CLASS TEST

Sl. 02 IG_CE_A+B_060822 [E+G]

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STRUCTURAL ANALYSIS

CIVIL ENGINEERING

Date of Test : 06/08/2022

ANSWER KEY

1.	(d)	7.	(d)	13.	(d)	19.	(a)	25.	(a)
2.	(b)	8.	(c)	14.	(c)	20.	(b)	26.	(b)
3.	(c)	9.	(d)	15.	(c)	21.	(a)	27.	(b)
4.	(b)	10.	(a)	16.	(c)	22.	(c)	28.	(a)
5.	(a)	11.	(b)	17.	(c)	23.	(a)	29.	(a)
6.	(d)	12.	(a)	18.	(c)	24.	(b)	30.	(b)

DETAILED EXPLANATIONS

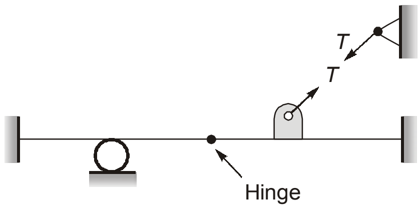
1. (d)

$$\frac{M_A}{M_B} = \frac{I_A}{I_B} \times \frac{L_B}{L_A}$$

$$= \frac{2I}{I} \times \frac{L}{L/2} = 4 : 1$$

2. (b)

$$D_S = 3m + r_e - 3j - r_r$$
$$= 3 \times 4 + 8 - 3 \times 5 - 1$$
$$= 4$$



4. (b)

Degree of kinematic indeterminacy for a plane rigid frame having inextensible member is given by

$$D_k = 3j - r_e - m$$

where m = Total number of inextensible members

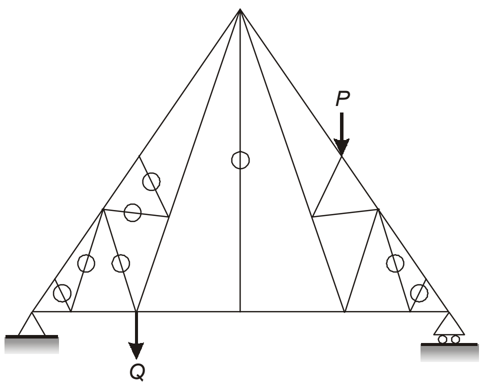
Here, $j = 9$

$r_e = 3 + 1 + 2 = 6$

$m = 10$

∴ $D_k = 3 \times 9 - 6 - 10 = 11$

5. (a)

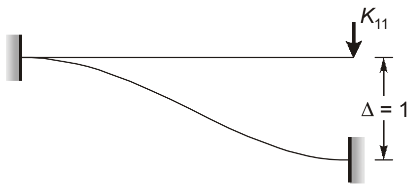


If three members meet at a joint and two of them are collinear, then the third member will carry zero force provided that there does not act any external load at the joint.

Thus using this statement, we arrive at 8 zero force members which are highlighted by '0' sign.

6. (d)

K_{11} = Force required along degree of freedom '1' to produce unit displacement in the direction of degree of freedom '1' and degree of freedom '2' must be locked.



$$\therefore \qquad K_{11} = \frac{12EI}{L^3}$$

7. (d)

Joint	Member	RS	TS	DF
O	OA	$\frac{3}{4}\frac{I}{L}$	$\frac{7}{4}\frac{I}{L}$	$\frac{3}{7}$
	OB	0		0
	OC	$\frac{I}{L}$		$\frac{4}{7}$

Distribute the moments, on respective members,according to the distribution factor.

8. (c)

Maximum bending moment occurs when full span is loaded

$$\text{B.M}_{max} = \frac{1}{2} \times 16 \times 3 \times 2 = 48 \text{ kN-m}$$

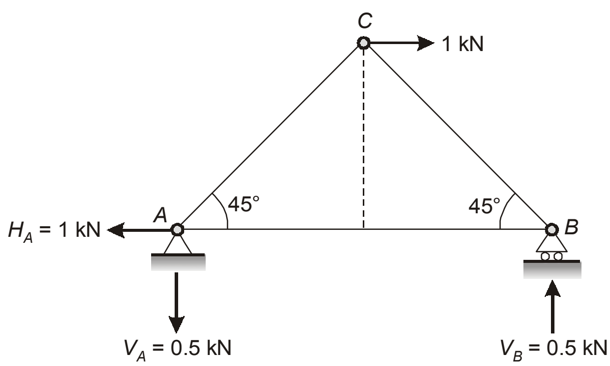
10. (a)

An indeterminate structure develops less maximum bending moment over the span. So it requires less cross-section to resist and more economical from material stand-point.

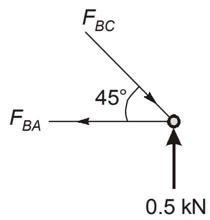
It is not necessary that removal of 'n' redundants result in stable structure.

11. (b)

Apply unit load at point C in the horizontal direction. The truss is then analysed for this unit load and member focus are formed out



At joint B:



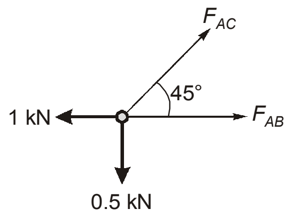
$$F_{BC} \times \sin 45° = 0.5$$

$$\therefore \quad F_{BC} = 0.5\sqrt{2}\,\text{kN}$$

and $\quad F_{BC} \times \cos 45° = F_{BA}$

$$\therefore \quad F_{BA} = 0.5\text{ kN}$$

At joint A:



$$F_{AC} \times \sin 45° = 0.5$$

$$\therefore \quad F_{AC} = 0.5\sqrt{2}\,\text{kN}$$

Member	k	Δ (in mm)	$k\Delta$ (in mm)
AB	0.5	5	2.5
BC	$-0.5\sqrt{2}$	0	0
CA	$0.5\sqrt{2}$	0	0
			$\sum k\Delta = 2.5$ mm

12. (a)

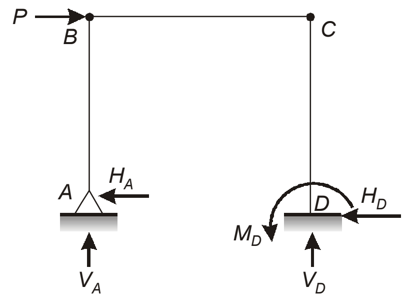
Vertical reaction at both support = $\frac{P}{2}$

Now, taking moment about hinge at crown from LHS = 0

$$\Rightarrow \quad \frac{P}{2} \times \frac{R}{\sqrt{2}} - H \times \left(R - \frac{R}{\sqrt{2}}\right) = 0$$

$$\Rightarrow \quad H = \frac{P}{2(\sqrt{2}-1)}$$

13. (d)



$$M_B = 0$$

$\Rightarrow$ $$H_A \times L = 0$$

$\therefore$ $$H_A = 0$$

$\therefore$ $$\Sigma F_x = 0$$

$\Rightarrow$ $$P - H_A - H_D = 0$$

$\Rightarrow$ $$H_D = P$$

Since $$M_C = 0$$

$\Rightarrow$ $$H_D \times L - M_D = 0$$

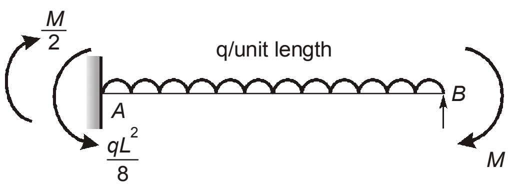
$\Rightarrow$ $$M_D = H_D.L$$

$\therefore$ $$M_D = P.L$$

14. (c)

Let the clockwise moment required to make the slope of the deflection curve equal to zero at B be M. Thus a carry over moment of magnitude $\frac{M}{2}$ will be induced at A in clockwise direction.

Taking moment about $B = 0$



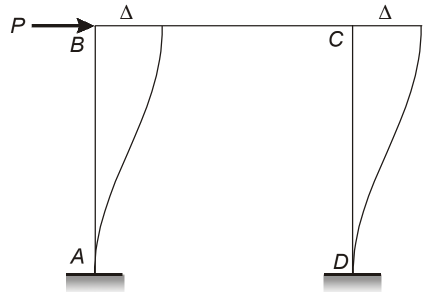
$\Rightarrow$ $$\frac{qL^2}{8} - \frac{M}{2} - M = 0$$

$\Rightarrow$ $$\frac{3}{2}M = \frac{qL^2}{8}$$

$\Rightarrow$ $$M = \frac{qL^2}{12}$$

15. (c)

$(EI)_b \rightarrow \infty$, means beam will not bend any more because its flexural rigidity is infinite so, column AB will behave as a column shown in figure below



As we know that

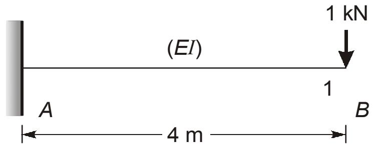
$$\frac{24EI}{L^3} = P$$

$\Rightarrow$ $$\Delta = \frac{PL^3}{24EI}$$

Now $$M_{AB} = \frac{6EI\Delta}{L^2}$$

$$= \frac{6EI}{L^2} \times \frac{PL^3}{24EI} = \frac{PL}{4}$$

16. (c)



Applying unit load in the direction of 1

Deflection at B, $$f_{11} = \frac{1 \times 4^3}{3EI} = \frac{64}{3EI}$$

Rotation at B, $$f_{21} = \frac{1 \times 4^2}{2EI} = \frac{8}{EI}$$

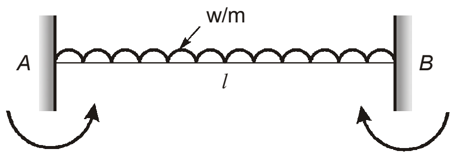
Applying unit moment in the direction of 2,

Deflection at B, $$f_{12} = \frac{1 \times 4^2}{2EI} = \frac{8}{EI}$$

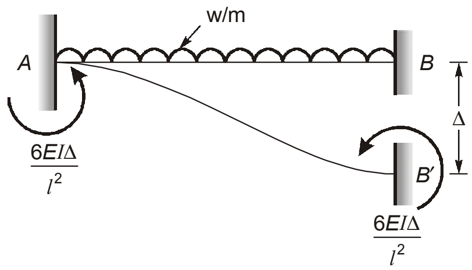
Rotation at B, $$f_{22} = \frac{1 \times 4}{EI} = \frac{4}{EI}$$

So, flexibility matrix $$= \begin{bmatrix} \frac{64}{3EI} & \frac{8}{EI} \\ \frac{8}{EI} & \frac{4}{EI} \end{bmatrix}$$

17. (c)



If support B settles by Δ then



$$\text{FEM at } B = \frac{wl^2}{12}$$

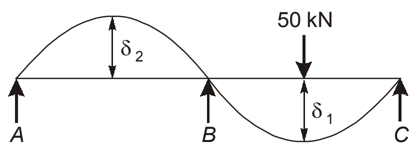
$$\Rightarrow \quad \text{Moment due to sinking of support} = \frac{6EI\Delta}{l^2}$$

For zero moment at B.

$$\frac{wl^2}{12} = \frac{6EI\Delta}{l^2}$$

$$\Rightarrow \quad \Delta = \frac{wl^4}{72EI}$$

18. (c)



Using Maxwell's reciprocal theorem,

Deflection at middle of span BC

$$= \frac{\delta_1}{50}\times 30 - \frac{\delta_2}{50}\times 20 = \left(\frac{0.05}{50}\times 30 - \frac{0.02}{50}\times 20\right) \text{ m}$$

$$= 0.022 \text{ m}$$

19. (a)

$$\bar{M}_{ab} = \bar{M}_{ba} = 0$$

$$\bar{M}_{bc} = \frac{-60\times 3}{8} = -22.5 \text{ kNm}$$

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}\left(2i_a + i_b - \frac{3\delta}{l}\right) \qquad \text{where } l = 4 \text{ m}$$

$$= \frac{1}{2}EIi_b - \frac{3}{8}EI\delta$$

20. (b)

$$H = \frac{120}{\pi}\sin^2 30° + \frac{100}{\pi} + \frac{80}{\pi}\sin^2 60° = \frac{190}{\pi} \text{ kN}$$

21. (a)

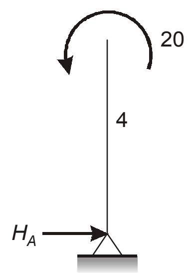
$$\overline{M}_{BA} = +\frac{6\times 3^2\times 1}{4^2} = +3.375\,\text{kNm}$$

$$M_{BA} = \overline{M}_{BA} + \frac{2EI}{4}(2\theta_B + \theta_A) = +3.375 + \frac{1}{2}EI(2\theta_B + 0)$$

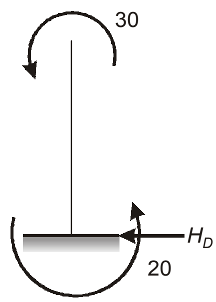
$$= 3.375 + EI\theta_B$$

(Note that the frame and loading are symmetrical).

22. (c)



$$-H_A \times 4 = 20$$
$$H_A = -5 \text{ kN}$$
$$H_A = 5 \text{ kN} \leftarrow$$



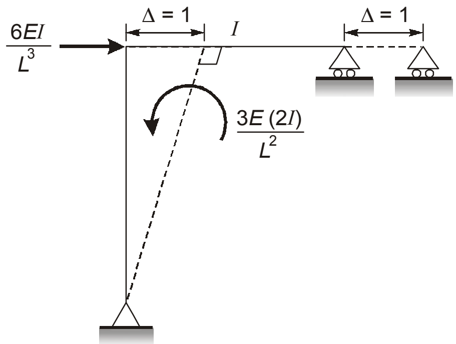
$$H_D \times 4 = 20 + 30$$

$$H_D = \frac{50}{4} = 12.5\,\text{kN} \leftarrow$$

$$H_A + H_D = 5 + 12.5 = 17.5 \text{ kN}$$
$$P = 17.5\,\text{kN}$$

23. (a)

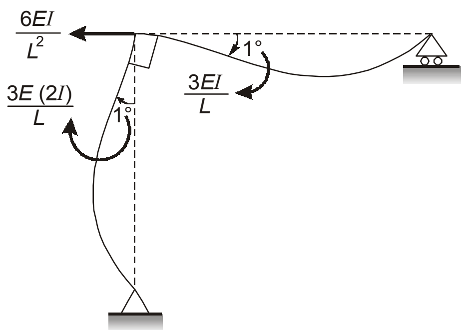
(i) When we provide unit deflection in the direction of (1)



$$k_{11} = \frac{6EI}{L^3}$$

$$k_{12} = -\frac{6EI}{L^2}$$

(ii) When we provide unit rotation in the direction of (2)



$$k_{12} = -\frac{6EI}{L^2}$$

$$k_{22} = \frac{3E(2I)}{L} + \frac{3EI}{L} = \frac{9EI}{L}$$

∴ Stiffness matrix, $k = \begin{bmatrix} \frac{6EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{9EI}{L} \end{bmatrix} = \frac{EI}{L^2}\begin{bmatrix} \frac{6}{L} & -6 \\ -6 & 9L \end{bmatrix}$

24. (b)

$$\overline{M}_{AB} = -\frac{16 \times 8}{8} = -16\,\text{kNm}$$

$$\overline{M}_{BA} = \frac{16 \times 8}{8} = 16\ \text{kNm}$$

When supports '*B*' sinks by 1 cm,

$$\frac{\Delta}{l} = \frac{1}{800}$$

$$M_{BA} = 0$$

$$M_{BA} = \overline{M}_{BA} + \frac{2EI}{l}\left(2\theta_B - \frac{3\Delta}{l}\right)$$

$$0 = 16 + \frac{2 \times 2.8 \times 10^2}{8}\left(2\theta_B - 3 \times \frac{1}{800}\right)$$

$$\theta_B = -0.112$$

$$M_{AB} = -16 + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -16 + \frac{2 \times 2.8 \times 10^2}{8}\left(-0.112 - 3 \times \frac{1}{800}\right)$$

$\therefore$ $$M_{AB} = -24.1 \text{ kNm}$$

25. (a)

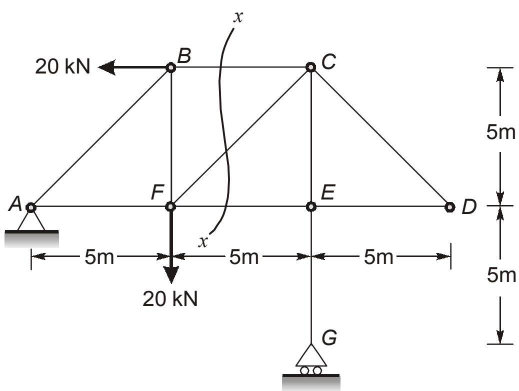
$$\Sigma M_A = 0$$

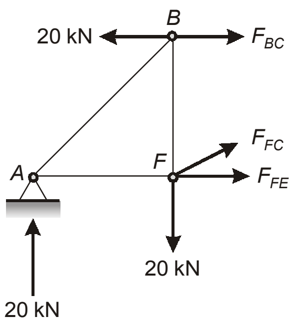
$\Rightarrow$ $$R_G \times 10 = 20 \times 5 - 20 \times 5$$

$\Rightarrow$ $$R_G = 0$$

From method of section $\rightarrow$

$\therefore$ $$R_A = 20 \text{ kN}$$





Taking moment about F,

$$20 \times 5 - 20 \times 5 + F_{BC} \times 5 = 0$$

$$F_{BC} = 0$$

Alternatively,

by inspection, $R_G = 0$ $\{BM_E = 0\}$

$\therefore$ Force in member $EG = 0$

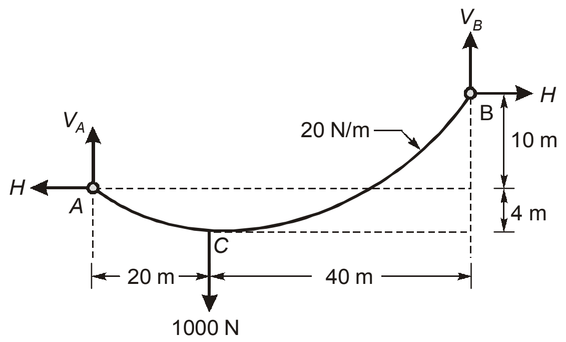
Forces in member ED and CD are already 0.

Thus, member EF and EC is zero.

Similarly, forces in members BC and CF are zero.

Also, force in member $AF = 0$

26. (b)



$$\Sigma V = 0$$

$$V_a + V_b = 20 \times 60 + 1000 = 2200 \text{ N} \quad \text{...(i)}$$

Taking moment about C of the forces on left side of C,

$$V_a \times 20 = 4H + \frac{20 \times 20^2}{2}$$

$$\Rightarrow \quad V_a = \frac{H}{5} + 200 \quad \text{...(ii)}$$

Taking moment about C of the forces on right side of C,

$$V_b \times 40 = 14H + \frac{20 \times 40^2}{2}$$

$$\Rightarrow \quad V_b = \frac{7H}{20} + 400 \quad \text{...(iii)}$$

Adding equation (ii) and (iii), we get

$$V_a + V_b = \frac{11H}{20} + 600$$

$$2200 = \frac{11H}{20} + 600$$

$$\Rightarrow \quad H = 2909 \text{ kN}$$

Substituting in equation (ii), we get

$$V_a = \frac{H}{5} + 200 = \frac{2909}{5} + 200 = 781.8 \text{ kN}$$

$$\therefore \quad V_b = 1418.2 \text{ kN}$$

Maximum tension $\rightarrow$

$$T_{max} = \sqrt{V^2 + H^2} = \sqrt{(1418.2)^2 + (2909)^2} = 3236.29 \text{ kN}$$

27. (b)

Fixed end moments →

$$\overline{M}_{BA} = \frac{Pa^2b}{l^2} = 24\,\text{kNm}$$

$$\overline{M}_{BC} = 0$$

Slope deflection equation →

$$M_{BA} = 24 + \frac{2EI}{5}[2\theta_B] = 24 + \frac{4EI\theta_B}{5}$$

$$M_{BC} = \frac{2EI}{3}[2\theta_B] = \frac{4EI\theta_B}{3}$$

Apply joint equilibrium condition at joint B,

$$M_{BA} + M_{BC} = 0$$

$$\Rightarrow \quad 24 + \frac{4EI\theta_B}{5} + \frac{4EI\theta_B}{3} = 0$$

$$\Rightarrow \quad \theta_B = \frac{-11.25}{EI}$$

28. (a)

Let force in inclined member and horizontal member be F_1 and F_2 respectively.

$$\Sigma V = 0$$

$$F_1 \sin 45° = P$$

$$\Rightarrow \quad F_1 = 100\sqrt{2}\ \text{kN} = P\sqrt{2}\ \text{kN}$$

$$\Sigma H = 0$$

$$F_1 \cos 45° + F_2 = 0$$

$$\Rightarrow \quad F_2 = -\frac{100\sqrt{2}}{\sqrt{2}} = -100\,\text{kN} = -P\,\text{kN}$$

Strain energy of the truss is given by,

$$\because \quad U = U_{AC} + U_{BC}$$

$$\Rightarrow \quad U = \frac{F_1^2 L_{AC}}{2AE} + \frac{F_2^2 L_{BC}}{2AE}$$

$$\Rightarrow \quad U = \frac{\left(\sqrt{2}P\right)^2 \times 3000}{2AE} + \frac{(-P)^2 \times \left(\frac{3000}{\sqrt{2}}\right)}{2AE}$$

$$\Rightarrow \quad U = \frac{2P^2 \times 3000}{2AE} + \frac{P^2 \times 3000}{2\sqrt{2}\,AE}$$

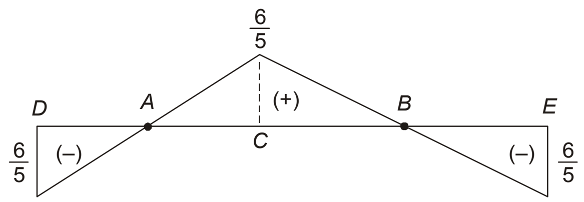
$$\Rightarrow \quad U = \frac{3000}{AE}\left(P^2 + \frac{P^2}{2\sqrt{2}}\right) = \frac{3000\,P^2}{AE}(1.3535)$$

$$\Rightarrow \quad \delta = \frac{\partial U}{\partial P} = \frac{6000\,P}{AE}(1.3535) = \frac{6000 \times 100 \times 1000}{3000 \times 2 \times 10^5}(1.3535)$$

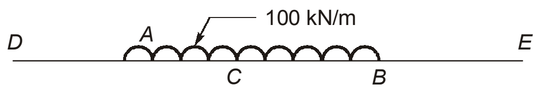
$$\delta = 1.3535\ \text{mm} \simeq 1.35\ \text{mm}$$

29. (a)

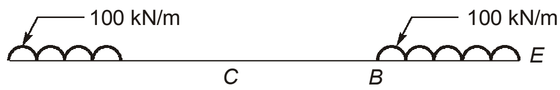
Influence line diagram →



Position of load for maximum positive BM at C



Position of load for maximum negative BM at C



Maximum positive *BM* at *C*,

$$= +100 \times \frac{1}{2} \times 5 \times \frac{6}{5} = +300\,\text{kNm}$$

Maximum negative *BM* at *C*,

$$= -\left[100 \times \frac{1}{2} \times 2 \times \frac{6}{5} + 100 \times \frac{1}{2} \times \frac{6}{5} \times 3\right] = -300\text{ kNm}$$

$\therefore$ Required ratio $= \frac{300}{300} = 1$

30. (0)

$$\Delta = \frac{5}{96}\frac{WL^3}{EI};$$

Moment at *B*, $\frac{3EI\Delta}{L^2} = \frac{3EI\,5WL^3}{96L^2\,EI} = \frac{5WL}{32}$

A	B	B	C	
	0.5	0.5		
$-\frac{WL}{12}$	$\frac{WL}{12}$	$-\frac{WL}{8}$	$\frac{WL}{8}$	
	$-\frac{5WL}{32}$	$\frac{5WL}{32}$		
$-\frac{WL}{12}$	$-\frac{7WL}{96}$	$\frac{WL}{32}$	$\frac{WL}{8}$	
$+\frac{WL}{12} \rightarrow$	$\frac{WL}{24}$	$-\frac{WL}{16}$	$\leftarrow \frac{WL}{8}$	
0	$-\frac{WL}{32}$	$-\frac{WL}{32}$	0	
	$+\frac{WL}{32}$	$+\frac{WL}{32}$		
0	0	0	0	Final end moments

Therefore, moment at *B* is 0.

■■■■