CLASS TEST

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ENGINEERING MATHEMATICS

CIVIL ENGINEERING

Date of Test : 14/08/2022

ANSWER KEY

1.	(a)	7.	(a)	13.	(b)	19.	(b)	25.	(c)
2.	(d)	8.	(c)	14.	(a)	20.	(c)	26.	(b)
3.	(a)	9.	(b)	15.	(c)	21.	(d)	27.	(b)
4.	(a)	10.	(a)	16.	(c)	22.	(d)	28.	(b)
5.	(c)	11.	(d)	17.	(a)	23.	(a)	29.	(d)
6.	(a)	12.	(a)	18.	(d)	24.	(c)	30.	(d)

1. (a)

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

2. (d)

For function to be differentiable it should be continuous $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

$$f(0^-) = \lim_{x\to 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)} = \lim_{x\to 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3}$$

$$f(0^+) = \lim_{x\to 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)}$$

$$= \lim_{x\to 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\dfrac{5}{3}$

3. (a)

$$\frac{dy}{dx} = e^{ax} \times e^{by}$$

$$\frac{dy}{e^{by}} = e^{ax} \times dx$$

$$\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$$

$y(0) = 0$

$$\Rightarrow \qquad c = -\left[\frac{1}{b} + \frac{1}{a}\right] = -\left[\frac{a+b}{ab}\right]$$

4. (a)

$$\int f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 \ldots) + 2(y_2 + y_4 \ldots)\right]$$

Here $h = \dfrac{1}{2}$. So, $\displaystyle\int_0^2 f(x)dx = \frac{1}{2\times 3}\left[(0+4) + 4(0.25 + 2.25) + 2(1)\right]$

$$= \frac{1}{6}\left[4 + 10 + 2\right] = 2.667$$

5. (c)

Case-I: White ball is transferred from urn A to urn B

Probability of drawing white ball from urn $B = \frac{2}{2+4} \times \frac{6}{13} = \frac{2}{13}$

Case-II: Black ball is transferred from A to B

Probability of drawing white ball from urn $B = \frac{4}{2+4} \times \frac{5}{13} = \frac{10}{39}$

$$\text{Required probability} = \frac{2}{13} + \frac{10}{39} = \frac{16}{39}$$

6. (a)

$$\nabla \cdot \vec{F} = 0 \qquad \text{[For solenoidal vector]}$$

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $a = 2$

7. (a)

$$(2y - 3x)dx + xdy = 0$$

$$\frac{dy}{dx} + \frac{2}{x}y = 3$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$y.x^2 = 3\int x^2 dx = x^3 + c$$

For $x = 0$, $y = 0$

$\Rightarrow$ $0 = 0 + c$

$\Rightarrow$ $c = 0$

For $x = 2$, $y \times 2^2 = 2^3$

$$y = 2$$

8. (c)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

$$f(u) = e^u = \frac{x^4 + y^4}{x + y}$$

From here $n = 3$.

$\Rightarrow$ $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\frac{e^u}{e^u} = 3$$

9. (b)

The roots of auxiliary equation are 2, ± 2i

$$a = -(2 + 2i - 2i) = -2$$
$$b = 2 \times (2i) + 2 \times (-2i) + 2i \times (-2i) = 4$$
$$c = -(2 \times 2i \times (-2i) = -8$$
$$a + b + c = -2 + 4 - 8 = -6$$

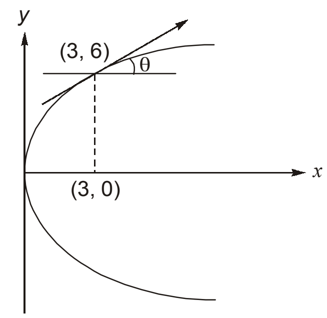
10. (a)

Let the numbers appeared on dices are X_1 and X_2.

$$\text{Expectation of the sum} = E[X_1 + X_2] = E[X_1] + E[X_2]$$
$$= 2E[X]$$
$$E[X] = \sum_{i=1}^{6} x_i p(x_i) = \frac{1}{6}(1+2+3+...+6)$$
$$= \frac{1}{6} \times \frac{6 \times 7}{2} = \frac{7}{2}$$

So, $$\text{Expected sum} = 2E[X] = 2 \times \frac{7}{2} = 7$$

11. (d)



Direction of velocity at point (3, 6) is in the direction of tangent at that point.

$\Rightarrow$ $$\text{Slope of tangent} = \text{Slope of velocity}$$

$$\tan\theta = \left.\frac{dy}{dx}\right|_{(3,6)} = \frac{12}{2y} = \frac{12}{2 \times 6} = 1$$

$\Rightarrow$ $$\theta = 45°$$

$$v = v_x\hat{i} + v_y\hat{j}$$

$$|v| = \sqrt{v_x^2 + v_y^2} = 10 \text{ m/s}$$

$$v_x = 10 \cos 45° = 5\sqrt{2} \text{ m/s}$$

$$v_y = 10 \sin 45° = 5\sqrt{2} \text{ m/s}$$

12. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)dx = 1$$

$$\left.\frac{kx^2}{2}\right|_0^2 + 2kx\Big|_2^4 + \left.\left(\frac{-kx^2}{2} + 6kx\right)\right|_4^6 = 1$$

$$\frac{k}{2}(2^2 - 0) + 2k(4-2) - \frac{k}{2}(6^2 - 4^2) + 6k(6-4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \qquad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2 dx + \int_2^4 \frac{1}{4}x dx + \int_4^6 \left(-\frac{1}{8}x^2 + \frac{3}{4}x\right)dx$$

$$= \left.\frac{1}{8}\frac{x^3}{3}\right|_0^2 + \left.\frac{1}{4}\frac{x^2}{2}\right|_2^4 - \left.\frac{1}{8}\frac{x^3}{3}\right|_4^6 + \left.\frac{3}{4}\frac{x^2}{2}\right|_4^6$$

$$= \frac{1}{3} + \frac{3}{2} - \frac{19}{3} + \frac{15}{2} = 3$$

13. (b)

If one of the eigen values is zero, then

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

$$R_1 \to R_1 + R_2 + R_3 : \begin{vmatrix} x+1+\omega+\omega^2 & x+1+\omega+\omega^2 & x+1+\omega+\omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

Since ω is cube root of unity,

$$1 + \omega + \omega^2 = 0$$

$$x\begin{vmatrix} 1 & 1 & 1 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

$$x[(x+\omega^2)(x+\omega) + \omega^2 + \omega - \omega^2(x+\omega^2) - 1 - \omega(x+\omega)] = 0$$

$$x[(x^2 + (\omega+\omega^2)x + \omega^3 + \omega^2 + \omega - x\omega^2 - \omega^4 - 1 - \omega x - \omega^2] = 0$$

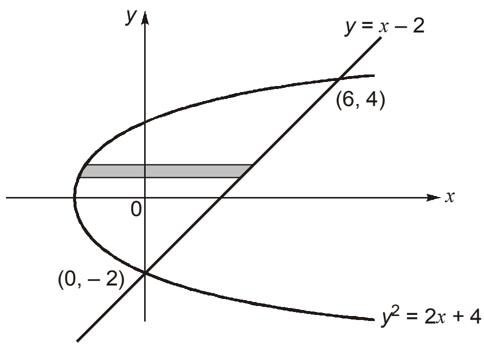
$$x[x^2 + \omega^3 + \omega - \omega^4 - 1] = 0 \qquad (\because \omega^4 = \omega \text{ and } \omega^3 = 1)$$

$$x^3 = 0$$

$$x = 0$$

14. (a)

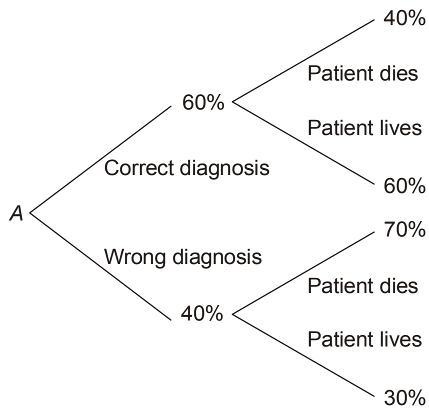
The point of intersection of line and parabolic are (0, –2) and (6, 4).



$$\text{Area} = \int_{-2}^{4}\int_{\left(\frac{y^2-4}{2}\right)}^{y+2} dx\,dy = \int_{-2}^{4} x\Big|_{\frac{y^2-4}{2}}^{y+2} dy$$

$$= \int_{-2}^{4}\left(y+2-\frac{y^2}{2}+2\right)dy = \left(\frac{y^2}{2}+4y-\frac{y^3}{6}\right)\Bigg|_{-2}^{4} = 18$$

15. (c)



Probability that patient dies who had diseases X is

$$= \frac{40}{100}\times\frac{60}{100}+\frac{70}{100}\times\frac{40}{100} = \frac{52}{100}$$

Probability that he dies of correct diagnosis

$$= \frac{60\times 40}{100\times 100} = \frac{24}{100}$$

$$\text{Required probability} = \frac{24/100}{52/100} = \frac{6}{13}$$

16. (c)

$$\begin{vmatrix} 1-\lambda & 2 \\ p & 5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(5-\lambda) - 2p = 0$$

$$\lambda^2 - 6\lambda + 5 - 2p = 0$$

Let the roots are λ_1 and λ_2.

From the characteristic equation,

$$\lambda_1 + \lambda_2 = 6$$

$$\lambda_1\lambda_2 = 5 - 2p \geq 0 \qquad \text{[For roots to be positive]}$$

$$p \leq \frac{5}{2} \qquad \dots \text{(i)}$$

For roots to be real,

$$6^2 - 4(5 - 2p) \geq 0$$

$$36 - 20 + 8p \geq 0$$

$$p \geq -2 \qquad \dots \text{(ii)}$$

From equations (i) and (ii),

$$-2 \leq p \leq \frac{5}{2}$$

17. (a)

$$h = 0.2$$

$$\int_{4.0}^{5.2} f(x)dx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 \dots y_5)]$$

$$= \frac{0.2}{2}\left[\begin{array}{l}(2.3863 + 2.6484) + 2(2.4351 + 2.4816 \\ \qquad\qquad + 2.5261 + 2.5686 + 2.6094)\end{array}\right]$$

$$= 3.0276$$

18. (d)

Probability of showing even number $= \frac{2}{1+2} = \frac{2}{3}$

Probability of showing odd number $= \frac{1}{1+2} = \frac{1}{3}$

For sum to be odd = (Even + Even + odd)/(Even + Odd + Even)/(Odd + Even + Even)/(odd + odd + odd)

$$\text{Required probability} = \left(\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\right)\times 3 + \left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right) = \frac{12+1}{27} = \frac{13}{27}$$

19. (b)

For non-trivial solution to exist,

$$\begin{vmatrix} 1 & -2 & 1 \\ k & -1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$-1 - 8 - k + 2 + 2 + 2k = 0$$

$$k = 5$$

For $k = 5$,

$$x - 2y + z = 0$$

$$5x - y + 2z = 0$$

$$\frac{x}{-4+1} = \frac{y}{5-2} = \frac{z}{-1+10}$$

$$\frac{x}{-3} = \frac{y}{3} = \frac{z}{9}$$
$$x : y : z = -1 : 1 : 3$$

20. (c)

There are 3 ways of selecting 1 girl and 2 boys.

	Group 1	Group 2	Group 3	Probability
Case I	Girl	Boy	Boy	$\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{18}{64}$
Case II	Boy	Girl	Boy	$\frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{6}{64}$
Case III	Boy	Boy	Girl	$\frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{2}{64}$

$$\text{Total probability} = \frac{18}{64} + \frac{6}{64} + \frac{2}{64} = \frac{26}{64} = 0.40625 \simeq 0.41$$

21. (d)

$$\oint_C \vec{F} \cdot \vec{dr} = \oint_C (yzdx + zxdy + xydz)$$
$$= \oint_C (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$
$$\oint_C \vec{F} \cdot \vec{dr} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds$$

where S is the surface bounded by the circle C

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = (x - x)\hat{i} - (y - y)\hat{j} + (z - z)\hat{k} = 0$$

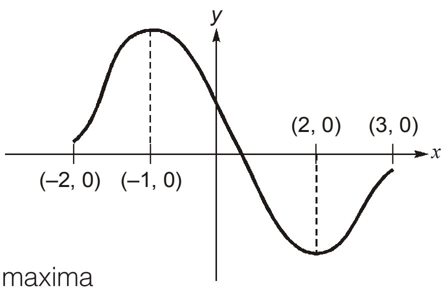
So, $$\oint_C \vec{F} \cdot \vec{dr} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS = 0$$

22. (d)

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$
$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$
$$x^2 - x - 2 = 0$$
$$(x + 1)(x - 2) = 0$$
$$x = -1, 2$$
$$f''(x) = 12x - 6$$
$$f''(-1) = -12 - 6 = -18 < 0 \Rightarrow \text{maxima}$$
$$f''(2) = 24 - 6 = 18 > 0 \Rightarrow \text{minima}$$



The function has maxima at $x = -1$ and minima at $x = 2$.

The function is decreasing between –1 and 2.

23. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by $\cos x \cos y$, we get ,

$$\tan x\, dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$

$$\log \frac{1}{\cos x \cos y} = C_1$$

$$\cos x \cos y = C$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

$\Rightarrow$ The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

24. (c)

$$\frac{\partial M}{\partial y} = 3xy^2 + 1$$

$$\frac{\partial N}{\partial x} = 4xy^2 + 2$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, the given equation is not exact.

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2 + 1)} = \frac{1}{y}$$

$$IF = e^{\int \frac{1}{y} dy} = e^{\log y} = y$$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

25. (c)

If the two curves intersects, then at point of intersection,

$$3x^3 + 2x^2 + 8x - 5 = 2x^3 + 3x + 2$$

$$x^3 + 2x^2 + 5x - 7 = 0$$

$$f(x) = x^3 + 2x^2 + 5x - 7$$

$$f(0) = 0 + 0 + 0 - 7 = -7 < 0$$

$$f(1) = 1 + 2 + 5 - 7 = 1 > 0$$

$\Rightarrow$ One root lies between 0 and 1. Let us assume 1 as initial value.

$$f'(x) = 3x^2 + 4x + 5$$

$$x_1 = 1 - \left.\frac{f(x)}{f'(x)}\right|_{x=1} = 1 - \frac{1^3 + 2 \times 1^2 + 5 \times 1 - 7}{3 \times 1^2 + 4 \times 1 + 5} = 0.9167$$

$$x_2 = x_1 - \left.\frac{f(x)}{f'(x)}\right|_{x=0.9167} = 0.9136$$

26. (b)

Number of ways of throwing 6 is five ⇒ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)

Number of ways of throwing 7 is six ⇒ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \frac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \frac{5}{36} = \frac{31}{36}$

Probability of throwing 7, $q_1 = \frac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \frac{6}{36} = \frac{30}{36}$

$$\begin{aligned}
\text{Probability of } B \text{ winning} &= p_2q_1 + p_2q_2\,p_2q_1 + p_2q_2p_2q_2p_2q_1 + \ldots \\
&= p_2q_1[1 + p_2q_2 + (p_2q_2)^2 + (p_2q_2)^3 + \ldots..] \\
&= \frac{p_2q_1}{(1-p_2q_2)} = \frac{\frac{31}{36}\times\frac{6}{36}}{1-\frac{31}{36}\times\frac{30}{36}} = \frac{31\times 6}{366} = \frac{31}{61}
\end{aligned}$$

27. (b)

The characteristic equation of matrix A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

or $\quad A^2 - 4A - 5I = 0$

Now the given polynomial can be written as,

$$A^5 - 4A^4 - 7A^3 + 11A^2 - 2A + kI = (A^3 - 2A + 3I)(A^2 - 4A - 5I) + (k + 15)I$$

Since, $\quad A^2 - 4A - 5I = 0$

For the given polynomial to be zero,

$$k + 15 = 0$$

$$k = -15$$

28. (b)

z varies from 0 to $\frac{x^2+y^2}{4}$; y varies from 0 to $\sqrt{16-x^2}$; x varies from 0 to 4.

$$\begin{aligned}
\text{Volume} &= \iiint dxdydz = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\frac{x^2+y^2}{4}} dzdydx \\
&= \frac{1}{4}\int_0^4 \int_0^{\sqrt{16-x^2}} (x^2+y^2)dydx = \frac{1}{4}\int_0^4 \left(x^2y + \frac{y^3}{3} \right)\Bigg|_0^{\sqrt{16-x^2}} dx \\
&= \frac{1}{4}\int_0^4 \left(x^2\sqrt{16-x^2} + \frac{\left(\sqrt{16-x^2}\right)^3}{3} \right) dx
\end{aligned}$$

Let, $\quad x = 4\sin\theta \qquad x \to 0 \text{ to } 4$

$$dx = 4\cos\theta\, d\theta \qquad \theta \to 0 \text{ to } \frac{\pi}{2}$$

$$\text{Volume} = \frac{1}{4}\left[4^4\int_0^{\pi/2}\sin^2\theta\cos^2\theta\, d\theta + \frac{4^4}{3}\int_0^{\pi/2}\cos^4\theta\, d\theta\right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{\Gamma\left(\frac{3}{2}\right)\times\Gamma\left(\frac{3}{2}\right)}{2\Gamma\left(\frac{6}{2}\right)} + \frac{4^4}{3}\times\frac{\Gamma\left(\frac{5}{2}\right)\times\Gamma\left(\frac{3}{2}\right)}{2\Gamma\left(\frac{6}{2}\right)}\right]$$

$$= \frac{1}{4}\left[4^4\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2!}\pi + \frac{4^4}{3}\times\frac{3}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2!}\times\pi\right]$$

$$= \frac{1}{4}\left[16\pi + 16\pi\right] = 8\pi \text{ unit}^3$$

29. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF\cdot f(x)\cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)}\cdot f(x)\cdot f'(x)dx$$

Let $f(x) = t$

$$f'(x)\ dx = dt$$

$$y \times e^t = \int e^t \cdot t dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log\,[y + 1 - f(x)] + f(x) = c'$$

30. (d)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$$

$$\vec{v}\Big|_{t=1} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

Component of velocity in direction $\hat{i} - 3\hat{j} + 2\hat{k}$ will be,

$$\frac{\vec{v}\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{(4\hat{i} - 2\hat{j} + 3\hat{k})\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}} = \frac{4+6+6}{\sqrt{14}} = \frac{16}{\sqrt{14}}$$

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