}$$

and $$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \qquad \Rightarrow \text{BIBO stable}$$

6. (c)

Given transfer function

$$H(z) = \frac{1}{1+K\left[\frac{z}{z-3}\right]} = \frac{z-3}{z-3+Kz}$$

$$= \frac{z-3}{(K+1)z-3} = \frac{1}{1+K}\left[\frac{z-3}{z-\frac{3}{K+1}}\right]$$

$\therefore$ pole at $$z = \frac{3}{1+K}$$

for the system to be stable, the poles lies inside the unit circle

$$|z| < 1$$

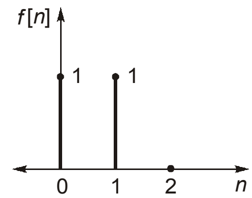
or $$\left|\frac{3}{1+K}\right| < 1$$

$$3 < |K+1|$$

$$K > 2 \quad \text{or} \quad K < -4$$

7. (b)

Given input sequence $\{\underset{\uparrow}{1}, \ 1\}$



$$f[n] = u[n] - u[n-2]$$
$$u[n] \rightarrow S[n]$$
$$u[n-2] \rightarrow S[n-2] \rightarrow \text{Time invariant}$$
$$u[n] - u[n-2] \rightarrow S[n] - S[n-2] \rightarrow \text{Linear}$$
$$\alpha^n u[n] - \alpha^{n-2} u[n-2]$$

at $n = 1$; $\alpha^1 u[1] - \alpha^{-1} u[-1] = \alpha$

8. (c)

$$y[n] = x[n] \otimes h[n]$$

$$\sum_{n=-\infty}^{\infty} y[n] = \sum_{n=-\infty}^{\infty} x[n] \cdot \sum_{n=-\infty}^{\infty} h[n]$$

$$\sum_{n=-\infty}^{\infty} x[n] = 2 + 4 + 5 + 7 = 18 \qquad \text{for given } x[n]$$

$$\sum_{n=-\infty}^{\infty} y[n] = 144$$

so,
$$144 = (18) \cdot \sum_{n=-\infty}^{\infty} h[n]$$

$$\Rightarrow \quad \sum_{n=-\infty}^{\infty} h[n] = \frac{144}{18} = 8$$

$\therefore$ only signal given in option (c) satisfies

$$\because \quad \sum_{n=-\infty}^{\infty} h[n] = 2 + 2 + 2 + 2 = 8$$

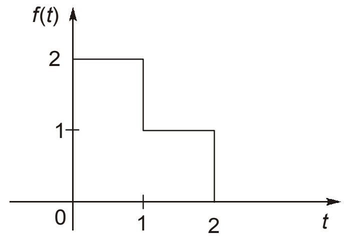
9. (d)

The complex magnitude spectrum is always even symmetric.
The spectrum of real Fourier series is one sided.
The complex phase spectrum is odd symmetry.
If Real of $x(t)$ is even then $b_n = 0$.

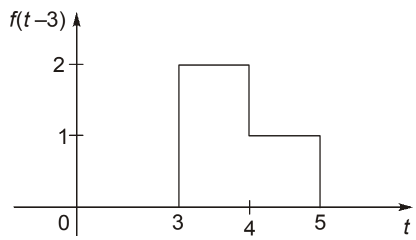
$\therefore$ Phase defined as $-\tan^{-1}\left[\frac{b_n}{a_n}\right] = 0$

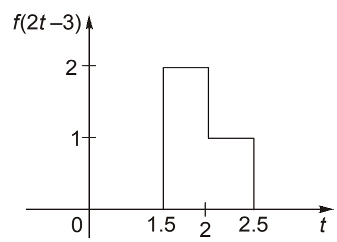
10. (c)

Given signal $f(t) =$



The signal $f(2t-3)$ involves time scaling and time shifting.
Then follow the order of time shifting first and then time scaling.





11. (b)
by using Taylor series we can expand the sin(*z*) into polynomial components

i.e. $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...$$

thus $$\sin(z^2) = z^2 - \frac{z^6}{3!} + \frac{z^{10}}{5!} - ....$$

now, from the above equation, we can deduce that $x(-10) = \dfrac{1}{5!}$

which is nothing but the coefficient of z^{10}

12. (d)

$$Y(z) = \sum_{n=-\infty}^{\infty} y[n]z^{-n} = \sum_{n=-\infty}^{\infty} x[n]z^{-nL} = \sum_{n=-\infty}^{\infty} x[n]\left(z^{L}\right)^{-n} = X(z^{L})$$

now, the previous ROC was, $\alpha < |z| < \beta$
then after passing through the system the ROC will be

$$\alpha < |z|^{L} < \beta$$
$$(\alpha)^{1/L} < |z| < (\beta)^{1/L}$$

13. (a)

$\because$ $$x^*(t) \overset{F}{\longleftrightarrow} X^*(-j\omega)$$

and $$\int_{-\infty}^{t} x(\tau)\,d\tau \overset{F}{\longleftrightarrow} \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

14. (c)

$$y(t) = 3x\left(\frac{2t+15}{30}\right)$$

$$\int_{-10}^{10} x(t)^2 dt = 100$$

energy of $y(t)$

$\Rightarrow$ Since $x(t)$ exist for –10 to 10

so $y(t)$ exist for –157.5 to 142.5

$$\text{energy of } y(t) = \int_{-157.5}^{142.5} 9\left(x\left(\frac{2t+15}{30}\right)\right)^2 dt$$

Let $\frac{2t+15}{30} = \tau$

$$dt = 15\, d\tau$$

$$\Rightarrow 9 \times 15 \int_{-10}^{10} (x(\tau))^2 d\tau$$

$$\Rightarrow \quad 100 \times 9 \times 15 = 13500$$

15. (a)

For a minimum phase system all the zeros must be inside the unit circle

zeros for $H_1(z) = \frac{1}{2}, \frac{1}{3}$

zeros for $H_2(z) = 2, \frac{1}{2}$

zeros for $H_3(z) = 2, 3$

hence, option (a).

16. (d)

Given, $x(t) = 2 + \cos(50\pi t)$

Frequency of signal $\omega_{sig} = 50\pi$

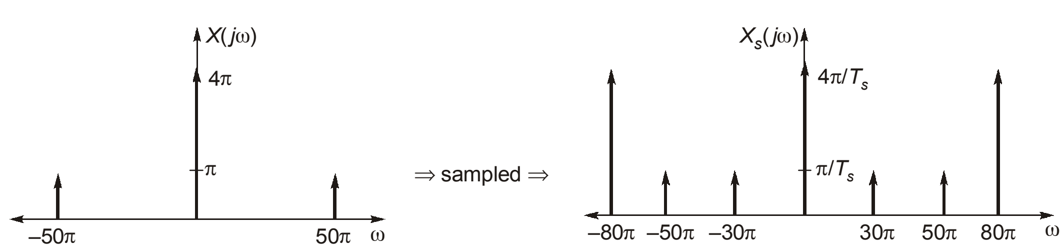
$T_s = 0.025$ sec

$\therefore$ sampling frequency $\omega_s = \frac{2\pi}{T_s} = 80\,\pi$ rad/sec

then, $X(j\omega) = 4\pi\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

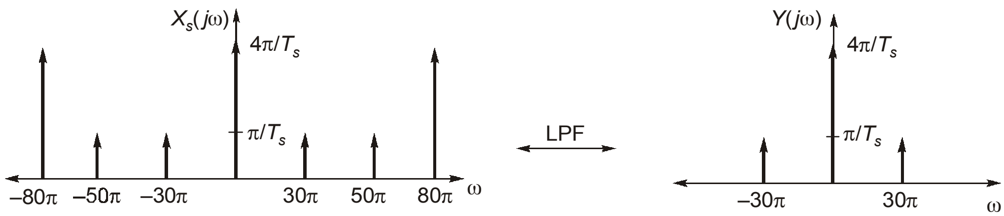
Let the sampled signal be represented as $X_s(j\omega)$, where $X_s(j\omega)$ is given as

$$X_s(j\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X(j(\omega - n\omega_s))$$

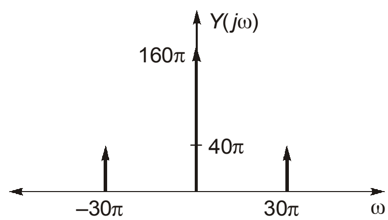


$$X_s(j\omega) = 40 \sum_{m=-\infty}^{\infty} [4\pi\delta(\omega - 80\pi)] + \pi\delta(\omega - 50\pi - 80\pi n) - \pi\delta(\omega + 50\pi - 80\pi n)]$$

now, the sampled input $X_s(j\omega)$ is passed through a low passed filter having cut-off frequency at $\omega = 40\pi$. Therefore the output $Y(j\omega)$ will contain only the components which are less than $\omega = 40\pi$.



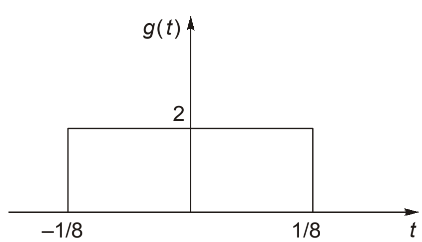
Now by putting $T_s = 0.025$, we will get



17. (d)

$$g(t) = \text{rect}(4t) * 4\delta(-2t)$$
$$= 4\,\text{rect}(4t) * \delta(-2t) \qquad (\because \delta(-t) = \delta(t))$$
$$= 2\,\text{rect}(4t) \qquad \left(\because \delta(at) = \frac{1}{|a|}\delta(t)\right)$$

thus $g(t)$ is given as



now,
$$\text{rect}(t) \xleftrightarrow{F.T} \text{sinc}(f)$$
$$2\text{rect}(t) \xleftrightarrow{F.T} 2\,\text{sinc}(f)$$
$$2\text{rect}(4t) \xleftrightarrow{F.T} 2 \cdot \frac{1}{4}\text{sinc}\left(\frac{f}{4}\right) \qquad \text{(scaling property)}$$
$$\therefore \quad 2\text{rect}(4t) \xleftrightarrow{F.T} \frac{1}{2}\text{sinc}\left(\frac{f}{4}\right)$$

18. (b)

Given
$$X(s) = In\left[1 + \frac{\omega^2}{s^2}\right]$$

Let
$$x(t) = L^{-1}[X(s)] = L^{-1}\left[In\left(1 + \frac{\omega^2}{s^2}\right)\right]$$

$$\therefore \quad L[x(t)] = In\left[1+\frac{\omega^2}{s^2}\right] = In\left[\frac{s^2+\omega^2}{s^2}\right]$$

$$= In[s^2+\omega^2] - In\, s^2$$

$$L[tx(t)] = \frac{-d}{dS}\left[In(s^2+\omega^2) - In\, s^2\right] = \frac{-1}{s^2+\omega^2}\cdot 2s + \frac{1}{s^2}2s = \frac{2}{s} - \frac{2s}{s^2+\omega^2}$$

$$\therefore \quad tx(t) = L^{-1}\left[\frac{2}{s} - \frac{2s}{s^2+\omega^2}\right] = (2-2\cos\omega t)u(t) = 2(1-\cos\omega t)u(t)$$

$$\therefore \quad x(t) = \frac{2(1-\cos\omega t)}{t}u(t)$$

19. (d)

We know that

$$FT[e^{-t}u(t)] = \frac{1}{1+j\omega}$$

Using duality property

$$x(t) \xleftrightarrow{FT} X(\omega)$$

$$X(t) \xleftrightarrow{FT} 2\pi x(-\omega)$$

we have $$FT\left[\frac{1}{1+jt}\right] \xleftrightarrow{FT} 2\pi e^{-(-\omega)}\, u(-\omega)$$

$$\xleftrightarrow{FT} 2\pi e^{\omega}\, u(-\omega)$$

using the time reversal property,

i.e. $x(-t) = X(-\omega)$, we have

$$FT\left[\frac{1}{1-jt}\right] \xleftrightarrow{FT} 2\pi e^{-\omega}\, u(\omega)$$

$$\therefore \quad e^{-\omega}u(\omega) \xleftrightarrow{IFT} \frac{1}{2\pi}FT\left[\frac{1}{1-jt}\right]$$

$$\therefore \quad FT^{-1}\left[e^{-\omega}u(\omega)\right] = \frac{1}{2\pi(1-jt)}$$

20. (b)

Given $N = 13$, $C_3 = 2 + 3j$

$C_k \rightarrow$ periodic with period $N = 13$

$$C_{55} = C_{4\times 13+3} = C_3 = 2+3j$$

$$C_{-29} = C_{-2\times 13-3} = C_{-3} = C_3^* = 2-3j$$

$$\therefore \quad C_{-29} + C_{55} = 2-3j+2+3j = 4$$

21. (b)

Given unit impulse response is

$$h[n] = \delta(n) - \alpha\delta[n-1]$$

The frequency response is

$$H(e^{j\omega}) = 1 - \alpha e^{-j\omega} = 1 - \alpha\cos\omega + j\alpha\sin\omega$$

The phase delay $$\phi_{ph}(\omega) = \frac{-\phi(\omega)}{\omega}$$

$\because \phi(\omega)$ is the phase of $H(e^{j\omega})$

$$\phi(\omega) = \tan^{-1}\left[\frac{\alpha\sin\omega}{1-\alpha\cos\omega}\right]$$

$\because$ phase delay $\quad \phi_{ph}(\omega)\Big|_{\omega=\frac{\pi}{2}} = \dfrac{-\tan^{-1}\alpha}{\frac{\pi}{2}}$

$\therefore$ phase delay, $\phi_{ph}\left(\dfrac{\pi}{2}\right) = \dfrac{-2}{\pi}\tan^{-1}\alpha$

22. (d)

Given discrete-time signal

$$x[n] = n2^n \sin\left(\frac{\pi}{2}n\right)u[n]$$

we know that

$$Z\left[\sin\left(\frac{\pi}{2}n\right)u[n]\right] = \frac{z\sin\left(\frac{\pi}{2}\right)}{z^2 - 2z\cos\left(\frac{\pi}{2}\right)+1} = \frac{z}{z^2+1}$$

Using the multiplication by exponential property, we have

$$\left[\begin{array}{l} x[n] \longleftrightarrow X(z) \\ a^n x[n] \longleftrightarrow X\left(\frac{z}{a}\right) \end{array}\right]$$

$$Z\left[2^n\sin\left(\frac{\pi}{2}n\right)u[n]\right] = Z\left[\sin\left(\frac{\pi}{2}n\right)u[n]\right]\Bigg|_{z\to\left(\frac{z}{2}\right)}$$

$$= \frac{z}{z^2+1}\Bigg|_{z\to\frac{z}{2}} = \frac{2z}{z^2+4}$$

Using differentiation in z-domain property

$$\left[\begin{array}{l} x[n] \longleftrightarrow X(z) \\ nx[n] \longleftrightarrow -z\frac{d}{dz}X(z) \end{array}\right]$$

$$Z\left[n2^n\sin\left(\frac{\pi}{2}n\right)u[n]\right] = -z\frac{d}{dz}\left\{Z\left[n2^n\sin\left(\frac{\pi}{2}n\right)u[n]\right]\right\}$$

$$= -z\frac{d}{dz}\left(\frac{2z}{z^2+4}\right) = -z\left[\frac{(z^2+4)(2)-2z(2z)}{(z^2+4)^2}\right]$$

$$= -z\left[\frac{-2z^2+8}{(z^2+4)^2}\right]$$

$$Z\left[n2^n\sin\left(\frac{\pi}{2}n\right)u[n]\right] = \frac{2z(z^2-4)}{(z^2+4)^2}$$

23. (b)

Let the exponential Fourier series coefficient of $g(t)$ are C_k then,

$$g(t) = \sum_{-\infty}^{\infty} C_k e^{j\omega_0 kt}$$

Since, $$C_0 = \frac{1}{T}\int_0^T g(\tau)d\tau = \frac{1}{2}\int_0^2 g(\tau)d\tau = 1$$

So, $$g(t) = 1 + \sum_{k=-\infty}^{-1} C_k e^{j\omega_0 kt} + \sum_{k=1}^{\infty} C_k e^{j\omega_0 kt}$$

To find C_k, let $$f(t) = \frac{d}{dt}g(t)$$

So, $$f(t) = 1 - \sum_{k=-\infty}^{\infty} 2\delta(t-2k)$$

The exponential Fourier series coefficient (F_k) of $f(t)$ = Exponential Fourier coefficient A_k of signal $(1\ \forall t)$ – Exponential Fourier coefficient B_k of signal $\left(\sum_{k=-\infty}^{\infty} 2\delta(t-2k)\right)$

Let define A_k : $$A_k = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

Let define B_k : $$B_k = (1)\ \forall\ k$$

So, $$F_k = A_k - B_k = \begin{cases} 0 & k = 0 \\ -1 & k \neq 0 \end{cases}$$

Since, $$f(t) = \frac{d}{dt}g(t) \quad \Rightarrow \quad F_k = j\omega_0 k\, C_k$$

$\Rightarrow$ $$C_k = \frac{F_k}{j\omega_0 k} = \frac{-1}{j\omega_0 k} \quad (\text{for } k \neq 0)$$

$\Rightarrow$ $$C_k = \frac{-1}{j\omega_0 k}$$

From the definition $$C_k = X_m$$

So, $$X_m = \frac{1}{\pi m}e^{j\pi/2}$$ (Since period of signal is 2, $\omega_0 = \pi$ rad/sec.)

24. (c)

Since $$H(\omega) = 2\cos\omega\left(\frac{\sin 2\omega}{\omega}\right)$$

$$= (e^{-j\omega} + e^{j\omega})\left(\frac{\sin 2\omega}{\omega}\right)$$

Since

$$\frac{\sin 2\omega}{\omega} \xrightarrow[\text{Fourier Transform}]{\text{Inverse}} \frac{1}{2}\text{rect}\left(\frac{t}{4}\right)$$

$$e^{-j\omega}\frac{\sin 2\omega}{\omega} \xrightarrow[\text{Fourier Transform}]{\text{Inverse}} \frac{1}{2}\text{rect}\left(\frac{t-1}{4}\right)$$

$$e^{+j\omega}\frac{\sin 2\omega}{\omega} \xrightarrow[\text{Fourier Transform}]{\text{Inverse}} \frac{1}{2}\text{rect}\left(\frac{t+1}{4}\right)$$

$$\Rightarrow \qquad h(t) = \frac{1}{2}\text{rect}\left(\frac{t+1}{4}\right) + \frac{1}{2}\text{rect}\left(\frac{t-1}{4}\right)$$

Thus, $\qquad h(0) = 1$

25. (a)

Given that $h[n] = \left(\frac{1}{2}\right)^n u(n)$ and $g[n]$ is a causal sequence.

$$y[n] = h[n] * g[n]$$

$$h[n] = \left[\underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} .....\right]$$

$$g[n] = \left[\underset{\uparrow}{\alpha}, \beta, \gamma, ...\right]$$

$$y[n] = h[n] * g[n]$$

$$\begin{array}{ccccc}
..... & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 1 \\
 & .... & \gamma & \beta & \alpha \\
\hline
..... & \frac{\alpha}{8} & \frac{\alpha}{4} & \frac{\alpha}{2} & \alpha \\
..... & \frac{\beta}{4} & \frac{\beta}{2} & \beta & \times \\
..... & \frac{\gamma}{2} & \gamma & \times & \times \\
\hline
y[n] = & & .... & \frac{1}{2} & 1
\end{array}$$

Now, $\qquad \alpha = 1$

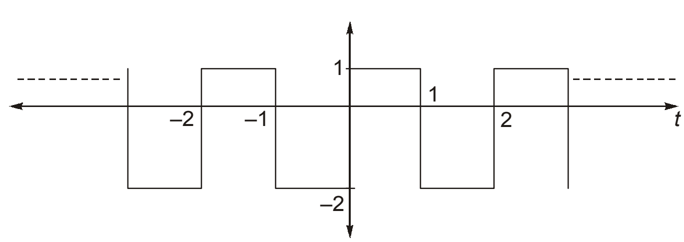
Also, $\qquad \frac{\alpha}{2} + \beta = \frac{1}{2}$

or, $\qquad \beta = 0$

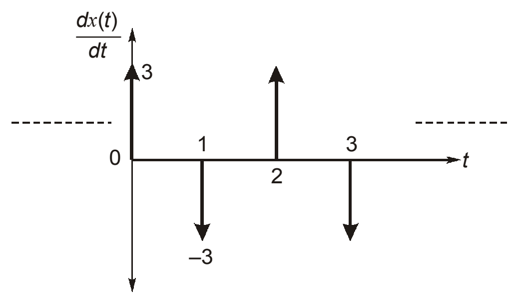
$\therefore \qquad g[1] = 0$

26. (a)

The above signal can be represented as



Then differentiating the signal we get



$$\frac{dx(t)}{dt} = 3g(t) - 3g(t-1)$$

thus $A_1 = 3, \quad A_2 = -3$

$T_1 = 0, \quad T_2 = 1$

27. (d)

$$Y(s) = H(s)\,X(s)$$

Since, it is asked in the question to find the forced response thus, we will take the initial conditions to be equal to zero.

$$Y(s) = \frac{1}{(s+2)(s+3)(s+1)}$$

Taking partial fraction, we get

$$Y(s) = \frac{1/2}{(s+1)} + \frac{1/2}{s+3} - \frac{1}{s+2}$$

$$\therefore \quad y(t) = \frac{1}{2}e^{-t} + \frac{1}{2}e^{-3t} - e^{-2t}$$

$\because$ We are taking the Laplace transform with zero initial condition thus the response so obtained is the forced response.

28. (c)

Given $X(z) = \ln\left(\frac{\alpha}{\alpha - z^{-1}}\right)$; ROC; $|z| > \frac{1}{\alpha} = -\ln\left(1-(\alpha z)^{-1}\right)$

now expanding it by Taylor series, we get

$$X(z) = \left[(\alpha z)^{-1} + \frac{(\alpha z)^{-2}}{2} + \frac{(\alpha z)^{-3}}{3} + \ldots\ldots\right] = \sum_{k=1}^{\infty} \frac{\left[(\alpha z)^{-1}\right]^k}{k}$$

$$\therefore \quad X(z) = \sum_{k=1}^{\infty} \frac{\alpha^{-k}}{k} \cdot z^{-k}$$

Taking the inverse z-transform, we get

$$x[n] = \sum_{k=1}^{\infty} \frac{\alpha^{-k}}{k}\delta(n-k) \qquad [\because \delta[n-k] \leftrightarrow z^{-k}]$$

$$\therefore \quad x[n] = \left(\frac{\alpha^{-n}}{n}\right)u[n-1]$$

29. (a)

Applying initial value theorem (since the $F(s)$ function is proper we can apply initial value theorem)

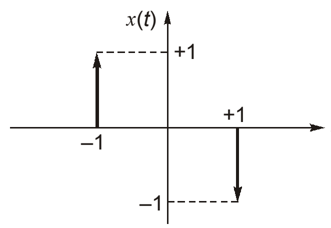
$$f(0) = \text{initial value} = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{3s^2}{s^2+5s+6} = 3$$

Now, we can apply final value theorem because the poles of $F(s)$ are in left half of s-plane

So, $$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{3s^2}{s^2+5s+6} = 0$$

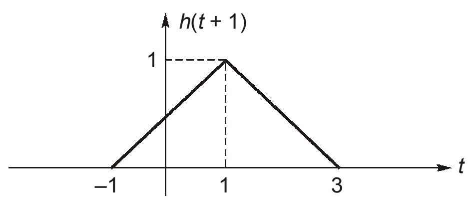
So, $$f(0) = 3,$$
$$f(\infty) = 0$$

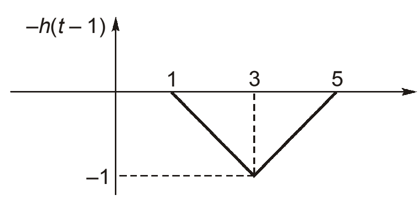
30. (d)



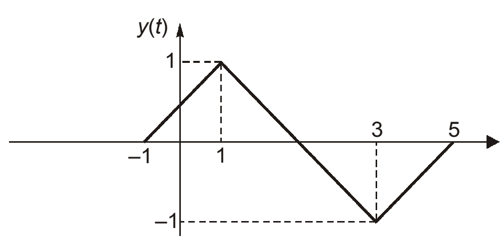
$\Rightarrow$ $$x(t) = \delta(t+1) - \delta(t-1)$$

so, $$x(t) * h(t) = y(t) = h(t+1) - h(t-1)$$





so, $$y(t) = h(t+1) - h(t-1)$$



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