CLASS TEST

S.No. : 01 JP_CS_010822

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DISCRETE MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 01/08/2022

ANSWER KEY

1.	(b)	7.	(a)	13.	(c)	19.	(a)	25.	(d)
2.	(d)	8.	(b)	14.	(a)	20.	(b)	26.	(c)
3.	(a)	9.	(d)	15.	(d)	21.	(b)	27.	(b)
4.	(d)	10.	(d)	16.	(b)	22.	(c)	28.	(c)
5.	(d)	11.	(b)	17.	(a)	23.	(a)	29.	(d)
6	(c)	12.	(a)	18.	(b)	24.	(b)	30.	(d)

DETAILED EXPLANATIONS

1. **(b)**

G has 4 vertices

$$\text{Maximum \# of edges} = \frac{4(4-1)}{2} = 6 \text{ Edges}$$

$$2*2+1+3 = 2|E|$$

$$\Rightarrow \quad 4+1+3 = 2|E|$$

$$\Rightarrow \quad |E| = 4$$

G has 4 edges

$\bar{G}$ has ${}^4C_2 - 4 = 6 - 4 = 2$ Edges

With 4 vertices and 2 edges, the graph is always disconnected.

2. **(d)**

$\sim\forall x\ \exists y\ P(x, y) \equiv \exists x\ \forall y\ (\sim P(x, y))$

$\sim\forall x\ P(x) \equiv \exists x\ [\sim P(x)]$

$\sim\exists x\ \forall y\ [P(x, y) \vee Q(x, y) \equiv \forall x\ \exists y\ [\sim P(x,y) \wedge \sim Q(x, y)]$

$\therefore$ All logical equivalents are correct.

3. **(a)**

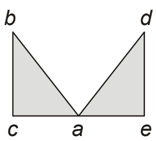
$$d * c = d * (a * b)[\text{Given, } c = a * b] = (d * a) * b$$

[Associative holds in semigroup]

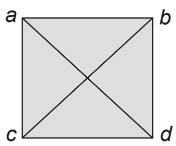
$$= b * b[\text{Given, } b * b = a] = a$$

4. **(d)**

If graph contain Euler circuit, it need not contain Hamiltonian cycle and vice-versa.

1. 

$\begin{bmatrix} a-b-c-a-d-e-a \text{ is Euler circuit} \\ \text{no hamiltonian cycle.} \end{bmatrix}$

2. 

$\begin{bmatrix} \text{No Euler circuit (all vertices must even degree)} \\ a-b-d-c-a \text{ is hamiltonian cycle.} \end{bmatrix}$

(1) is Euler but not Hamiltonian graph.

(2) is Hamiltonian but not Euler graph.

5. **(d)**

$$a_n = -4a_{n-1} + 12\,a_{n-2}$$

$$a_n + 4a_{n-1} - 12\,a_{n-2} = 0$$

$$x^2 + 4x - 12 = 0$$

$$(x+6)(x-2) = 0$$

$$x = -6, x = 2$$

$$\therefore \quad a_n = A(-6)^n + B \cdot (2)^n$$

6 **(c)**

Number of vertices = 11

In K_n each vertex can contain degree 10.

In complement of G:

(10-1, 10-1,10-2, 10-2, 10-3, 10-3, 10-3, 10-3, 10-4, 10-5, 10-5) = (9, 9, 8, 8, 7, 7, 7, 7, 6, 5, 5) is degree sequence.

So option (c) is correct.

7. **(a)**

1. S = $\{(x, y) : y = x + 1$ and $0 < x < 2\}$

- Check for Reflexive Relation:

  $(x, x) : x = x + 1$ but $x \neq x + 1$

  Hence cannot be reflexive S is not equivalence relation on R.

2. T = $\{(x, y) : x - y$ is an integer$\}$

- Check for Reflexive Relation:

  $(x, x) : x - x$ is integer $x - x = 0$ and $0 \in$ integer

  So, T is reflexive.

- Check for Symmetric Relation:

  $(x, y) : x - y$ is integer and $(y, x) : y - x$ also an integer.

  So, T is symmetric relation.

- Check for Transitive Relation:

  $(x, y) : x - y$ is integer and $(y, z) : y - z$ is integer then $(x, z) : x - z$ is also integer.

  So, T is transitive.

  Hence T is equivalence relation but S is not.

8. **(b)**

"No students are allowed to carry smartphone"

Can be written as: Not a student are allowed to carry smartphone

$\equiv \neg[\exists x(\text{student}(x) \wedge \text{carry\_smartphone}(x))]$

$\equiv \forall x(\neg \text{student}(x) \vee \neg \text{carry\_smartphone}(x))$

$\equiv \forall x(\text{student}(x) \rightarrow \neg \text{carry\_smartphone}(x))$

So, option (b) is correct representation only.

9. **(d)**

$$f(x) = \frac{x}{x-1}$$

$$f \circ f(x) = \frac{\left(\frac{x}{x-1}\right)}{\left(\frac{x}{x-1}\right)-1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}}$$

i.e. $$\underbrace{f \circ f}_{2\,\text{times}}(x) = x$$

So, $$f \circ \underbrace{\left(f \circ f \circ f \circ \ldots.. f\right)}_{20\text{ times}}(x) = f(x)$$

$$= \frac{x}{x-1}$$

10. (d)

- If a graph is connected, then its complement may or may not be disconnected.
  Example: cyclic graph on 5 vertices.
- Chromatic number of complete graph with n vertices is n.
- If two graph G_1 and G_2 are isomorphic, then their complements will always be isomorphic.
- If any simple graph with n nodes with nodes > 1, there are atleast two vertices of same degree.

11. (b)

Let
p : GATE rank is needed
q : I will write the GATE exam
r : I will join in MADEEASY.

Given arguments:

P_1: If GATE rank is needed, i will not write GATE exam, if i do not join MADEEASY.

$$p \to (\sim r \to \sim q) = (p \wedge \sim r) \to \sim q$$

P_2: GATE rank is needed : p
P_3: I will join MADEEASY : r
Q: I will write the GATE exam : q

Inference is: $(p \wedge \neg r) \to \neg q$

$$\begin{array}{c} p \\ r \\ \hline q \\ \hline \end{array}$$

We can also write the above inference as following: $(p \wedge \neg r)$

$$\left([(p \wedge \neg r) \to \neg q] \wedge p \wedge r\right) \to q$$

If above proposition is tautology then given inference is valid.

$((pr')' + q')' + p' + r' + q$

$= pr'q + p' + r' + q$

$= p' + r' + q$ which is consistency hence invalid.

12. (a)

Total number of terms $= 8 + 1 = 9$

The middle term is : 5^{th} term

$(x + y)^n$ has $(r + 1)^{th}$ term as : ${}^nC_r\, x^{n-r}\, y^r$

$[(4 + 1)^{th}$ term] 5^{th} term is:

$${}^8C_4 \left(\frac{y\sqrt{x}}{3}\right)^{8-4} \left(\frac{-3}{x\sqrt{y}}\right)^4$$

$$= {}^8C_4 \cdot \frac{y^4 \cdot x^2}{3^4} \cdot \frac{3^4}{x^4 \cdot y^2}$$

$$= {}^8C_4 \cdot \frac{y^2}{x^2}$$

$$= 70\left(\frac{y}{x}\right)^2$$

13. (c)

$\forall x \in N\ [(x \neq 7 \wedge \text{Prime}\ (x)) \rightarrow \neg\ \text{Divisibleby7}(x)]$

$\cong$

$\forall x \in N\ [x = 7 \vee \neg\ \text{Prime}\ (x) \vee \neg\ \text{Divisibleby7}(x)]$

$\cong$

$\neg \exists x \in N\ [x \neq 7 \wedge \text{Prime}\ (x) \wedge \text{Divisibleby7}(x)]$

All represents that "no prime except 7 is divisible by 7".

14. (a)

Put $x = y$ and $y = x$ at the and to get inverse function

$$y = 2.2^x + 4^x$$

$$\Rightarrow \quad x = 2.2^y + 4^y$$

$$\Rightarrow \quad x = 2.2^y + (2^y)^2$$

$$\Rightarrow \quad x+1 = (2^y)^2 + 2.2^y + 1$$

$$\Rightarrow \quad x+1 = (2^y + 1)^2$$

$$\Rightarrow \quad \sqrt{x+1} = 2^y + 1$$

$$\Rightarrow \quad 2^y = \sqrt{x+1} - 1$$

$$\Rightarrow \quad \log 2^y = \log\left(\sqrt{x+1} - 1\right)$$

$$\Rightarrow \quad y \log 2 = \log\left(\sqrt{x+1} - 1\right)$$

$$\Rightarrow \quad y = \frac{\log\left(\sqrt{x+1} - 1\right)}{\log 2}$$

15. (d)

- $R = \{\langle x, y\rangle \mid x \equiv y \bmod m\}$ when $x = y$ then $x \equiv x \bmod m$ is always reflexive.
- $R = \{\langle x, y\rangle \mid x \equiv y \bmod m\}$ $(x - y)$ mod m is always equal to $(y - x)$ mod m. So relation is always symmetric.
- $R = \{\langle x, y\rangle \mid x \equiv y \bmod m\}$ if $(x - y)$ mod m is always equal to $(y - z)$ mod m. Which is equal to $(x - z)$ mod m so relation is always transitive.

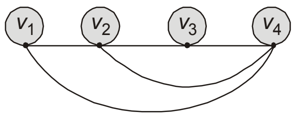
The given relation $R = \{\langle x, y\rangle \mid x \equiv y \bmod m\}$ is equivalence relation (reflexive, symmetric and transitive).

16. (b)

Let (a) $n = 2$;

\# edge = 1

(b) $n = 4$



\# edges = 5

So option (b) is correct.

17. (a)

Everybody loves India: $\forall x$ Loves (x, India)

Everybody loves somebody: $\forall x\ \exists y$ Loves (x, y)

There is somebody whom everybody loves: $\exists y\ \forall x$ Loves (x, y)

There is somebody whom no one loves: $\exists y\ \forall x\ \neg$ Love (x, y)

18. (b)

- Set A is countable. Since Q (set of rational numbers) is countable and every subset of countable set is also countable.
- Set B is uncountable. Since every subset of real number is uncountable.
- Set C is countable because it is Cartesian product of two countable sets i.e. $N \times Z$.
- Set D is countable. Since one to one correspondence with set of natural number Cantor's theorem.

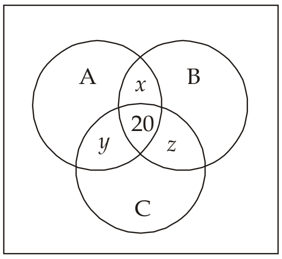
19. (a)

$$\text{Total children} = 75$$
$$\therefore \quad \text{Total receipt} = ₹\ 70\ (₹\ 0.50/\text{ride})$$
$$\therefore \quad \text{Total rides} = 70 \times 2 = 140$$

20 children had taken all the 3 rides

$\therefore$ 55 had taken at least 2 rides (2 or 3 rides).

So, 55 – 20 = 35 had taken exactly 2 rides.



Let, $x + y + z = 35$

Children who had taken exactly one ride

$$\text{Total single ride} = 140 - (35 \times 2 + 20 \times 3)$$
$$= 140 - (70 + 60) = 10$$

So, total number of students who took exactly singe ride = 10

$$\text{Children who took no ride} = 75 - (35 + 20 + 10)$$
$$= 75 - (65) = 10$$

20. (b)

- R is not reflexive since ϕ is an element of power, set of any subset of A and $\phi \cap \phi = \phi$ and belongs to R.
- R is symmetric because intersection ($\cap$) is commutative, thus $a \cap b \neq \phi$ the $b \cap a \neq \phi$.
- R is not transitive because $a \cap b \neq \phi$ and $b \cap c \neq \phi$ does not assure $a \cap c \neq \phi$. e.g., $a = \{1, 2\}$, $b = \{2, 3\}$ and $c = \{3, 4\}$

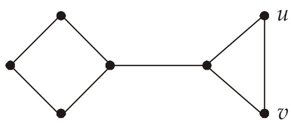
So, $\{1, 2\} \cap \{2, 3\} \neq \phi$

$\{2, 3\} \cap \{3, 4\} \neq \phi$

but $\{1, 2\} \cap \{3, 4\} = \phi$ so fail.

21. (b)

(a) Consider a graph:



$$n(G) \geq d(u) + d(v)$$
$$7 \geq 2 + 2$$
$$7 \geq 4 \quad \text{satisfied but } u/v \text{ is not cut edge. So false}$$

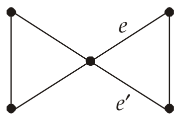
(b) Let, G be a graph such that $|E_G| < |V_G|$ further, suppose G_1, G_2, G_2 ________ G_k are connected components of G, and if no connected component of G is a tree.

Hence, for each $1 \le i \le k$, $|E_{Gi}| \ge |V_{Gi}|$. Thus,

$$|E_G| = \sum_{i=1}^{k} |E_{Gi}| \ge \sum_{i=1}^{k} |V_{Gi}| \ge |V_G|$$

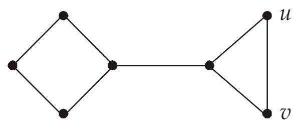
Which is a contradiction. Hence, there exists a component of G which is tree.

(c) Consider a graph:



Since graph is Eulerian graph but don't have eulerian circuit. So false

(d) Consider a graph:



$$\text{Average degree } (G) = \frac{2e}{n} \qquad \text{Average degree } (G) = \frac{2e}{n}$$

$$\text{Before removal of 'v'} = \frac{20}{7} = 2.857 \qquad \text{After removal of 'v'} = \frac{16}{6} = 2.66$$

So false

22. (c)

$$a_n = a_{n-1} + 3^{n-1}$$

$$a_n = \sum_{i=0}^{n} a_i x^i$$

$$= 1 + \sum_{i=1}^{n} a_i x^i$$

$$= 1 + \sum_{i=1}^{n} (a_{i-1} + 3^{i-1}) x^i$$

$$= 1 + \sum_{i=1}^{n} (a_{i-1} + x^i) + \sum_{i=1}^{n} (3^{i-1} x^i)$$

$$= 1 + x\left[\sum_{i=0}^{n} a_i x^i\right] + x\left(\sum_{i=0}^{n} 3^i x^i\right)$$

$$a_n = 1 + x a_n + \frac{x}{1-3x}$$

$$a_n(1 - x) = 1 + \frac{x}{1-3x}$$

$$a_n = \frac{1-3x+x}{(1-3x)(1-x)} = \frac{1-2x}{(1-x)(1-3x)}$$

$$= \frac{A}{1-x} + \frac{B}{1-3x} = \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1-3x}$$

$$a_n = \frac{1}{2}(1+x+x^2+x^3.....) + \frac{1}{2}(1+3x+(3x^2)+.....)$$

$$= \frac{1}{2}(1+3^n)$$

23. (a)

P_1 : $xyz^{-1}w = 1$, then $y = x^{-1}w^{-1}z$

Put $y = x^{-1}w^{-1}z$ in $xyz^{-1}w = 1$

$$x(x^{-1}w^{-1}z)z^{-1}w = 1$$

$$w^{-1}z\,z^{-1}w = 1$$

$$w^{-1}w = 1$$

$$1 = 1 \quad \text{Hence true}$$

P_2 : $xyz = 1$, then $xyz = 1$

Assume, $x = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, z = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$xyz = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$yxz = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

And $xyz \neq yxz$ Hence false

24. (b)

- Every cyclic group is Abelien group but every Abelien group is not cyclic group.
- Every group of prime order is cyclic group and we know that every cyclic group is Abelien group hence, every group of prime order is Abelien group.
- If $(G, *)$ be a cyclic group of even order, then there exist atleast one elements other than identity element such that $a^{-1} = a$.

25. (d)

- On empty set ϕ is an equivalence relation therefore S_1 is false.
- In S_2 and relation is not transitive.

  e.g.

  $(2, 5) \in R$ as $10 \geq 1$

  $\left(5, \frac{1}{4}\right) \in R$ as $\frac{5}{4} \geq 1$

  but $\left(2, \frac{1}{4}\right) \notin R$ as $\frac{2}{4} < 1 \Rightarrow$ Not transitive

So, both S_1 and S_2 are false.

26. **(c)**

$$a_k = -8a_{k-1} - 15a_{k-2}$$
$$k-2 = 1$$
$$k-1 = n$$
and $$k = n^2$$
Using characteristics equation
$$n^2 + 8 + 15 = 0$$
$$n = -3$$
and $$n = -5$$
So, $$a_k = (-3)^k C_1 + (-5)^k C_2$$
$$= (-3)^0 C_1 + (-5)^0 C_2 = 0$$
$$C_1 + C_2 = 0 \quad \text{...(i)}$$
$$a_1 = (-3)^1 C_1 + (-5)^1 C_2$$
$$= -3\,C_1 + (-5)C_2 = 2 \quad \text{...(ii)}$$
Solving equation (i) and (ii), we get,
$$C_1 = 1$$
and $$C_2 = -1$$
then $$a_n = (-3)^k - (-5)^k$$

27. **(b)**

- I is not D_{42} because the divisor 7 is missing. So, there is no way for I to be isomorphic to $(P\{a,b,c\},\subseteq)$ as it needs to have 8 divisors but right now it has only 7.
- II is D_{66} a well known boolean algebra and has 8 vertices and its masses diagram will be isomorphic $(P(\{a,b,c\}),\subseteq)$.
- III is not isomorphic even though it looks like D_{70}, it is on the relation $\le$, resulting in a chain, which won't be boolean algebra.

28. **(c)**

- G_1 has 6 items and G_2 has 5 cycles. Hence it can not be isomorphic.
- G_3 and G_4 are also isomorphic.
- Hence the option (c) is correct.

29. **(d)**

Each ambulance "covers" the adjacent roads, and all roads are covered in this way.

30. **(d)**

This says that there is a bound, m, such that any twin prime is below m. In other words, that there only finitely many twin primes.

■■■■