CLASS TEST

S.No. : 03_SK_EE_BCD_210922

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ENGINEERING MATHEMATICS

ELECTRICAL ENGINEERING

Date of Test : 21/09/2022

ANSWER KEY

1.	(c)	7.	(d)	13.	(b)	19.	(b)	25.	(c)
2.	(a)	8.	(c)	14.	(d)	20.	(b)	26.	(c)
3 .	(a)	9.	(a)	15.	(a)	21.	(a)	27.	(a)
4.	(c)	10.	(b)	16.	(c)	22.	(c)	28.	(d)
5.	(c)	11.	(b)	17.	(b)	23.	(b)	29.	(c)
6.	(a)	12.	(c)	18.	(a)	24.	(d)	30.	(d)

DETAILED EXPLANATIONS

1. (c)

For the system to be consistent,

$$\begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$$

$$1 + (-abc) + (-abc) - b^2 - a^2 - c^2 = 0$$

$$a^2 + b^2 + c^2 + 2abc = 1$$

2. (a)

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

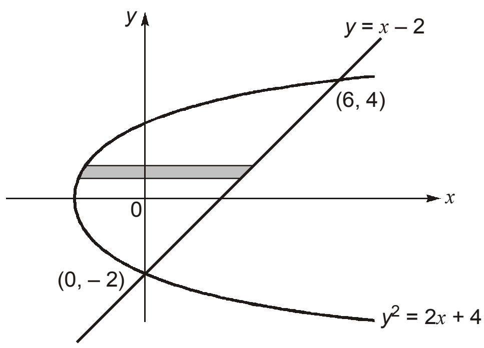
3. (a)

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 (2-x)x\,dx$$

$$= \left.\frac{x^3}{3}\right|_0^1 + \left.\left(x^2 - \frac{x^3}{3}\right)\right|_1^2 = \frac{1}{3} + 4 - 1 - \frac{8-1}{3} = 1$$

4. (c)

The point of intersection of line and parabolic are (0, –2) and (6, 4).



$$\text{Area} = \int_{-2}^{4}\int_{\left(\frac{y^2-4}{2}\right)}^{y+2} dx\,dy = \int_{-2}^{4} x\Big|_{\frac{y^2-4}{2}}^{y+2} dy$$

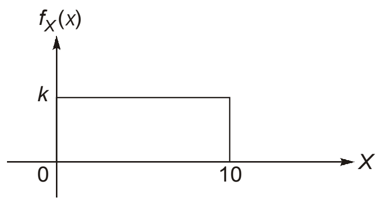
$$= \int_{-2}^{4}\left(y+2-\frac{y^2}{2}+2\right)dy = \left(\frac{y^2}{2}+4y-\frac{y^3}{6}\right)\Bigg|_{-2}^{4} = 18$$

5. (c)

$$p = 0.1$$
$$q = 0.9$$
$$n = 400$$
$$\text{Standard deviation} = \sqrt{npq} = \sqrt{400\times 0.1\times 0.9} = 6$$

6. (a)

$$\int_0^{10} k\,dx = 1$$



$$kx\Big|_0^{10} = 1$$
$$10k = 1$$
$$k = \frac{1}{10}$$
$$P(2.5\le X\le 7.5) = \int_{2.5}^{7.5}\frac{1}{10}dx = \frac{1}{10}x\Big|_{2.5}^{7.5} = \frac{1}{10}(7.5-2.5) = \frac{1}{2}$$

Mean square value,

$$\int\frac{1}{10}x^2dx = \frac{1}{10}\frac{x^3}{3}\Bigg|_0^{10} = \frac{10^3-0^3}{30} = \frac{1000}{30} = \frac{100}{3}$$

7. (d)

The roots of auxiliary equation are 2, $\pm\, 2i$

$$a = -(2+2i-2i) = -2$$
$$b = 2\times(2i)+2\times(-2i)+2i\times(-2i) = 4$$
$$c = -(2\times 2i\times(-2i)) = -8$$
$$a+b+c = -2+4-8 = -6$$

8. (c)

$$x\,dy - y\,dx + 2x^3dx = 0$$
$$\Rightarrow \quad \frac{dy}{dx}-\frac{y}{x} = -2x^2$$
$$\Rightarrow \quad \text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = \frac{1}{x}$$

9. (a)

$$\text{Mean} = \frac{1+3+2+1+5+6+3+3}{8} = 3$$
$$\text{Standard deviation} = \sqrt{\frac{(1-3)^2+(3-3)^2+(2-3)^2\ldots(3-3)^2}{8}} = 1.66$$

10. (b)

$$\frac{\partial z}{\partial x} = f(x^2 - y^2)\, 2x$$

$$\frac{\partial z}{\partial y} = f(x^2 - y^2)\, (-2y)$$

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$$

11. (b)
According to question $A \times B = C$
Matrix C is a unit matrix. So matrix B will be inverse of A.

$$B = A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

12. (c)
Given,

P(Scoring a century) = 0.20

P(Scoring a century) $\cap$ runs > 150) = 0.05

The desired probability

$= P(> 150 \mid \text{Scoring a century})$

$$= \frac{0.05}{0.20} = \frac{1}{4} = 0.25$$

13. (b)

$$\begin{bmatrix} (4-\lambda) & 1 \\ 0 & (7-\lambda) \end{bmatrix} = 0$$

$\Rightarrow$ $(4-\lambda)(7-\lambda) = 0$

$\therefore$ $\lambda = 4, 7$

Putting the value of $\lambda = 4$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ p \end{bmatrix} = 0$$

$\Rightarrow$ $p = 0$

Putting the value of $\lambda = 7$

$$\Rightarrow \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ q \end{bmatrix} = 0$$

$\Rightarrow$ $q = 3$

$\therefore$ $p + q = 3$

14. (d)

For the given matrix to be non singular,

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix} \neq 0$$

$\Rightarrow 1 - (a + c)\omega + a\,c\omega^2 \neq 0$

$\Rightarrow (1 - a\omega)(1 - c\omega) \neq 0$

$\Rightarrow a \neq \omega^2$ and $c \neq \omega^2$ where ω is complex cube root of unity ...($\because \omega^3 = 1$)

As a, b and c are complex cube roots of unity

$\therefore$ a and c can take only one value i.e. ω while b can take 2 values i.e. ω and ω^2.

$\therefore$ Total number of distinct matrices = $1 \times 1 \times 2 = 2$

15. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x\, dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$

$$\log \frac{1}{\cos x \cos y} = C_1$$

$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0)\cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

16. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$IF = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

17. (b)

Number of ways of throwing 6 is five $\Rightarrow$ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)

Number of ways of throwing 7 is six $\Rightarrow$ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \frac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \frac{5}{36} = \frac{31}{36}$

Probability of throwing 7, $q_1 = \frac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \frac{6}{36} = \frac{30}{36}$

$$\begin{aligned}
\text{Probability of } B \text{ winning} &= p_2q_1 + p_2q_2\,p_2q_1 + p_2q_2p_2q_2p_2q_1 + \ldots \\
&= p_2q_1[1 + p_2q_2 + (p_2q_2)^2 + (p_2q_2)^3 + \ldots] \\
&= \frac{p_2q_1}{(1-p_2q_2)} = \frac{\frac{31}{36}\times\frac{6}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{31\times 6}{366} = \frac{31}{61}
\end{aligned}$$

18. (a)

The equation is $4y''(x) + 64\,y(x) = 0$

The auxiliary equation is

$$\begin{aligned}
4m^2 + 64 &= 0 \\
m^2 + 16 &= 0 \\
m &= \pm 4i
\end{aligned}$$

Solution is $\quad y = C_1\cos 4x + C_2\sin 4x$

Given that $\quad y(0) = 0$

$\therefore \quad 0 = C_1$

$$\begin{aligned}
y' &= -4C_1\sin 4x + 4C_2\cos 4x \\
y'(0) &= 1024 \\
1024 &= 4C_2
\end{aligned}$$

$\Rightarrow \quad C_2 = 256$

$\therefore$ Solution is y = 256 sin4x

At $x = 1$, $y = 256 \sin 4 = -193.74$

19. (b)

$$\frac{d^2y}{dx^2} = 0$$

Let, $\quad y = C_1x + C_2$

$$C_1 = \frac{dy}{dx} = 3$$

At $x = 0$, $\quad y = 7 = C_2$

$\therefore \quad y = C_1x + C_2 = 3x + 7$

At $x = 18$, $\quad f(18) = 3 \times 18 + 7 = 54 + 7 = 61$

20. (b)

$$\frac{dx}{dt} = 3x$$

$$\int \frac{dx}{x} = \int 3dt$$

$$\ln x = 3t + C$$

at $t = 0$, $\quad x = 5$

$$\ln 5 = C$$

So, $\quad \ln x = 3t + \ln 5$

$$\ln \frac{x}{5} = 3t$$

$$\frac{x}{5} = e^{3t}$$

$$x = 5e^{3t}$$

At $t = 4$, $\quad x = 5e^{12}$

21. (a)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$$

$$\vec{v}\big|_{t=1} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

Component of velocity in direction $\hat{i} - 3\hat{j} + 2\hat{k}$ will be,

$$\frac{\vec{v}\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{(4\hat{i} - 2\hat{j} + 3\hat{k})\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}} = \frac{4+6+6}{\sqrt{14}} = \frac{16}{\sqrt{14}} = 4.276$$

22. (c)

$$\log\sqrt{\frac{1+x}{1-x}} = \log\left(\frac{1+x}{1-x}\right)^{1/2}$$

$$= \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$$

$$= \frac{1}{2}\{\log(1+x) - \log(1-x)\}$$

$$= \frac{1}{2}\left\{\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}\ldots\right) - \left[-\left(x + \frac{x^2}{2} + \frac{x^3}{3}\ldots\right)\right]\right\}$$

$$= x + \frac{x^3}{3} + \frac{x^5}{5}\ldots..$$

23. (b)

$$p = \frac{d\,f(x,y)}{dx} = 2x+6$$

$\Rightarrow$ $$p = 0 \quad \text{at } x = -3$$

$$q = \frac{df(x,y)}{dy} = 2y$$

$\Rightarrow$ $$q = 0 \quad \text{at } y = 0$$

$\therefore$ (–3, 0) is a stationary point

$$r = \frac{d^2f(x,y)}{dx^2} = 2$$

$$s = \frac{d^2f(x,y)}{dx\,dy} = 0$$

$$t = \frac{d^2f(x,y)}{dy^2} = 2$$

At (–3, 0), $rt - s^2 = 4 > 0$ and $r = 2 > 0$

$\therefore$ $f(x, y)$ has a minimum value at (–3, 0)

$\therefore$ $f(-3, 0) = 1$

24. (d)

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & \alpha \\ \beta & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - \alpha\beta = 0$$

$$\lambda^2 - 3\lambda + 2 - \alpha\beta = 0$$

$$\lambda = \frac{3 \pm \sqrt{9-4(2-\alpha\beta)}}{2}$$

$$\lambda_1\lambda_2 = 2 - \alpha\beta$$

For eigen values to be real,

$$9 - 4(2-\alpha\beta) \geq 0$$

$$9 - 8 + 4\alpha\beta \geq 0$$

$$4\alpha\beta \geq -1$$

$$\alpha\beta \geq -\frac{1}{4} \quad \text{...(i)}$$

For eigen values to be positive,

$$\lambda_1\lambda_2 \geq 0$$

$$2 - \alpha\beta \geq 0$$

$$\alpha\beta \leq 2 \quad \text{...(ii)}$$

So, for eigen values to be real and positive,

$$-\frac{1}{4} \leq \alpha\beta \leq 2$$

25. (c)

$$x^2y'' + xy' + y = 0$$
$$\downarrow$$
$$\Rightarrow \quad (\theta(\theta - 1) + \theta + 1)\, y = 0$$
$$\Rightarrow \quad (\theta^2 - \theta + \theta + 1)\, y = 0$$
$$\Rightarrow \quad (\theta^2 + 1)\, y = 0$$
$$AE \text{ is } m^2 + 1 = 0$$
$$\Rightarrow \quad m = \pm i$$
$$\Rightarrow \quad CF = C_1 \cos z + C_2 \sin z$$
$$\therefore \quad \text{Solution is } y = C_1 \cos(\ln x) + C_2 \sin(\ln x)$$

26. (c)

$$\text{Standard deviation} = \sqrt{E(X)^2 - \{E(X)\}^2}$$

Mean,
$$E(X) = 2\left(\frac{1}{3}\right) + (-1)\left(\frac{2}{3}\right) = 0$$
$$\Rightarrow \quad E(X)^2 = \Sigma X^2 P(X)$$
$$= (2)^2 \times \frac{1}{3} + (-1)^2\left(\frac{2}{3}\right) = 2$$
$$\therefore \quad \text{Standard deviation} = \sqrt{(2) - 0} = \sqrt{2}$$

27. (a)

Let
$$f(x) = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x}\, dx \;(a > 0) \qquad \text{...(i)}$$

We know
$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$
$$\therefore \quad f(x) = \int_{-\pi}^{\pi} \frac{a^x \cos^2 x}{1 + a^x}\, dx \qquad \text{...(ii)}$$

from (i) and (ii)

$$\Rightarrow \quad 2f(x) = \int_{-\pi}^{\pi} \cos^2 x\, dx = 2\int_0^{\pi} \cos^2 x\, dx$$
$$\Rightarrow \quad 2f(x) = 2 \times 2\int_0^{\pi/2} \cos^2 x\, dx$$
$$\Rightarrow \quad 2f(x) = 4 \times \frac{1}{2} \times \frac{\pi}{2}$$
$$\left[\text{By using } \int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \times \frac{\pi}{2} \text{ if } n \text{ is even}\right]$$
$$f(x) = \frac{\pi}{2}$$

28. (d)

$$\overline{PQ} = (5-1)\bar{i} + (0-2)\bar{j} + (4-3)\bar{k} = 4\bar{i} + 2\bar{j} + \bar{k}$$

$$\nabla f = 2x\bar{i} - 2y\bar{j} + 4z\bar{k}$$

$$\Rightarrow \quad (\nabla f)_p = 2\bar{i} - 4\bar{j} + 12\bar{k}$$

The directional derivative of f at P along $\overline{PQ} = (\nabla f)_p \cdot \frac{\overline{PQ}}{|\overline{PQ}|}$

$$= \frac{(2\bar{i} - 4\bar{j} + 12\bar{k})\cdot(4\bar{i} - 2\bar{j} + \bar{k})}{\sqrt{16+4+1}} = \frac{28}{\sqrt{21}} \simeq 6.11$$

29. (c)

$$np = 3$$

$$npq = \sigma^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

from here $\quad q = \frac{3}{4}$

$$p = \left(1 - \frac{3}{4}\right) = \frac{1}{4}$$

$$n \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{4}$$

$$n = 12$$

30. (d)

$$y^2 = 12x$$

$$2y\frac{dy}{dx} = 12$$

$$\frac{dy}{dx} = \frac{12}{2y} = \frac{6}{y} = \text{slope of tangent } (m_1)$$

slope of normal $(m_2) = -\frac{y}{6}$ $\quad$ [as $m_1 m_2 = -1$]

slope of line given is –1

$$-\frac{y}{6} = -1$$

$$y = 6$$

for $y = 6$

$$6^2 = 12x$$

$$x = 3$$

for (3, 6) to lie on the given line

$$\lambda = x + y = 3 + 6 = 9$$

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