CLASS TEST

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 09/09/2022

ANSWER KEY

1.	(c)	7.	(d)	13.	(b)	19.	(b)	25.	(a)
2.	(a)	8.	(a)	14.	(b)	20.	(c)	26.	(c)
3.	(b)	9.	(d)	15.	(d)	21.	(d)	27.	(c)
4.	(a)	10.	(b)	16.	(b)	22.	(d)	28.	(c)
5.	(c)	11.	(c)	17.	(d)	23.	(c)	29.	(b)
6.	(b)	12.	(c)	18.	(b)	24.	(a)	30.	(a)

DETAILED EXPLANATIONS

1. (c)

The characteristic equation $[A - \lambda I] = 0$

i.e. $\begin{bmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{bmatrix} = 0$

or $(4-\lambda)(8-\lambda) - 12 = 0$

or $32 - 8\lambda - 4\lambda + \lambda^2 - 12 = 0$

$\Rightarrow \lambda^2 - 12\lambda + 20 = 0$

$\Rightarrow \lambda^2 - 10\lambda - 2\lambda + 20 = 0$

$\Rightarrow (\lambda - 10)(\lambda - 2) = 0$

$\Rightarrow \lambda = 10, 2$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $-6x + 6y = 0$

$\Rightarrow x = y$

$2x - 2y = 0$

$\Rightarrow x = y$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A - \lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x + 6y = 0$i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

2. (a)

The product of eigen values is always equal to the determinant value of the matrix.

$\lambda_1 = 5,$ $\lambda_2 = \text{Unknown}$

$|A| = 30$

$\lambda_1\lambda_2 = 30$

$5(\lambda_2) = 30$

$\Rightarrow \lambda_2 = 6$

3. (b)

$$\lim_{x\to 0}\left(\frac{3\cos^2 x - 2\sin^2 x}{2\sin x + 3\cos x}\right) = \lim_{x\to 0}\left(\frac{3-0}{0+3}\right) = \frac{3}{3} = 1$$

4. (a)

Probability of atleast one meeting the specification

$= 1 - \left(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D}\right)$

$= 1 - (0.4 \times 0.3 \times 0.2 \times 0.1)$

$= 1 - (0.0024) = 0.9976$

5. (c)

Case-I: White ball is transferred from urn A to urn B

Probability of drawing white ball from $B = \frac{2}{2+4} \times \frac{6}{13} = \frac{2}{13}$

Case-II: Black ball is transferred from A to B

Probability of drawing black ball from $B = \frac{4}{2+4} \times \frac{5}{13} = \frac{10}{39}$

$$\text{Required probability} = \frac{2}{13} + \frac{10}{39} = \frac{16}{39}$$

6. (b)

$$\frac{\partial M}{\partial y} = \frac{x}{xy} = \frac{1}{y}$$

$$\frac{\partial N}{\partial x} = \frac{m}{y}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\Rightarrow \quad m = 1$$

7. (d)

$$\frac{e^x}{(1-e^x)}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

Integrating on both sides, we get,

$$-\ln(1-e^x) + \ln(\tan y) = C_1$$

$$\ln\left(\frac{\tan y}{(1-e^x)}\right) = C_1$$

$$\frac{\tan y}{(1-e^x)} = e^{C_1} = C$$

$$\tan y = C(1-e^x)$$

8. (a)

$$(D^2 + D)y = x^2 + 2x + 8$$

The particular integral is,

$$PI = \frac{x^2+2x+8}{D(1+D)}$$

$$= \frac{1}{D}(1+D)^{-1}(x^2+2x+8) = \frac{1}{D}(1-D+D^2-D^3+\ldots)(x^2+2x+8)$$

$$= \frac{1}{D}(x^2+2x+8-2x-2+2) = \frac{1}{D}(x^2+8) = \frac{x^3}{3}+8x$$

9. (d)

Probability of showing even number $= \frac{2}{1+2} = \frac{2}{3}$

Probability of showing odd number $= \frac{1}{1+2} = \frac{1}{3}$

For sum to be odd = (Even + Even + odd)/(Even + Odd + Even)/(Odd + Even + Even)/(odd + odd + odd)

$$\text{Required probability} = \frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times 3+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3} = \frac{12+1}{27} = \frac{13}{27}$$

10. (b)

$$y = e^x \cos x$$

In series $e^x \cos x$, coefficient of x^2 is $\frac{1}{2}\left(\frac{d^2e^x \cos x}{dx^2}\right)\Big|_{x=0}$

$$y' = \frac{de^x \cos x}{dx} = e^x \cos x - e^x \sin x = y - e^x \sin x$$

$$\frac{d^2e^x \cos x}{dx^2} = y' - e^x \sin x - e^x\cos x = y - e^x \sin x - e^x \sin x - e^x \cos x$$

At $x = 0$, $\left(\frac{d^2e^x \cos x}{dx^2}\right)\Big|_{x=0} = \left(e^x \cos x - 2e^x \sin x - e^x \cos x\right)\Big|_{x=0}$

$$= \left(-2e^{-x} \sin x\right)\Big|_{x=0} = 0$$

So, The coefficient of $x^2 = \frac{1}{2}(0) = 0$

11. (c)

Given matrix is $M = \begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

Determinant of $M = \begin{vmatrix} 12+9i & -i \\ i & 12-9i \end{vmatrix} = (12 + 9i)(12 - 9i) + i^2$

$$= (12^2 - 9^2 i^2) + i^2$$
$$= 225 - 1$$
$$= 224$$

$\therefore$ Inverse of $M = M^{-1} = \frac{1}{|M|}(adjM) = \frac{1}{224}\begin{bmatrix} 12-9i & i \\ -i & 12+9i \end{bmatrix}$

12. (c)

$$\begin{vmatrix} 1-\lambda & 2 \\ p & 5-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)(5 - \lambda) - 2p = 0$$

$$\lambda^2 - 6\lambda + 5 - 2p = 0$$

Let the roots are λ_1 and λ_2.

From the characteristic equation,

$$\lambda_1 + \lambda_2 = 6$$

$$\lambda_1\lambda_2 = 5 - 2p \geq 0 \qquad \text{[For roots to be positive]}$$

$$p \leq \frac{5}{2} \qquad \text{... (i)}$$

For roots to be real,

$$6^2 - 4(5-2p) \geq 0$$

$$36 - 20 + 8p \geq 0$$

$$p \geq -2 \quad \text{... (ii)}$$

From equations (i) and (ii),

$$-2 \leq p \leq \frac{5}{2}$$

13. (b)

For non-trivial solution to exist,

$$\begin{vmatrix} 1 & -2 & 1 \\ k & -1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$-1 - 8 - k + 2 + 2 + 2k = 0$$

$$k = 5$$

For $k = 5$,

$$x - 2y + z = 0$$

$$5x - y + 2z = 0$$

$$\frac{x}{-4+1} = \frac{y}{5-2} = \frac{z}{-1+10}$$

$$\frac{x}{-3} = \frac{y}{3} = \frac{z}{9}$$

$$x : y : z = -1 : 1 : 3$$

14. (b)

At extremum value $\quad \dfrac{dy}{dx} = 0$

So, $\quad \dfrac{dy}{dx} = \dfrac{p}{(x-4)(x-1)} - \dfrac{(px+q)[(x-4)+(x-1)]}{(x-4)^2(x-1)^2} = 0$

$x \neq 4,\ x \neq 1$

$$\frac{p(x-4)(x-1) - (px+q)\left[(x-4)+(x-1)\right]}{(x-4)^2(x-1)^2} = 0$$

At $x = 2$,

$p(2-4)(2-1) - (2p+q)\ [2-4+2-1] = 0$

$$-2p + 2p + q = 0$$

$$\Rightarrow \quad q = 0$$

$$y(2) = \frac{2p}{(2-4)(2-1)} = -1$$

$$p = 1$$

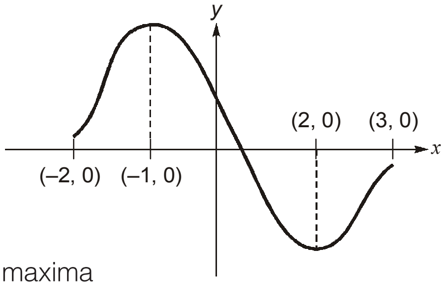
$\Rightarrow$ The value of p & q are 1 and 0 respectively.

15. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x+1)(x-2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$



The function has maxima at $x = -1$ and minima at $x = 2$.
The function is decreasing between –1 and 2.

16. (b)

$$f(x) = \frac{\log_e(1+ax) - \log_e(1-bx)}{x}$$

For function to be continuous

$$f(0) = \lim_{x \to 0} f(x)$$
$$= \lim_{x \to 0} \frac{\log_e(1+ax) - \log(1-bx)}{x} = \lim_{x \to 0} \frac{\log_e(1+ax) \times a}{ax} + \frac{\log(1-bx) \times b}{-bx}$$
$$= a + b$$

17. (d)

$$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$$
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 3x^2 - 3y^2 + 6x$$
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = -6xy - 6y$$
$$dv = \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$
$$= (6xy + 6y)dx + (3x^2 - 3y^2 + 6x)dy$$
$$v = 3x^2 y + 6xy - y^3 + C$$

18. (b)

$$\phi_1 = ax^2 - byz - (a+2)x$$
$$\nabla \phi_1 = [2ax - (a+2)]\hat{i} - bz\hat{j} - by\hat{k}$$
$$\nabla \phi_1(1, -1, 2) = (a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$
$$\phi_2 = 4x^2 y + z^3 - 4$$
$$\nabla \phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$
$$\nabla \phi_2(1, -1, 2) = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

Since surfaces are orthogonal to each other at (1, –1, 2)

$$\nabla \phi_1 \cdot \nabla \phi_2 = 0$$
$$[(a-2)\hat{i} - 2b\hat{j} + b\hat{k}] \cdot [-8\hat{i} + 4\hat{j} + 12\hat{k}] = 0$$

$$-8(a-2) - 8b + 12b = 0 \quad \text{... (i)}$$

Also point (1, –1, 2) lies on the surface.

$$\Rightarrow \quad a \times 1 + 2b = (a+2)1$$

$$b = 1$$

Putting this in equation 1, we get,

$$-8(a-2) - 8 + 12 = 0$$

$$a - 2 = -\frac{1}{8} \times (-4) = 0.5$$

$$a = 2.5$$

19. (b)

z varies from 0 to $\frac{x^2+y^2}{4}$; y varies from 0 to $\sqrt{16-x^2}$; x varies from 0 to 4.

$$\text{Volume} = \iiint dxdydz = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\frac{x^2+y^2}{4}} dzdydx$$

$$= \frac{1}{4}\int_0^4 \int_0^{\sqrt{16-x^2}} (x^2+y^2)dydx = \frac{1}{4}\int_0^4 \left(x^2 y + \frac{y^3}{3} \right)\Bigg|_0^{\sqrt{16-x^2}} dx$$

$$= \frac{1}{4}\int_0^4 \left(x^2\sqrt{16-x^2} + \frac{\left(\sqrt{16-x^2}\right)^3}{3} \right) dx$$

Let, $\quad x = 4\sin\theta \qquad x \to 0 \text{ to } 4$

$$dx = 4\cos\theta d\theta \qquad \theta \to 0 \text{ to } \frac{\pi}{2}$$

$$\text{Volume} = \frac{1}{4}\left[4^4 \int_0^{\pi/2} \sin^2\theta \cos^2\theta d\theta + \frac{4^4}{3}\int_0^{\pi/2} \cos^4\theta d\theta \right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{\left|\frac{3}{2} \times \right|\frac{3}{2}}{2\left|\frac{6}{2}\right.} + \frac{4^4}{3} \times \frac{\left|\frac{5}{2} \times \right|\frac{3}{2}}{2\left|\frac{6}{2}\right.} \right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2!}\pi + \frac{4^4}{3} \times \frac{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2!} \times \pi \right]$$

$$= \frac{1}{4}[16\pi + 16\pi] = 8\pi = 25.13 \text{ unit}^3$$

20. (c)

$$\frac{dT}{dt} = k(T-25)$$

T = Temperature of the body in °C and t = time in minutes.

$$\frac{dT}{T-25} = kdt$$

$$\log(T-25) = kt + C_1$$

$$T - 25 = Ce^{kt}$$

At $t = 0$, $T = 100$°C and at $t = 1$ minute, $T = 75$°C.

$$(100-25) = Ce^0$$

$$\Rightarrow \qquad C = 75°C$$

$$50 = 75\, e^k \quad \Rightarrow \quad e^k = \frac{2}{3}$$

At $t = 5$ minutes, $\quad T - 25 = 75\, e^{k \times 5}$

$$T = 25 + 75 \times \left(\frac{2}{3}\right)^5 \approx 34.88°C$$

21. (d)

$$AX = B$$

Augmented matrix, $[A : B] = \begin{bmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 1 & 1 & -2 & : & n \end{bmatrix}$

$R_3 \to R_3 + R_2 + R_1$:

$$|A : B| = \begin{vmatrix} -2 & 1 & 1 & : & l \\ 1 & -2 & 1 & : & m \\ 0 & 0 & 0 & : & l+m+n \end{vmatrix}$$

Since, $\quad l + m + n = 0$

Rank of $[A : B] = 2$

Rank of $[A]$ = Rank of $[A : B] = 2 < 3$ (Number of variables)

$\Rightarrow$ Infinitely many solutions are possible.

22. (d)

$$c(y + c)^2 = x^3 \qquad \text{...(i)}$$

Differentiating, we get

$$2c(y+c)\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{2x^3}{(y+c)^2}(y+c)\frac{dy}{dx} = 3x^2 \qquad \left(\because c = \frac{x^3}{(y+c)^2}\right)$$

$$\Rightarrow \quad \frac{2x^2}{y+c}\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \quad \frac{2x}{3}\left(\frac{dy}{dx}\right) = y + c$$

$$\Rightarrow \quad c = \frac{2x}{3}\left(\frac{dy}{dx}\right) - y$$

Putting this value of 'c' in equation (i)

$$\left[\frac{2x}{3}\left(\frac{dy}{dx}\right) - y\right]\left[\frac{2x}{3}\frac{dy}{dx}\right]^2 = x^3$$

23. (c)

Suppose $$y = \lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$$

$$\Rightarrow \quad y = \lim_{x\to\infty}\left[\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{\left[\frac{5(x+4)}{x+1}\right]}$$

$$\Rightarrow \quad \ln y = \lim_{x\to\infty}\frac{5(x+4)}{(x+1)}\ln\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}} \qquad ...(i)$$

$\lim_{x\to\infty}\frac{5(x+4)}{(x+1)}$ is in the form of $\frac{\infty}{\infty}$ and $\lim_{x\to\infty}\ln\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}$ is in the form of 0^0.

Calculating the limits of both terms separately

$$\lim_{x\to\infty}5\frac{(x+4)}{(x+1)} = \lim_{x\to\infty}5\frac{\left(1+\frac{4}{x}\right)}{\left(1+\frac{1}{x}\right)} = 5\frac{(1+0)}{(1+0)}$$

$$= 5$$

We can use direct result of $\lim_{t\to 0}(1+t)^{1/t} = e$ ...(ii)

$$\Rightarrow \quad \lim_{x\to\infty}\ln\left[1+\frac{5}{x+1}\right]^{\frac{x+1}{5}} = \ln(e)$$

$$= 1 \qquad ...(iii)$$

$$\therefore \quad \ln y = 5(1)$$

$$\Rightarrow \quad y = e^5$$

24. (a)

Required probability = Chosen a defective bolt from Machine A + Chosen a defective bolt from Machine B + Chosen a defective bolt from Machine C

$$= \frac{1}{2}\times\frac{2}{100}+\frac{1}{4}\times\frac{2}{100}+\frac{1}{4}\times\frac{4}{100} = 0.25\times10^{-1} = 0.025$$

25. (a)

$$\text{Probability} = \int_2^\infty f(x)dx$$

$$= \int_2^\infty\left[\frac{1}{2}e^{\frac{-x}{2}}\right]dx$$

$$= \left[-e^{\frac{-x}{2}}\right]_2^\infty = e^{-1} = 0.368$$

26. (c)

$$\sin y \frac{dy}{dx} = \cos y\,(1 - x\cos y)$$

$$\Rightarrow \quad \sin y \frac{dy}{dx} - \cos y = -x\cos^2 y$$

$$\Rightarrow \quad \frac{\sin y}{\cos^2 y}\frac{dy}{dx} - \frac{\cos y}{\cos^2 y} = -x$$

$$\Rightarrow \quad \tan y \sec y \frac{dy}{dx} - \sec y = -x \qquad ...(i)$$

Let $\sec y = t$

$$\Rightarrow \quad \sec y \tan y \frac{dy}{dx} = \frac{dt}{dx} \qquad ...(ii)$$

From equation (i) and (ii), we get

$$\frac{dt}{dx} - t = -x$$

$$\text{Integrating factor} = e^{\int -dx} = e^{-x}$$

Multiplying the equation by the integrating factor

$$\Rightarrow \quad e^{-x}\frac{dt}{dx} - e^{-x}t = -xe^{-x}$$

$$\Rightarrow \quad te^{-x} = \int -xe^{-x}dx = xe^{-x} + e^{-x} + c$$

$$\Rightarrow \quad t = (x + 1) + ce^{x}$$

$$\therefore \quad \sec y = (x + 1) + ce^{x}$$

27. (c)

$$f(t) = (t + 1)^2$$

$$\Rightarrow \quad \text{Laplace}\{f(t)\} = L(t^2) + L(2t) + L(1)$$

$$= \frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}$$

28. (c)

The matrix formed by the coefficients is $\begin{bmatrix} a & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & a \end{bmatrix}$

$$\text{Determinant} = 2a^2 - 2a - 4$$

$$\therefore \quad D = 0 \text{ for } a = 2 \text{ or } a = -1$$

(A) If $D \neq 0$, then the system will have unique solution.

(B) If $a = 2$, the matrix formed by the coefficients is $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

The rank of matrix is 2.
Considering 'z' as side unknown.

The characteristic determinant will be $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this is 0.
The system will have infinite solutions when $a = 2$.

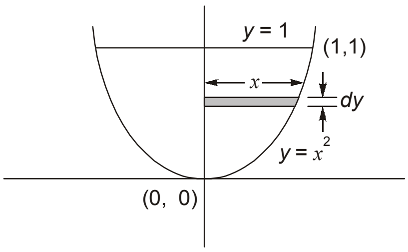
(C) If $a = -1$, the matrix formed by the coefficients is $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Its rank is 2.
Considering 'z' as side unknown.

The characteristic matrix is $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & b \\ 2 & 1 & 0 \end{bmatrix}$

The determinant of this matrix is $3b$.
The system will have no solution if $b \neq 0$
$\therefore$ For $a = -1$ and $b \neq 0$, the system will have no solution.

29. (b)



$y = x^2$ and $y = 1$ intesect at (1, 1)
Small disk of radius 'x' and depth 'dy' are integrated to compute the volume

$$\text{Volume} = \int_0^1 \pi x^2 dy$$

$$= \int_0^1 \pi y\, dy = \pi \left[\frac{y^2}{2}\right]_0^1 = \frac{\pi}{2} \qquad (\because \ y = x^2)$$

30. (a)

$$\text{Required probability} = \frac{\text{Favorable outcomes}}{\text{Total possible outcomes}}$$

Favorable outcomes = A false coin is chosen and flipped every time

Probability of selecting a false coin = $\frac{1}{4}$

Probability of getting a tail on every flip of false coin = 1.

$\therefore$ Favorable outcomes = $\frac{1}{4} \times 1 = \frac{1}{4}$

Total possible outcomes = Favourable outcomes + Unfavourable outcomes

Unfourable outcomes = A fair coin is chosen and flipped everytime to get tail

Probability of selecting a fair coin = $\frac{3}{4}$

Probability of flipping a fair coin 4 times and getting

tails every time = $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$\therefore$ Unfavourable outcomes = $\frac{3}{4} \times \frac{1}{16} = \frac{3}{64}$

Total possible outcomes = $\frac{1}{4} + \frac{3}{64} = \frac{19}{64}$

$\therefore$ Required probability = $\frac{\frac{1}{4}}{\frac{19}{64}} = \frac{16}{19} \approx 0.84$

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