

CLASS TEST

S.No. : 06 SP1_ME_U+I_230619

Engineering Mathematics



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CLASS TEST 2019-2020

MECHANICAL ENGINEERING

Date of Test : 23/06/2019

ANSWER KEY > Engineering Mathematics

1. (d)	7. (c)	13. (b)	19. (b)	25. (c)
2. (b)	8. (a)	14. (d)	20. (c)	26. (d)
3. (d)	9. (d)	15. (c)	21. (c)	27. (b)
4. (d)	10. (b)	16. (d)	22. (d)	28. (a)
5. (a)	11. (c)	17. (a)	23. (b)	29. (a)
6. (d)	12. (c)	18. (b)	24. (d)	30. (a)

DETAILED EXPLANATIONS

1. (d)

$$\begin{aligned} \text{rank of } [AB] &\leq \text{rank of } [A] \\ \text{rank of } [AB] &\leq \text{rank of } [B] \\ \text{rank of } [AB] &\leq \min[\text{rank of } A, \text{rank of } B] \end{aligned}$$

2. (b)

eigen values of $(A + 5I)$ are $\alpha + 5$ and $\beta + 5$

$$\text{eigen values of } (A + 5I)^{-1} = \frac{1}{\alpha + 5} \text{ and } \frac{1}{\beta + 5}$$

3. (d)

Let
$$I = \lim_{t \rightarrow 0} \frac{1}{t} \cdot \int_0^t f(x) dx$$

By applying L-Hospital's rule

$$\begin{aligned} I &= \lim_{t \rightarrow 0} \frac{f(t)}{1} \\ &= f(0) \end{aligned}$$

4. (d)

(a) If A and B are independent

$$P(A \cap B) = P(A) \cdot P(B)$$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(c) If A and B are mutually exclusive

$$P(A \cap B) = 0$$

(d)
$$P(B) = \frac{P(A \cap B)}{P(A)} \leq 1$$

$$P(A \cap B) \leq P(A)$$

5. (a)

Auxillary equation

$$D^4 + 2D^2 + 1 = 0$$

$$(D^2 + 1)^2 = 0$$

$$D = \pm i, \pm i$$

$$CF = (C_1 + C_2x) \cos x + (C_3 + C_4x) \sin x$$

6. (d)

Given function is $y = \frac{1}{x}$ [hyperbolic function]

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

hence, option (d) is correct.

7. (c)

Since $\lim_{x \rightarrow 0} (1+0)^0 = 1^\infty \rightarrow$ indeterminate

$$f(0) = \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}$$

$$\begin{aligned} \ln f(0) &= \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{\ln(1 + \sin x)}{x} \quad (\text{Apply L' Hospital rule}) \end{aligned}$$

$$\ln f(0) = \lim_{x \rightarrow 0} \frac{\cos x}{1 + \sin x} \times \frac{1}{1}$$

$$= \frac{1}{1+0} = 1$$

$$\ln f(0) = 1$$

$$f(0) = e^1 = e$$

8. (a)

minimum (2, 3) \Rightarrow highest possible rank = 2

if rank of $A = 2$, it will consistent. In order to be inconsistent, maximum rank of A is '1'.

9. (d)

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{\pi}{4}$$

10. (b)

$$\frac{\ln(1+i\sqrt{3})}{2} = \frac{\ln\left(2e^{\frac{i\pi}{3}}\right)}{2} = \frac{\ln 2 + \frac{i\pi}{3}}{2}$$

$$= \frac{\ln 2}{2} + \frac{i\pi}{6} = \frac{0.693}{2} + \frac{i\pi}{6} = 0.35 + \frac{i\pi}{6}$$

11. (c)

$$\frac{dy}{dx} - y \cos x = \sin x \cos x$$

$$\text{IF} = e^{-\int \cos x dx} = e^{-\sin x}$$

$$ye^{-\sin x} = \int \sin x \cos x e^{-\sin x} dx$$

$$ye^{-\sin x} = -(1 + \sin x)e^{-\sin x} + C_0$$

$$y + 1 + \sin x = C_0 e^{\sin x}$$

12. (c)

For $f(x)$ to be probability density function $\Rightarrow \int_{-\infty}^{\infty} f(x) dx = 1$

$$\frac{1}{A} \int_2^4 (2x+3) dx = 1$$

$$\frac{1}{A} \left[2 \frac{x^2}{2} + 3x \right]_2^4 = 1$$

$$\begin{aligned} A &= (4^2 - 2^2) + 3(4 - 2) \\ &= 16 - 4 + 3 \times 2 = 18 \end{aligned}$$

13. (b)

Let $\tan^{-1}(x) = \theta$, $x = \tan \theta$

$$g(x) = \tan^{-1} \left(\frac{\sqrt{1 + \tan^2 \theta} - 1}{\tan \theta} \right) = \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right) = \tan^{-1} \left(\frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right) = \frac{\theta}{2}$$

$$\frac{df(x)}{dg(x)} = \frac{d\theta}{d(\theta/2)} = 2$$

14. (d)

$$A^{-1} = \frac{(\text{adj } A)}{|A|}$$

$$|A| = -6 \times 3 = -18$$

$$|A| \cdot (A^{-1}) = (\text{adj } A)$$

$$\lambda \text{ of adj } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-18}{-6}, \frac{-18}{3}$$

$$= 3, -6$$

15. (c)

Here

$$u = e^{xyz}$$

 \Rightarrow

$$\frac{\partial u}{\partial x} = e^{xyz} \cdot yz$$

$$\begin{aligned} \frac{\partial^2 u}{\partial x \partial y} &= ze^{xyz} + yze^{xyz} \cdot xz \\ &= e^{xyz} (z + xyz^2) \end{aligned}$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz} (1 + 3xyz + x^2 y^2 z^2)$$

16. (d)

$$f(z) = u + iv$$

$$|f(z)| = \sqrt{u^2 + v^2} = c \quad (\text{constant given})$$

$$u^2 + v^2 = c_1 \quad \dots(i)$$

differentiating equation (i) with respect to x

$$2u \frac{\partial u}{\partial x} + 2v \frac{\partial v}{\partial x} = 0 \quad \left[\begin{array}{l} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \end{array} \right]$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} = 0 \quad \dots(ii)$$

differentiating equation (i) with respect to y

$$\text{similarly} \quad u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} = 0$$

$$-u \frac{\partial v}{\partial x} + v \frac{\partial u}{\partial x} = 0 \quad \dots(iii)$$

squaring and adding equation (ii) and (iii)

$$(u^2 + v^2) \left(\frac{\partial u}{\partial x} \right)^2 + (u^2 + v^2) \left(\frac{\partial v}{\partial x} \right)^2 = 0$$

$$(u^2 + v^2) \left(\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 \right) = 0$$

$$\text{since} \quad u^2 + v^2 \neq 0$$

$$\Rightarrow \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} \right)^2 = 0$$

$$\text{similarly} \quad \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 = 0$$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x}$$

$$\overline{f'(z)} = \frac{\partial u}{\partial x} - i \frac{\partial v}{\partial x}$$

$$|f'(z)|^2 = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 = 0$$

$$|f'(z)|^2 = 0$$

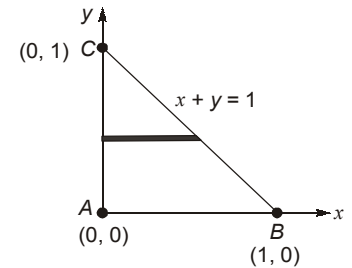
17. (a)

$$I = \int_0^1 \int_0^{1-x} 3y \, dy \, dx$$

$$= \int_0^1 \left[\frac{3y^2}{2} \right]_0^{1-x} dx$$

$$= \int_0^1 \frac{3}{2} (1-x)^2 dx$$

$$= -\frac{3}{2} \frac{(1-x)^3}{3} \Big|_0^1 = -\frac{1}{2} \frac{(1-x)^3}{1} \Big|_0^1 = \frac{1}{2}$$



18. (b)

$$M = (1 + xy)y$$

and

$$N = (1 + xy)x$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 4xy \neq 0$$

$$M_x - N_y = 2x^2 y^2 \neq 0$$

$$\frac{1}{M_x - N_y} = \frac{1}{2x^2 y^2}$$

$$\text{IF (Integrating factor)} = \frac{1}{2x^2 y^2}$$

$$\left(\frac{1}{x^2 y} + \frac{1}{x}\right) dx + \left(\frac{1}{xy^2} - \frac{1}{y}\right) dy = 0 \quad \dots(i)$$

equation (i) is exact

$$\frac{1}{y} \int \frac{1}{x^2} dx + \int \frac{1}{x} dx - \int \frac{1}{y} dy = C$$

$$\frac{-1}{xy} + \log x - \log y = C$$

or $\log\left(\frac{x}{y}\right) - \frac{1}{xy} = C$

19. (b)

$$\begin{aligned}
 Z &= \frac{(2-3i)(1+i)}{2+i} = \frac{(2-3i)(1-i)}{2+i} = \frac{-1-5i}{2+i} \\
 &= \frac{(-1-5i)(2-i)}{(2+i)(2-i)} = \frac{-7-9i}{5} = -\frac{7}{5} - \frac{9}{5}i \\
 |Z| &= \sqrt{\left(-\frac{7}{5}\right)^2 + \left(-\frac{9}{5}\right)^2} = \sqrt{\frac{130}{25}} = \sqrt{\frac{26}{5}}
 \end{aligned}$$

20. (c)

$$(D^2 + 1)y = \sin x$$

$$PI = \frac{\sin x}{D^2 + 1}$$

putting $D^2 = -1$

$$PI = \frac{\sin x}{-1+1} \quad [\text{Makes denominator zero}]$$

∴ Differentiating numerator and denominator

$$PI = x \cdot \frac{\sin x}{2D}$$

$$PI = \frac{1}{2} x \int \sin x \, dx$$

$$PI = -\frac{1}{2} x \cos x$$

21. (c)

$$\operatorname{Re} z = \frac{z + \bar{z}}{2}$$

$$\operatorname{Re}\{\tan^{-1}(z)\} = \frac{\tan^{-1}(x+iy) + \tan^{-1}(x-iy)}{2}$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{x+iy+x-iy}{1-(x+iy)(x-iy)}\right) = \frac{1}{2} \tan^{-1}\left(\frac{2x}{1-x^2-y^2}\right)$$

22. (d)

$$\vec{A} \times \vec{B} \times \vec{C} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

$$\therefore \nabla \times \nabla \times \vec{A} = (\nabla \cdot \vec{A})\nabla - (\nabla \cdot \nabla)\vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

23. (b)

$$x = (N)^{1/N}$$

$$x^N = N$$

$$x^N - N = 0$$

$$f(x) = x^N - N$$

$$f'(x) = N \cdot x^{N-1}$$

Newton Raphson Iteration

$$x_{k+1} = x_k - \frac{f(k)}{f'(k)} = x_k - \frac{x_k^N - N}{N x_k^{N-1}} = \frac{N x_k^N - x_k^N + N}{N x_k^{N-1}}$$

$$x_{k+1} = \left(\frac{N-1}{N} \right) x_k + x_k^{1-N}$$

24. (d)

$$[A]^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$$

$$\alpha^2 = 1 \quad ; \quad \alpha + 1 = 5$$

$$\alpha = \pm 1 \quad ; \quad \alpha = 4$$

Unique value of α is not possible.

25. (c)

$$\text{Let } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} \cdot \hat{i} = x, \quad \vec{r} \cdot \hat{j} = y, \quad \vec{r} \cdot \hat{k} = z$$

$$A = x(\vec{r} \times \hat{i}) + y(\vec{r} \times \hat{j}) + z(\vec{r} \times \hat{k})$$

$$= (\vec{r} \times x\hat{i}) + (\vec{r} \times y\hat{j}) + (\vec{r} \times z\hat{k}) = \vec{r} \times (x\hat{i} + y\hat{j} + z\hat{k}) = \vec{r} \times \vec{r}$$

$$A = 0 \quad (\text{always})$$

26. (d)

Rearranging the equation

$$\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$$

The equation is of the form

$$\frac{dy}{dx} + p(x)y = Q(x)$$

$$IF = e^{\int p(x)dx} = e^{\int \frac{-1}{x+1} dx} = e^{-[\ln(x+1)]}$$

$$= \frac{1}{x+1}$$

27. (b)

By Euler theorem

$$x \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y} = nz$$

$$x^2 \frac{\partial^2 Z}{\partial x^2} + 2xy \frac{\partial^2 Z}{\partial x \partial y} + y^2 \frac{\partial^2 Z}{\partial y^2} = n(n-1)z$$

28. (a)

$$f'(x) = 2x - 1$$

For minima/maxima

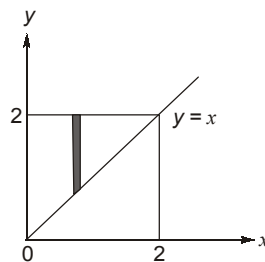
$$f'(x) = 0$$

$$2x - 1 = 0$$

$$x = \frac{1}{2}$$

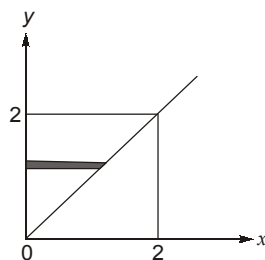
$$f''(x) = 2 > 0 \Rightarrow \text{minima}$$

29. (a)



$$x < y < 2$$

$$0 < x < 2$$



$$0 < x < y$$

$$0 < y < 2$$

$$I = \int_0^2 \int_0^{2-y} f(x,y) dx dy$$

$$r = p = 0$$

$$q = y$$

$$s = 2$$

30. (a)

Since the probability of occurrence is very small, this follows Poisson distribution

$$\begin{aligned} \text{mean} = m &= np \\ &= 2000 \times 0.001 = 2 \end{aligned}$$

Probability that more than 2 will get a bad reaction

$$= 1 - p(0) - p(1) - p(2)$$

$$= 1 - \left[e^{-m} + \frac{e^{-m} \cdot m^1}{1!} + \frac{e^{-m} \cdot m^2}{2!} \right]$$

$$= 1 - \left[e^{-2} + \frac{e^{-2} \cdot 2}{1} + \frac{2^2 \cdot e^{-2}}{2} \right] = 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2} \right] = 1 - \frac{5}{e^2}$$

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