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NETWORK THEORY

ELECTRICAL ENGINEERING

Date of Test: 04/09/2022

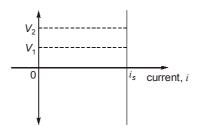
ANSWER KEY 1. (c) 7. (d) 13. (c) 19. (c) 25. (a) 2. (a) (c) 14. (a) 20. (b) 26. (b) 3. 15. (d) 21. (d) 27. (d) (b) (c) 4. (a) 10. (d) 16. (b) 22. (a) 28. (b) 5. (b) 11. (a) 17. (b) 23. (b) 29. (c) 12. (d) 6. (b) 18. (b) 24. (a) 30. (b)

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DETAILED EXPLANATIONS

1. (c)

The ideal independent current source is a two terminal element which supplies its specified current to the circuit in which it is placed independently of the value and direction of the voltage appearing across its terminals.



2. (a)

When switch s is closed, the equivalent resistance

$$R_{\text{eq}} = \left(\frac{R_1 \times R_3}{R_1 + R_3}\right) \text{ is less than } R_1 \text{ or } R_3 \text{ individually}$$

$$V_1 \qquad V_2 \qquad V_2 \qquad V_3 \qquad V_4 \qquad V_4 \qquad V_4 \qquad V_5 \qquad V_7 \qquad V_8 \qquad V_9 \qquad V$$

for constant current, $V \propto R$.

So incase of $R_{\rm eq}$, the resistance is decreased so voltage V_1 is decreased so obviously V_2 is increased.

3. (b)

Across the load
$$Z_L$$
, $Z_{TH} = \frac{(-j40)(80 + j100)}{80 + j60} = 12.8 - j49.6 \Omega$

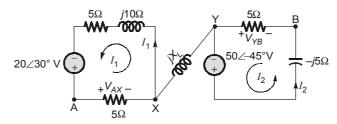
For maximum average power transfer

$$Z_L = Z_{\text{TH}}^*,$$

: $Z_L = 12.8 + j49.6 \text{ W}$

4. (a)

The given figure with current direction is shown below.



We can notice that no current will flow through the inductor jX_L .

Thus,
$$V_{XY} = 0 \text{ V}$$



Now, using KVL, from mesh-1, we have:

$$I_1 = \frac{20\angle 30^{\circ}}{10 + i10} = 1.414\angle - 15^{\circ}A$$

Using KVL, from mesh-2

and
$$I_2 = \frac{50 \angle -45^\circ}{5-j5} = 7.07 \angle 0^\circ A$$
 Thus,
$$V_{AX} = I_1 \ (5) = (1.414 \ \angle -15^\circ) \times 5$$

$$V_{AX} = 7.07 \ \angle -15^\circ \ V$$
 and
$$V_{YB} = -I_2 (5) = -(7.07 \ \angle 0^\circ) \ 5$$

$$= -35.35 \ \angle 0^\circ \ V$$
 Hence,
$$V_{AB} = V_{AX} + V_{XY} + V_{YB} = V_A - V_B$$

$$= 7.07 \ \angle -15^\circ + 0 - 35.35 \ \angle 0^\circ$$

$$V_{AB} = -28.52 - j1.83 = 28.579 \angle -176.3^\circ \ V \approx 28.6 \ \angle 183.7^\circ \ V$$

5. (b)

Since the network is passive, current I is due to the two sources V_1 and V_2 . By principle of superposition,

$$I = K_1V_1 + K_2V_2$$

$$1 = 4K_1 + 0$$

$$K_1 = 0.25$$
and
$$-1 = 0 + 5K_2$$

$$K_2 = \frac{-1}{5} = -0.2$$
Given,
$$V_1 = 10 \text{ V}$$

$$V_2 = 5 \text{ V}$$

$$Current, I = (10 \times 0.25) + (5 \times -0.2)$$

$$I = 2.5 - 1 = 1.5 \text{ A}$$

6. (b)

From given figure, we have:

$$I_{\Delta ph} = \frac{V_{ph}}{100\Omega} = \frac{400}{100} = 4 \,\mathrm{A}$$

Therefore,

$$P_{\Delta} = 4^2 \times 100$$

$$P_{\Delta} = 1600 \text{ W}$$

For star connected resistors,

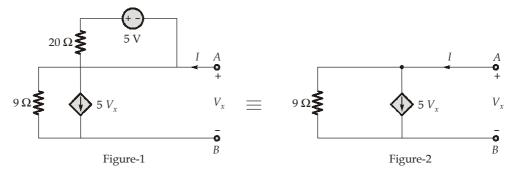
$$P_{Y} = 3I_{ph}^{2}R$$
 Now,
$$I_{Yph} = \frac{V_{Yph}}{R}$$

$$I_{Yph} = \frac{400/\sqrt{3}}{R}$$
 Given,
$$P_{A} = P_{Y}$$

or,
$$1600 = \left(\frac{400 / \sqrt{3}}{R}\right)^2 \times 3 \times R$$
or,
$$R = 100 \Omega$$

7. (d)

For calculation of R_{Th} , the circuit can be redrawn as



Using KCL in figure-2, we get,

$$\frac{V_x}{9} + 5V_x = I$$

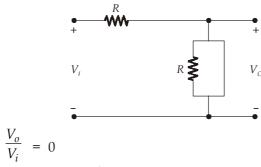
$$46V_x = 9I$$

$$\frac{V_x}{I} = \frac{9}{46} = 0.195 \Omega \approx 195.65 \text{ m}\Omega$$

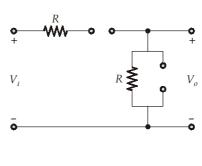
8. (c)

or

At $\omega \to \infty$, Capacitor \to short circuited Circuit looks like,

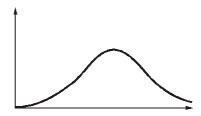


At $\omega \to 0$, Capacitor \to open circuited Circuit looks like



So frequency response of the circuit will be,

 $\frac{V_0}{V_i} = 0$



So the circuit is Band pass filter

9.

Current is lagging voltage by 53.13°,

$$\phi = 53.13^{\circ}$$

 \therefore the elements will be *R* and *L*

$$\tan \phi = \frac{\omega L}{R}$$

$$\tan (53.13^{\circ}) = 1.33 = \frac{5000L}{R} \qquad ...(i)$$

[Since $\omega = 5000 \text{ rad/sec}$]

$$\frac{V}{I} = |Z| = \frac{50}{10} = \sqrt{R^2 + (\omega L)^2}$$

$$5 = \sqrt{R^2 + (\omega L)^2}$$

$$25 = R^2 + (5000L)^2 \qquad \dots(ii)$$

From equation (i),

$$L = \frac{R \times 1.33}{5000}$$
 substituting in equation (ii),

$$25 = R^2 + (R \cdot 1.33)^2$$

$$R = 3 \Omega$$

Substituting in equation (ii), we get

$$L = \frac{3 \times 1.33}{5000} = 0.8 \text{ mH}$$

10. (d)

From the first circuit

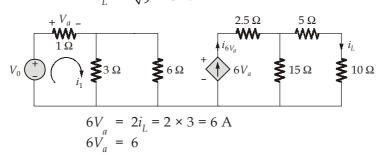
$$V_0 = i_1 \left[1 + \frac{3 \times 6}{3 + 6} \right] = 3i_1$$
 ...(i)

From the second circuit,

Power across $10 \Omega = 90 W$.

$$i_L^2 \times 10 = 90$$

 $i_L = \sqrt{9} = 3A$



Hence

From the figure,
$$V_a = 1 \text{ V}$$

 $V_a = i_1 \times 1 = i_1$
So, $i_1 = 1 \text{ A}$
From equation (i), $V_0 = 3i_1 = 3 \times 1 = 3 \text{ V}$

$$i_1 = 1$$

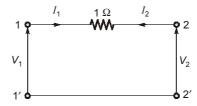
$$V_0 = 3i_1 = 3 \times 1 = 3 \text{ V}$$

11.

For parallel connected 2 ports.

$$[Y] = [Y_1] + [Y_2]$$

$$[Y_1] = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$$



$$I_1 = -I_2$$

$$V_1 = I_1 + 1$$

$$\begin{split} I_1 &= -I_2 \\ V_1 &= I_1 + V_2 \\ I_1 &= V_1 - V_2 \\ I_2 &= V_2 - V_1 \end{split} \qquad ...(i)$$

or and

$$[Y_2] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

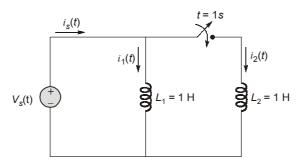
e can write
$$\begin{bmatrix} Y_2 \end{bmatrix} = \begin{bmatrix} -1 \end{bmatrix}$$

Now

$$[Y] = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 0 & 2 \end{bmatrix}$$

12. (d)

The initial conditions on the inductors, since no voltage is applied to either inductor for t < 0.

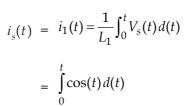


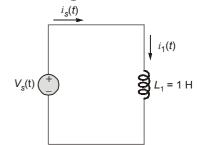
$$\Rightarrow$$

$$i_1(0) = i_2(0) = 0$$

Further, no voltage appears across the second inductor until $t \ge 1$. Hence $i_2(1) = 0$.

 $\Rightarrow i_1(t)$ for $0 \le t \le 1$;

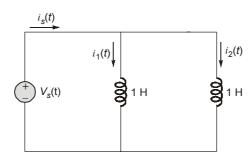




$$i_{s}(t) = i_{1}(t) = \sin(t) \text{ A}$$
 (for $t < 1$)

At t = 1, the switch is closed. The two inductors are then in parallel as shown below and the source voltage appears across each.

 $\Rightarrow i_1(t)$ and $i_2(t)$ for $t \ge 1$;



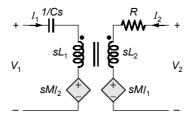
$$i_1(t) = i_1(1) + \int_1^t \cos(t) dt = \sin(1) + \sin(t) - \sin(1) = \sin(t) A$$

$$i_2(t) = i_2(1) + \int_1^t \cos(t) dt = \sin(t) - \sin(1) A$$

for KCL, the input current $i_s(t) = i_1(t) + i_2(t) = 2\sin(t) - \sin(1)$ A (for $t \ge 1$)

13. (c)

Drawing equivalent in s-domain



Applying KVL in loop I

$$V_1 = \left(\frac{1}{sC} + sL_1\right) I_1 + sMI_2$$

in loop II

$$V_2 = sMI_1 + (R + sL_2)I_2$$

comparing with standard equations

$$Z_{11} = \frac{1}{sC} + sL_1$$
 $Z_{12} = sM$ $Z_{21} = sM$ $Z_{22} = R + sL_2$

14. (a)

The voltage difference across capacitor is

$$V_d = V_a - V_b = \frac{30 \times 10}{30} - \frac{30 \times 2}{20}$$

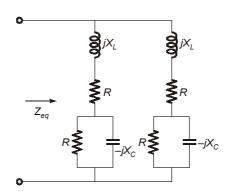
= 10 - 3
= 7 V

energy stored in capacitor =
$$\frac{1}{2}CV_d^2 = \frac{1}{2} \times 1 \times 10^{-6} \times 7^2$$

= 24.5 μJ

15. (d)

For current to be in phase with applied voltage imaginary part of impedance should be zero. Redrawing the circuit



$$Z_{eq} = \frac{\left(R + jX_L\right) + \left(R\| - jX_C\right)}{2} = \frac{\left(R + jX_L + \frac{-jR \cdot X_C}{R - jX_C}\right)}{2} = \frac{\left(R + jX_L - \frac{jR \cdot X_C}{R^2 + X_C^2}(R + jX_C)\right)}{2}$$

equating imaginary part to zero

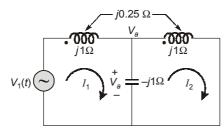
$$\frac{\left(X_{L} - \frac{R^{2}X_{C}}{R^{2} + X_{C}^{2}}\right)}{2} = 0$$

$$R^{2}X_{C} = X_{L} \cdot \left(R^{2} + X_{C}^{2}\right)$$

$$X_{L} = \frac{R^{2}X_{C}}{R^{2} + X_{C}^{2}}$$

16. (b)

Redrawing the circuit



$$jI_{1} + j\frac{1}{4}I_{2} - j(I_{1} - I_{2}) = V_{1}(t)$$

$$j\frac{5}{4}I_{2} = V_{1}(t)$$

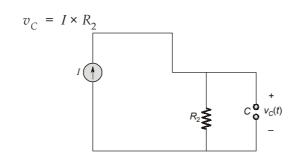
$$I_{2} = j\frac{4}{5} \cdot 2 \angle 0^{\circ} = 1.6 \angle -90^{\circ} A$$

$$jI_{2} + j\frac{1}{4}I_{1} - j(I_{2} - I_{1}) = 0$$

$$\Rightarrow$$
 $I_1 = 0$

 V_a = voltage across capacitor = voltage across inductor L_2 $=~I_2\times j1=j\times 1.6 \angle -90^\circ$ $= 1.6 \angle 0^{\circ} = 1.6 \cos t \text{ V}$

17. (b) at $t = 0^-$



at $t = 0^+$

$$v_{C}(0^{-}) = v_{C}(0^{+}) = IR_{2}$$

$$R_{1} R_{2} IR_{2}$$

$$R_{eq}$$

$$v_C(\infty) = 0$$

(: capacitor will discharge fully)

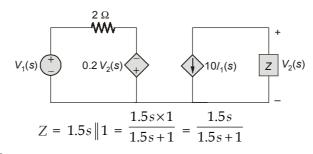
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$v_C(t) = [v_C(0) - v_C(\infty)] e^{-\frac{t}{\tau}} + v_C(\infty)$$

$$v_C(t) = IR_2 \cdot e^{\frac{-t(R_1 + R_2)}{R_1 R_2 \cdot C}} \text{Volts}$$

18. (b)

Redrawing the circuit



Applying KVL in loop *I*

$$\begin{split} V_1(s) &= 2I_1(s) - 0.2 \ V_2(s) \\ V_2(s) &= -10I_1(s) \times \frac{1.5s}{1.5 \ s + 1} \end{split} \ ...(i)$$

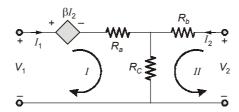


$$V_2(s) = \frac{-15 s I_1(s)}{1.5 s + 1} \qquad \dots (ii)$$

From equations (i) and (ii)

Input admittance =
$$\frac{I_1(s)}{V_1(s)} = \frac{1.5s + 1}{6s + 2}$$

19. (c)



Applying KVL

in loop 1

$$\beta I_2 + (R_a + R_c)I_1 + R_cI_2 = V_1$$

 $V_1 = (R_a + R_c)I_1 + (\beta + R_c)I_2$

Comparing this with standard equation

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$Z_{12} = \beta + R_c$$

20. (b)

When the switch is at position 1,

$$V_C(0^-) = 100 \text{ V}$$

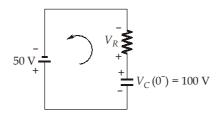
Voltage across capacitor at t > 0

$$V_C = V_{\infty} - (V_{\infty} - V_0)e^{-t/\tau}$$
 Where,
$$\tau = RC = 5000 \times 1 \times 10^{-6} = 5 \text{ msec}$$

$$V_C = (-50) - (-50 - 100)e^{-200t}$$

$$V_C = (150 e^{-200t} - 50) \text{ V}$$

Applying KVL in the loop



$$-V_C + V_R - 50 = 0$$

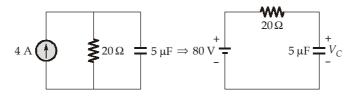
$$V_R = V_C + 50$$

$$= 150 e^{-200t} - 50 + 50$$

$$V_R = 150 e^{-200t} V$$

21. (d)

The circuit that exists for t < 0 is,



$$V = IR = 4 \times 20 = 80 \text{ V}$$

$$V_c(0^+) = V_c(0^-) = 80 \text{ V}$$
Time constant, $\tau = RC = 20 \times 5 \times 10^{-6} = 10^{-4} \text{ s}$
After $t = 0$,
$$V_c(t) = V_{\infty} - (V_{\infty} - V_0) e^{-t/\tau}$$

$$= 0 - (0 - 80) e^{-t/\tau}$$

$$V_c(t) = 80e^{-10^4t}$$

The half of initial voltage,

$$\frac{80}{2} = 40 \text{ V}$$

$$40 = 80e^{-10^4 t}$$

$$0.5 = e^{-10^4 t}$$
Time, $t = 69.3 \text{ us}$

22. (a)

:.

$$V_{ab} = V_{Th}$$

By applying KCL,

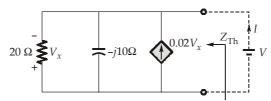
$$\frac{V_{Th} - 100}{20} + \frac{V_{Th}}{-j10} = 0.02 V_x$$

Where,

$$V_x = 100 - V_{\rm Th}$$

$$\frac{V_{Th} - 100}{20} + \frac{V_{Th}}{-j10} = 0.02(100 - V_{Th})$$

$$V_{\text{Th}} = \frac{7}{0.07 + j0.1} = 57.34 \angle -55^{\circ} \text{ V}$$



$$Z_{\text{Th}} = \frac{V}{I}$$

By apply KCL in the circuit gives,

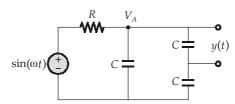
$$\frac{V}{20} + \frac{V}{-j10} - 0.02V_x = I$$

$$V_{x} = -V$$

$$\therefore \frac{V}{20} + \frac{V}{-j10} + 0.02V = I$$

$$Z_{\text{Th}} = \frac{V}{I} = \left[\frac{1}{\frac{1}{20} + \frac{1}{-j10} + 0.02} \right]$$
$$= (4.7 - j6.7)\Omega$$

23. (b)



Applying KCL at node A, we get

$$\frac{V_A - \sin \omega t}{R} + \frac{V_A}{\frac{1}{j\omega C}} + \frac{V_A}{\frac{2}{j\omega C}} = 0 \qquad \dots (i)$$

$$V_{A}\left[\frac{1}{R} + j\omega C + \frac{j\omega C}{2}\right] = \frac{\sin \omega t}{R} = \frac{1\angle 0^{\circ}}{R}$$

$$V_{A} = \frac{2}{2 + 3RC \cdot i\omega} \qquad \dots(ii)$$

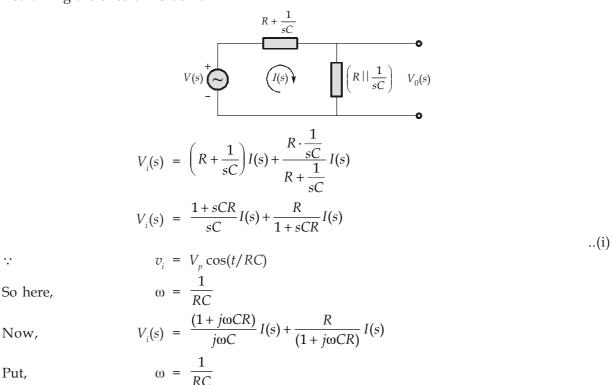
Also
$$Y = \frac{V_A}{2} = \frac{1}{2 + 3j\omega RC}$$
 ...(iii)

$$\therefore \qquad \frac{1}{4} = \frac{1}{\sqrt{4 + 9\omega^2 (RC)^2}}$$

or,
$$\omega = \frac{2}{\sqrt{3} RC}$$

24. (a)

Redrawing the circuit in s-domain



So,
$$V_{i}(s) = \left[\frac{(1+j)R}{j} + \frac{R}{1+j}\right]I(s)$$

$$\frac{V_{i}(s)}{I(s)} = \frac{3R}{(1+j)} \qquad ...(ii)$$

$$I(s) = \frac{V_{i}(s)}{3R} \times (1+j)$$
Now,
$$V_{0}(s) = \left(\frac{R}{sC}\right)I(s)$$

$$\Rightarrow V_{0}(s) = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}}I(s)$$

$$\Rightarrow V_{0}(s) = \frac{R}{1 + sCR} \cdot \frac{V_{i}(s)}{3R}(1+j)$$

$$\Rightarrow V_{0}(s) = \frac{R}{1+j} \cdot \frac{V_{i}(s)}{3R}(1+j)$$

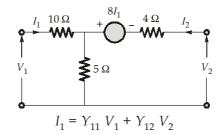
$$\Rightarrow V_{0}(s) = \frac{V_{i}(s)}{3R}$$
In time domain

In time domain,

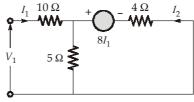
$$v_0(t) = \frac{1}{3}v_i(t)$$

$$v_0(t) = \frac{V_p}{3}\cos\left(\frac{t}{RC}\right)$$

25. (a)



Short-circuiting port-2 i.e. $V_2 = 0$, we get



Applying KVL in both loops, we get
$$V_1 = 10I_1 + 5I_1 + 5I_2 \qquad ...(i)$$
 and
$$4I_2 + 5I_2 + 5I_1 - 8I_1 = 0$$

$$\Rightarrow \qquad 9I_2 = 3I_1$$

$$\Rightarrow \qquad I_2 = \frac{I_1}{3} \qquad ...(ii)$$

From equations (i) and (ii); we get

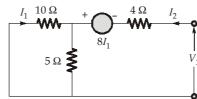
[Using equation (iii)]

$$V_1 = 10I_1 + 5I_1 + \frac{5I_1}{3}$$
$$= 15I_1 + \frac{5I_1}{3} = \frac{50I_1}{3}$$

$$\Rightarrow$$

$$\frac{I_1}{V_1}\Big|_{V_2=0} = Y_{11} = \frac{3}{50} \,\text{mho}$$

Short circuiting port-1; i.e. $V_1 = 0$, we get



Applying KVL in both loops; we have

$$10I_{1} + 5I_{1} + 5I_{2} = 0$$

$$15I_{1} = -5I_{2}$$

$$\Rightarrow I_{2} = -3I_{1} \qquad ...(iii)$$
and
$$V_{2} = 4I_{2} - 8I_{1} + 5I_{1} + 5I_{2}$$

$$\Rightarrow V_{2} = 9I_{2} - 3I_{1}$$

$$\Rightarrow V_{2} = 9(-3I_{1}) - 3I_{1}$$
[FIX: (iii)

$$\Rightarrow$$

$$V_2 = -30I_1$$

$$\therefore \frac{I_1}{V_2}\Big|_{V_1=0} = Y_{12} = -\frac{1}{30} \text{ mho}$$

26. (b)

The current lags the voltage by $50^{\circ} - 5^{\circ} = 45^{\circ}$

$$\omega L > \frac{1}{\omega C}$$

$$\tan 45^{\circ} = 1 = \frac{\omega L - \frac{1}{\omega C}}{R}$$
and
$$\frac{V_m}{I_m} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2}$$

$$\frac{100}{10} = \sqrt{2} R$$
or
$$R = 7.07 \Omega$$

$$\therefore \qquad R = \omega L - \frac{1}{\omega C}$$

$$\frac{1}{\omega C} = 314 \times 40 \times 10^{-3} - 7.07$$

$$\frac{1}{\omega C} = 12.56 - 7.07 = 5.49$$
or
$$C = \frac{1}{314 \times 5.49} \approx 580 \, \mu F$$

27. (d)

$$Z_{L} = R \| (-jX_{C}) = \frac{R(-jX_{C})}{R - jX_{C}} = \frac{R(-jX_{C})}{R - jX_{C}} \times \frac{R + jX_{C}}{R + jX_{C}}$$

$$= \frac{RX_{C}^{2}}{R^{2} + X_{C}^{2}} - j\frac{R^{2}X_{C}}{R^{2} + X_{C}^{2}} \qquad ...(i)$$

CT-2022-23

For maximum power transfer $Z_L = Z_s^*$

$$Z_s = 100 + j200$$

 $Z_L = 100 - j200$...(ii)

On comparing equation (i) and (ii), we get,

$$\frac{RX_C^2}{R^2 + X_C^2} = 100 \text{ and } \frac{R^2X_C}{R^2 + X_C^2} = 200$$

$$Ret X_C = 250 \Omega$$

on solving, we get, $X_C = 250 \Omega$

$$C = \frac{1}{500X_C} = 8 \,\mu\text{F}$$

and

$$R = 500 \Omega$$

28. (b)

Following switching, the differential equation representing the circuit is,

$$Ri(t) + \frac{1}{C} \int i(t) dt = V = 10$$

Taking Laplace transform, we get,

$$I(s) + \frac{I(s)}{2 \times 10^{-6} s} + \frac{Q_0}{2 \times 10^{-6} s} = \frac{10}{s}$$
 $\therefore Q_0 = -250 \,\mu\text{C}$

Therefore,

$$I(s) + \frac{I(s)}{2 \times 10^{-6} s} - \frac{250}{2s} = \frac{10}{s}$$

$$I(s) + \frac{I(s)}{2 \times 10^{-6} s} = \frac{135}{s}$$

$$I(s) = \frac{135}{(s + 5 \times 10^{5})}$$

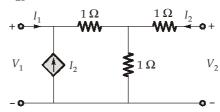
or

Taking inverse Laplace transform, we get,

$$i(t) = (135e^{-5 \times 10^5 t}) \text{ A}$$

29.

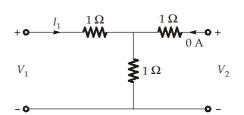
Let us first calculate z_{11} and z_{21} by open circuiting the output port,



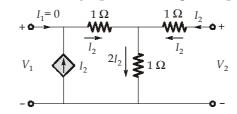
$$I_2 = 0$$

:. The circuit can be redrawn as

and $z_{11} = \frac{V_1}{I_1} = 2 \Omega$ *:*. $z_{21} = \frac{V_2}{I_1} = 1 \Omega$ and



Similarly z_{22} and z_{12} can be obtained by open circuiting the input port as,



and

$$V_{1} = I_{2} + 2I_{2} = 3I_{2}$$

$$V_{2} = I_{2} + 2I_{2} = 3I_{2}$$

$$z_{22} = \frac{V_{2}}{I_{2}} = 3 \Omega$$

$$z_{22} = \frac{V_2}{I_2} = 3 \,\Omega$$

and

$$z_{12} = \frac{V_1}{I_2} = 3 \Omega$$

z-parameter matrix = $\begin{bmatrix} 2\Omega & 3\Omega \\ 1\Omega & 3\Omega \end{bmatrix}$

30. (b)

For an RLC circuit,

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$
 ...(i)

and

$$f_2 - f_1 = \text{Bandwidth (BW)} = \frac{1}{2\pi} \times \frac{R}{L}$$
 ...(ii)

from equation (i) and (ii)

$$\frac{BW}{f_0^2} = \frac{\frac{1}{2\pi} \times \frac{R}{L}}{\frac{1}{4\pi^2} \times \frac{1}{LC}} = 2\pi RC$$

or

$$C = \frac{BW}{2\pi \times R \times f_0^2} = \frac{7.2 \times 10^3}{2 \times \pi \times 4.5 \times 8 \times 10^6 \times 8 \times 10^6} = 3.978 \text{ pF}$$