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# DISCRETE MATHEMATICS

## COMPUTER SCIENCE & IT

Date of Test : 06/08/2022

### ANSWER KEY >

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (a)  | 13. (c) | 19. (a) | 25. (d) |
| 2. (d) | 8. (b)  | 14. (a) | 20. (b) | 26. (c) |
| 3. (a) | 9. (d)  | 15. (d) | 21. (b) | 27. (b) |
| 4. (d) | 10. (d) | 16. (b) | 22. (c) | 28. (c) |
| 5. (d) | 11. (b) | 17. (a) | 23. (a) | 29. (d) |
| 6. (c) | 12. (a) | 18. (b) | 24. (b) | 30. (d) |

**DETAILED EXPLANATIONS**

1. (b)

G has 4 vertices

$$\text{Maximum \# of edges} = \frac{4(4-1)}{2} = 6 \text{ Edges}$$

$$2 * 2 + 1 + 3 = 2 |E|$$

$$\Rightarrow 4 + 1 + 3 = 2 |E|$$

$$\Rightarrow |E| = 4$$

G has 4 edges

$$\bar{G} \text{ has } {}^4C_2 - 4 = 6 - 4 = 2 \text{ Edges}$$

With 4 vertices and 2 edges, the graph is always disconnected.

2. (d)

$$\sim \forall x \exists y P(x, y) \equiv \exists x \forall y (\sim P(x, y))$$

$$\sim \forall x P(x) \equiv \exists x [\sim P(x)]$$

$$\sim \exists x \forall y [P(x, y) \vee Q(x, y)] \equiv \forall x \exists y [\sim P(x, y) \wedge \sim Q(x, y)]$$

∴ All logical equivalents are correct.

3. (a)

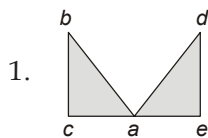
$$d * c = d * (a * b) [\text{Given, } c = a * b] = (d * a) * b$$

[Associative holds in semigroup]

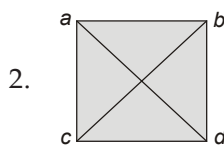
$$= b * b [\text{Given, } b * b = a] = a$$

4. (d)

If graph contain Euler circuit, it need not contain Hamiltonian cycle and vice-versa.



[ a - b - c - a - d - e - a is Euler circuit ]  
 [ no hamiltonian cycle. ]



[ No Euler circuit (all vertices must even degree) ]  
 [ a - b - d - c - a is hamiltonian cycle. ]

(1) is Euler but not Hamiltonian graph.

(2) is Hamiltonian but not Euler graph.

5. (d)

$$a_n = -4a_{n-1} + 12 a_{n-2}$$

$$a_n + 4a_{n-1} - 12 a_{n-2} = 0$$

$$x^2 + 4x - 12 = 0$$

$$(x + 6) (x - 2) = 0$$

$$x = -6, x = 2$$

$$\therefore a_n = A(-6)^n + B \cdot (2)^n$$

6 (c)

Number of vertices = 11

In  $K_n$  each vertex can contain degree 10.

In complement of G:

(10-1, 10-1, 10-2, 10-2, 10-3, 10-3, 10-3, 10-3, 10-4, 10-5, 10-5) = (9, 9, 8, 8, 7, 7, 7, 7, 6, 5, 5) is degree sequence.

So option (c) is correct.

7. (a)

1.  $S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$ • **Check for Reflexive Relation:** $(x, x) : x = x + 1$  but  $x \neq x + 1$ 

Hence cannot be reflexive S is not equivalence relation on R.

2.  $T = \{(x, y) : x - y \text{ is an integer}\}$ • **Check for Reflexive Relation:** $(x, x) : x - x$  is integer  $x - x = 0$  and  $0 \in \text{integer}$ 

So, T is reflexive.

• **Check for Symmetric Relation:** $(x, y) : x - y$  is integer and  $(y, x) : y - x$  also an integer.

So, T is symmetric relation.

• **Check for Transitive Relation:** $(x, y) : x - y$  is integer and  $(y, z) : y - z$  is integer then  $(x, z) : x - z$  is also integer.

So, T is transitive.

Hence T is equivalence relation but S is not.

8. (b)

"No students are allowed to carry smartphone"

Can be written as: Not a student are allowed to carry smartphone

 $\equiv \neg[\exists x(\text{student}(x) \wedge \text{carry\_smartphone}(x))]$  $\equiv \forall x(\neg \text{student}(x) \vee \neg \text{carry\_smartphone}(x))$  $\equiv \forall x(\text{student}(x) \rightarrow \neg \text{carry\_smartphone}(x))$ 

So, option (b) is correct representation only.

9. (d)

$$f(x) = \frac{x}{x-1}$$

$$f \circ f(x) = \frac{\left(\frac{x}{x-1}\right)}{\left(\frac{x}{x-1}\right)-1} = \frac{\frac{x}{x-1}}{\frac{x-x+1}{x-1}} = \frac{x}{x-1}$$

i.e.  $\underbrace{f \circ f(x)}_{2 \text{ times}} = x$

So,  $f \circ \underbrace{(f \circ f \circ f \circ \dots \circ f)}_{20 \text{ times}}(x) = f(x)$

$$= \frac{x}{x-1}$$

10. (d)

- If a graph is connected, then its complement may or may not be disconnected.  
**Example:** cyclic graph on 5 vertices.
- Chromatic number of complete graph with  $n$  vertices is  $n$ .
- If two graph  $G_1$  and  $G_2$  are isomorphic, then their complements will always be isomorphic.
- If any simple graph with  $n$  nodes with nodes  $> 1$ , there are atleast two vertices of same degree.

11. (b)

Let  $p$  : GATE rank is needed  
 $q$  : I will write the GATE exam  
 $r$  : I will join in MADEEASY.

Given arguments:

$P_1$ : If GATE rank is needed, i will not write GATE exam, if i do not join MADEEASY.

$$p \rightarrow (\sim r \rightarrow \sim q) = (p \wedge \sim r) \rightarrow \sim q$$

$P_2$ : GATE rank is needed :  $p$

$P_3$ : I will join MADEEASY :  $r$

**Q:** I will write the GATE exam :  $q$

Inference is:  $(p \wedge \sim r) \rightarrow \sim q$

$$\frac{p}{\frac{r}{q}}$$

We can also write the above inference as following:  $(p \wedge \sim r)$

$$([(p \wedge \sim r) \rightarrow \sim q] \wedge p \wedge r) \rightarrow q$$

If above proposition is tautology then given inference is valid.

$$((pr)' + q)' + p' + r' + q$$

$$= pr'q + p' + r' + q$$

$$= p' + r' + q \text{ which is consistency hence invalid.}$$

12. (a)

Total number of terms =  $8 + 1 = 9$

The middle term is : 5<sup>th</sup> term

$(x + y)^n$  has  $(r + 1)$ <sup>th</sup> term as :  ${}^n C_r x^{n-r} y^r$

[(4 + 1)<sup>th</sup> term] 5<sup>th</sup> term is:

$$\begin{aligned} & {}^8 C_4 \left( \frac{y\sqrt{x}}{3} \right)^{8-4} \left( \frac{-3}{x\sqrt{y}} \right)^4 \\ &= {}^8 C_4 \cdot \frac{y^4 \cdot x^2}{3^4} \cdot \frac{3^4}{x^4 \cdot y^2} \\ &= {}^8 C_4 \cdot \frac{y^2}{x^2} \\ &= 70 \left( \frac{y}{x} \right)^2 \end{aligned}$$

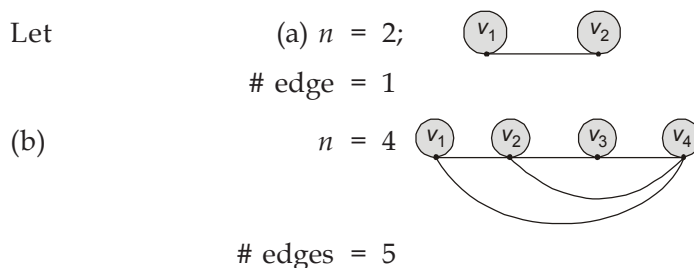
13. (c)  
 $\forall x \in N [(x \neq 7 \wedge \text{Prime}(x)) \rightarrow \neg \text{Divisibleby7}(x)]$   
 $\cong$   
 $\forall x \in N [x = 7 \vee \neg \text{Prime}(x) \vee \neg \text{Divisibleby7}(x)]$   
 $\cong$   
 $\neg \exists x \in N [x \neq 7 \wedge \text{Prime}(x) \wedge \text{Divisibleby7}(x)]$   
 All represents that "no prime except 7 is divisible by 7".

14. (a)  
 Put  $x = y$  and  $y = x$  at the end to get inverse function

$$\begin{aligned}
 & y = 2 \cdot 2^x + 4^x \\
 \Rightarrow & x = 2 \cdot 2^y + 4^y \\
 \Rightarrow & x = 2 \cdot 2^y + (2^y)^2 \\
 \Rightarrow & x+1 = (2^y)^2 + 2 \cdot 2^y + 1 \\
 \Rightarrow & x+1 = (2^y + 1)^2 \\
 \Rightarrow & \sqrt{x+1} = 2^y + 1 \\
 \Rightarrow & 2^y = \sqrt{x+1} - 1 \\
 \Rightarrow & \log 2^y = \log(\sqrt{x+1} - 1) \\
 \Rightarrow & y \log 2 = \log(\sqrt{x+1} - 1) \\
 \Rightarrow & y = \frac{\log(\sqrt{x+1} - 1)}{\log 2}
 \end{aligned}$$

15. (d)
- $R = \{(x, y) \mid x \equiv y \pmod{m}\}$  when  $x = y$  then  $x \equiv x \pmod{m}$  is always reflexive.
  - $R = \{(x, y) \mid x \equiv y \pmod{m}\}$   $(x - y) \pmod{m}$  is always equal to  $(y - x) \pmod{m}$ . So relation is always symmetric.
  - $R = \{(x, y) \mid x \equiv y \pmod{m}\}$  if  $(x - y) \pmod{m}$  is always equal to  $(y - z) \pmod{m}$ . Which is equal to  $(x - z) \pmod{m}$  so relation is always transitive.
- The given relation  $R = \{(x, y) \mid x \equiv y \pmod{m}\}$  is equivalence relation (reflexive, symmetric and transitive).

16. (b)



So option (b) is correct.

17. (a)  
**Everybody loves India:**  $\forall x \text{ Loves}(x, \text{India})$   
**Everybody loves somebody:**  $\forall x \exists y \text{ Loves}(x, y)$   
**There is somebody whom everybody loves:**  $\exists y \forall x \text{ Loves}(x, y)$   
**There is somebody whom no one loves:**  $\exists y \forall x \neg \text{Love}(x, y)$

18. (b)

- Set A is countable. Since Q (set of rational numbers) is countable and every subset of countable set is also countable.
- Set B is uncountable. Since every subset of real number is uncountable.
- Set C is countable because it is Cartesian product of two countable sets i.e.  $N \times Z$ .
- Set D is countable. Since one to one correspondence with set of natural number Cantor's theorem.

19. (a)

$$\text{Total children} = 75$$

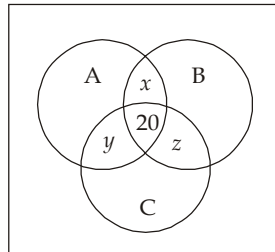
$$\therefore \text{Total receipt} = ₹ 70 \text{ (₹ 0.50/ride)}$$

$$\therefore \text{Total rides} = 70 \times 2 = 140$$

20 children had taken all the 3 rides

$$\therefore 55 \text{ had taken at least 2 rides (2 or 3 rides).}$$

So,  $55 - 20 = 35$  had taken exactly 2 rides.



Let,  $x + y + z = 35$

Children who had taken exactly one ride

$$\begin{aligned} \text{Total single ride} &= 140 - (35 \times 2 + 20 \times 3) \\ &= 140 - (70 + 60) = 10 \end{aligned}$$

So, total number of students who took exactly single ride = 10

$$\begin{aligned} \text{Children who took no ride} &= 75 - (35 + 20 + 10) \\ &= 75 - (65) = 10 \end{aligned}$$

20. (b)

- $R$  is not reflexive since  $\phi$  is an element of power, set of any subset of A and  $\phi \cap \phi = \phi$  and belongs to  $R$ .
- $R$  is symmetric because intersection ( $\cap$ ) is commutative, thus  $a \cap b \neq \phi$  the  $b \cap a \neq \phi$ .
- $R$  is not transitive because  $a \cap b \neq \phi$  and  $b \cap c \neq \phi$  does not assure  $a \cap c \neq \phi$ . **e.g.**,  $a = \{1, 2\}$ ,  $b = \{2, 3\}$  and  $c = \{3, 4\}$

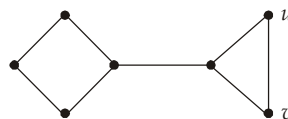
$$\text{So, } \{1, 2\} \cap \{2, 3\} \neq \phi$$

$$\{2, 3\} \cap \{3, 4\} \neq \phi$$

but  $\{1, 2\} \cap \{3, 4\} = \phi$  so **fail**.

21. (b)

(a) Consider a graph:



$$n(G) \geq d(u) + d(v)$$

$$7 \geq 2 + 2$$

$$7 \geq 4 \text{ satisfied but } u/v \text{ is not cut edge. So false}$$

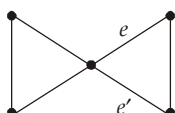
(b) Let,  $G$  be a graph such that  $|E_G| < |V_G|$  further, suppose  $G_1, G_2, G_2 \dots G_k$  are connected components of  $G$ , and if no connected component of  $G$  is a tree.

Hence, for each  $1 \leq i \leq k$ ,  $|E_{G_i}| \geq |V_{G_i}|$ . Thus,

$$|E_G| = \sum_{i=1}^k |E_{G_i}| \geq \sum_{i=1}^k |V_{G_i}| \geq |V_G|$$

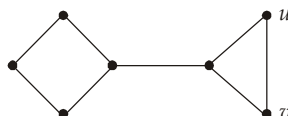
Which is a contradiction. Hence, there exists a component of  $G$  which is tree.

(c) Consider a graph:



Since graph is Eulerian graph but don't have eulerian circuit. So false

(d) Consider a graph:



Average degree ( $G$ ) =  $\frac{2e}{n}$

Average degree ( $G$ ) =  $\frac{2e}{n}$

Before removal of 'v' =  $\frac{20}{7} = 2.857$

After removal of 'v' =  $\frac{16}{6} = 2.66$

So false

22. (c)

$$a_n = a_{n-1} + 3^{n-1}$$

$$a_n = \sum_{i=0}^n a_i x^i$$

$$= 1 + \sum_{i=1}^n a_i x^i$$

$$= 1 + \sum_{i=1}^n (a_{i-1} + 3^{i-1}) x^i$$

$$= 1 + \sum_{i=1}^n (a_{i-1} + x^i) + \sum_{i=1}^n (3^{i-1} x^i)$$

$$= 1 + x \left[ \sum_{i=0}^n a_i x^i \right] + x \left( \sum_{i=0}^n 3^i x^i \right)$$

$$a_n = 1 + x a_n + \frac{x}{1-3x}$$

$$a_n(1-x) = 1 + \frac{x}{1-3x}$$

$$\begin{aligned}
 a_n &= \frac{1-3x+x}{(1-3x)(1-x)} = \frac{1-2x}{(1-x)(1-3x)} \\
 &= \frac{A}{1-x} + \frac{B}{1-3x} = \frac{\frac{1}{2}}{1-x} + \frac{\frac{1}{2}}{1-3x} \\
 a_n &= \frac{1}{2}(1+x+x^2+x^3+\dots) + \frac{1}{2}(1+3x+(3x^2)+\dots) \\
 &= \frac{1}{2}(1+3^n)
 \end{aligned}$$

23. (a)

$P_1$  :  $xyz^{-1}w = 1$ , then  $y = x^{-1}w^{-1}z$   
 Put  $y = x^{-1}w^{-1}z$  in  $xyz^{-1}w = 1$   
 $x(x^{-1}w^{-1}z)z^{-1}w = 1$   
 $w^{-1}z z^{-1}w = 1$   
 $w^{-1}w = 1$

$1 = 1$  Hence true

$P_2$  :  $xyz = 1$ , then  $xyz = 1$

Assume,  $x = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ,  $y = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ,  $z = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$xyz = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$yxz = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

And  $xyz \neq yxz$  Hence false

24. (b)

- Every cyclic group is Abelian group but every Abelian group is not cyclic group.
- Every group of prime order is cyclic group and we know that every cyclic group is Abelian group hence, every group of prime order is Abelian group.
- If  $(G, *)$  be a cyclic group of even order, then there exist atleast one elements other than identity element such that  $a^{-1} = a$ .

25. (d)

- On empty set  $\phi$  is an equivalence relation therefore  $S_1$  is false.
- In  $S_2$  and relation is not transitive.

e.g.

$$(2, 5) \in R \text{ as } 10 \geq 1$$

$$\left(5, \frac{1}{4}\right) \in R \text{ as } \frac{5}{4} \geq 1$$

but  $\left(2, \frac{1}{4}\right) \notin R \text{ as } \frac{2}{4} < 1 \Rightarrow \text{Not transitive}$

So, both  $S_1$  and  $S_2$  are false.



26. (c)

$$a_k = -8a_{k-1} - 15a_{k-2}$$

$$k - 2 = 1$$

$$k - 1 = n$$

and

$$k = n^2$$

Using characteristics equation

$$n^2 + 8n + 15 = 0$$

$$n = -3$$

and

$$n = -5$$

So,

$$a_k = (-3)^k C_1 + (-5)^k C_2$$

$$= (-3)^0 C_1 + (-5)^0 C_2 = 0$$

$$C_1 + C_2 = 0$$

...(i)

$$a_1 = (-3)^1 C_1 + (-5)^1 C_2$$

$$= -3 C_1 + (-5) C_2 = 2$$

...(ii)

Solving equation (i) and (ii), we get,

$$C_1 = 1$$

and

$$C_2 = -1$$

then

$$a_n = (-3)^k - (-5)^k$$

27. (b)

- I is not  $D_{42}$  because the divisor 7 is missing. So, there is no way for I to be isomorphic to  $(P\{a, b, c\}, \subseteq)$  as it needs to have 8 divisors but right now it has only 7.
- II is  $D_{66}$  a well known boolean algebra and has 8 vertices and its masses diagram will be isomorphic  $(P(\{a, b, c\}), \subseteq)$ .
- III is not isomorphic even though it looks like  $D_{70}$ , it is on the relation  $\leq$ , resulting in a chain, which won't be boolean algebra.

28. (c)

- $G_1$  has 6 items and  $G_2$  has 5 cycles. Hence it can not be isomorphic.
- $G_3$  and  $G_4$  are also isomorphic.
- Hence the option (c) is correct.

29. (d)

Each ambulance "covers" the adjacent roads, and all roads are covered in this way.

30. (d)

This says that there is a bound,  $m$ , such that any twin prime is below  $m$ . In other words, that there only finitely many twin primes.