CLASS TEST

S.No. : 01 SK_CS_ABCDE_250722

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ENGINEERING MATHEMATICS

COMPUTER SCIENCE & IT

Date of Test : 25/07/2022

ANSWER KEY

1.	(a)	7.	(b)	13.	(c)	19.	(c)	25.	(b)
2.	(d)	8.	(d)	14.	(a)	20.	(c)	26.	(d)
3.	(b)	9.	(d)	15.	(d)	21.	(c)	27.	(b)
4.	(c)	10.	(b)	16.	(b)	22.	(b)	28.	(c)
5.	(c)	11.	(b)	17.	(b)	23.	(a)	29.	(c)
6.	(d)	12.	(a)	18.	(b)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$R_2 + 2R_1$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or $$y = -4$$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

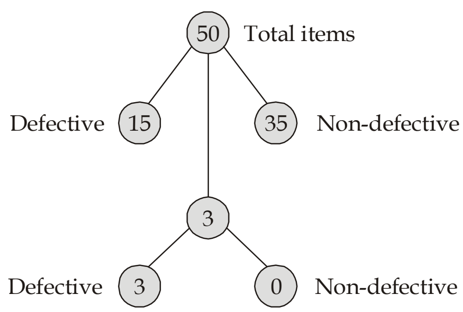
$\Rightarrow$ $$x = 12$$

$\therefore$ $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

2. (d)

Mean, median and mode are all same (μ) for normal distribution.

3. (b)



$$\text{Required probability} = \frac{{}^{15}C_3 \times {}^{35}C_0}{{}^{50}C_3}$$

$$= \frac{15 \times 14 \times 13}{50 \times 49 \times 48} = \frac{13}{560}$$

4. **(c)**

The characteristic equation $|A - \lambda I| = 0$

$$\text{i.e.} \qquad \begin{vmatrix} 4-\lambda & 6 \\ 2 & 8-\lambda \end{vmatrix} = 0$$

$$\begin{aligned}
\text{or} \quad (4-\lambda)(8-\lambda) - 12 &= 0 \\
\text{or} \quad 32 - 8\lambda - 4\lambda + \lambda^2 - 12 &= 0 \\
\Rightarrow \quad \lambda^2 - 12\lambda + 20 &= 0 \\
\Rightarrow \quad \lambda^2 - 10\lambda - 2\lambda + 20 &= 0 \\
\Rightarrow \quad (\lambda - 10)(\lambda - 2) &= 0 \\
\Rightarrow \quad \lambda &= 10, 2
\end{aligned}$$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]x = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{aligned}
\text{Which gives,} \quad -6x + 6y &= 0 \\
\Rightarrow \quad x &= y \\
2x - 2y &= 0 \\
\Rightarrow \quad x &= y
\end{aligned}$$

i.e. eigen vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Corresponding to $\lambda = 2$, we have

$$[A - \lambda I]x = \begin{bmatrix} 2 & 6 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Which gives, $2x + 6y = 0$ i.e. eigen vector $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

5. **(c)**

$$\lim_{x \to \infty}\left[1 + \frac{3}{2x}\right]^{5x}$$

Put limit $x \to \infty$

1^{∞} from create,

So, we know, for form 1^{∞}

$$\lim_{x \to \infty} f(x)^{g(x)} = e^{\left(\lim_{x \to \infty}(f(x)-1)\cdot g(x)\right)}$$

Apply in given function:

$$= e^{\lim_{x \to \infty}\left[1 + \frac{3}{2x} - 1\right]5x}$$

$$= e^{\lim_{x \to \infty}\left[\frac{3}{2x}\right]5x}$$

$$= e^{15/2}$$

6. **(d)**

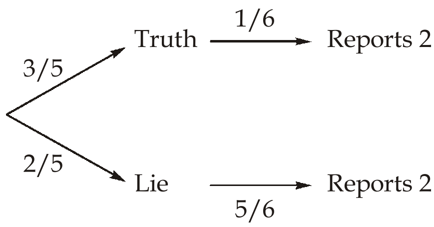
Check for continuous:

$$\begin{aligned} f(-2) &= -1.5 \times (-2)^2 = -6 \\ f(-2^+) &= 6(-2) - 5 = -17 \\ f(-2^-) &= -1.5 \times (-2)^2 = -6 \\ f(-2^-) &\neq f(-2^+) \end{aligned}$$

Function is not continuous, hence cannot be differentiable i.e. differentiable → continuous.

7. **(b)**

Applying Bayes Theorem:



So,

$$\text{P(spoke truth/reports 2)} = \frac{\text{P(spoke truth} \cap \text{reports 2)}}{\text{P(reports 2)}} = \frac{\frac{3}{5}\times\frac{1}{6}}{\frac{3}{5}\times\frac{1}{6}+\frac{2}{5}\times\frac{5}{6}} = \frac{3}{13}$$

8. **(d)**

$$D = -96 \text{ for the given matrix}$$

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

$$\therefore \quad \text{Det}(A) = (2)^3 \times D = 8 \times (-96) = -768$$

9. **(d)**

$$\lim_{x\to 4} \frac{(2x)^{1/3} - 2}{2x - 8}$$

Above form is $\left(\frac{0}{0}\right)$ by putting the value $x = 4$

Applying L' Hospital rule

$$= \lim_{x\to 4} \frac{\frac{1}{3}(2x)^{\left(\frac{1}{3}-1\right)} \times 2}{2} = \lim_{x\to 4} \frac{1}{3}(2x)^{\left(-\frac{2}{3}\right)}$$

$$= \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$$

10. (b)

$$\begin{aligned}
f(x) &= x^3 - 6x^2 + 9x + 1 \\
f'(x) &= 3x^2 - 12x + 9 = 0 \\
x^2 - 4x + 3 &= 0 \\
x^2 - 3x - x + 3 &= 0 \\
x(x-3) - 1(x-3) &= 0 \\
(x-1)(x-3) &= 0 \\
x &= 1, x = 3 \\
f''(x) &= 6x - 12
\end{aligned}$$

We need to check at all the extremum points i.e. 1, 3, 0, 5.

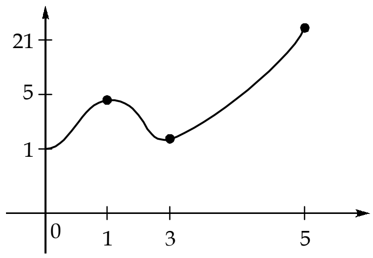
At 1, $f''(x) = -6 < 0$ (maximum)

At 3, $f''(x) = 6 > 0$ (minimum)

Taking into account all points:

$$\begin{aligned}
f(0) &= 1 \\
f(1) &= 5 \\
f(3) &= 1 \\
f(5) &= 21
\end{aligned}$$

Hence roughly graph can be drawn like:



Thus, maximum at 5 and minimum can be at 0 or 3.

11. (b)

We know that, $E(X) = 10$

and $Var(X) = 25$

Now, $E(Y) = E(aX - b) = 0$

$$aE(X) - b = 0$$

$$\Rightarrow \quad a(10) - b = 0$$

$$10a - b = 0 \quad \ldots(i)$$

Given, $Var(Y) = 1$

$$Var(aX - b) = a^2\, Var(X) = 1$$

$$\Rightarrow \quad 25a^2 = 1$$

i.e $a = \pm\dfrac{1}{5}$

$a = \dfrac{1}{5}$ (taking positive values only)

By putting value of 'a' in equation (i)

We get $b = 2$

12. (a)

For rectangular distribution

$$\text{Variance} = \frac{(b-a)^2}{12}$$

Here, $a = 0, b = \frac{1}{2}$

$$\therefore \quad \text{Variance} = \frac{\left(\frac{1}{2}-0\right)^2}{12} = \frac{\frac{1}{4}}{12} = \frac{1}{4\times 12}$$

$$\text{Then standard deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{1}{4\times 12}} = \frac{1}{2\sqrt{12}}$$

13. (c)

Given matrix is $M = \begin{bmatrix} 12+9i & -i \\ i & 12-9i \end{bmatrix}$

Determinant of $M = \begin{vmatrix} 12+9i & -i \\ i & 12-9i \end{vmatrix} = (12 + 9i)(12 - 9i) + i^2$

$$= (12^2 - 9^2 i^2) + i^2$$

$$= 225 - 1 = 224$$

$$\therefore \quad \text{Inverse of } M = M^{-1} = \frac{1}{|M|}(adjM)$$

$$= \frac{1}{224}\begin{bmatrix} 12-9i & i \\ -i & 12+9i \end{bmatrix}$$

14. (a)

Let, $'A' = \{a_1, a_2, a_3 \ldots\ldots\ldots a_n\}$

There is an element a_1 of 'A' and two subsets 'P' and 'Q', then four possibilities

$$\left.\begin{array}{l} \text{(a) } a_1 \in P \text{ and } a_1 \in Q \\ \text{(b) } a_1 \in P \text{ and } a_1 \notin Q \\ \text{(c) } a_1 \notin P \text{ and } a_1 \in Q \\ \text{(d) } a_1 \notin P \text{ and } a_1 \notin Q \end{array}\right\} \text{4 choices}$$

Total number of ways selecting 'P' and 'Q' = 2^n

$\Rightarrow \quad 2^n \times 2^n = 4^n$ ways

$\Rightarrow \quad n(S) = 4^n$

Number of favorable elements = 3^n

$$P(E) = \frac{n(E)}{n(S)} = \frac{3^n}{4^n}$$

$$= (0.75)^n$$

15. (d)

function $f(x)$ is continuous for every $x \neq 0$ (since $\frac{x-c}{1+c}$ and $x^2 + c$ are polynomials, and polynomials are continuous).

$$f(0) = \frac{0-c}{1+c} = \frac{-c}{1+c}$$

$$\lim_{x \to 0^-} \frac{0-c}{1+c} = \frac{-c}{1+c}$$

$$\lim_{x \to 0^+} 0^2 + c = c$$

Since $f(x)$ is continuous for every x, hence continuous for $x = 0$.

$$\Rightarrow \quad f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \quad \frac{-c}{1+c} = c$$

$$\Rightarrow \quad -c = c(1+c)$$

$$c^2 + 2c = 0$$

$$c = -2 \text{ or } c = 0$$

So option (d) is correct answer

16. (b)

To check matrix is LU decomposible by checking if principal minors have non-zero determinants.

Check (a):

$$|A_1| = |\,1\,| = 1 \neq 0$$

Now $$|A_2| = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = 0$$

So option (a) is not LU decomposible.

Check (b):

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \text{ here } |A_1| = 3, \; |A_2| = \begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

So LU decomposible.

Check (c):

$$\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} \text{ here } |A_1| = 0$$

So not LU decomposible.

Check (d):

$$\begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix} \text{ here } |A_1| = 1 \neq 0 \text{ but}$$

$$|A_2| = \begin{vmatrix} 1 & -3 \\ -2 & 6 \end{vmatrix} = |6 - 6| = 0$$

So not LU decomposible.

17. (b)

Let, P(G) represent given day mood is good.
P(S) represent given day is sunny.

		Weather		
		Sunny	Not sunny	
Mood	Good	12	9	21
	Not Good	4	5	9
		16	14	30

So, $$P(G\,|\,S) = \frac{P(G \cap S)}{P(S)}$$

$$P(G \cap S) = \frac{12}{30}$$

$$P(S) = \frac{16}{30}$$

So, $$P(G\,|\,S) = \frac{\frac{12}{30}}{\frac{16}{30}} = \frac{12}{16} = \frac{3}{4}$$

18. (b)

Consider, $$u = \cot x$$

$$\frac{du}{dx} = -\text{cosec}^2\, x$$

$$du = -\text{cosec}^2\, x\, dx$$

$$-du = \text{cosec}^2\, x\, dx$$

Now new limits:

$$x = \frac{\pi}{6} \to u = \cot\frac{\pi}{6} = \sqrt{3}$$

$$x = \frac{\pi}{3} \to u = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

Substitute new limits and $\text{cosec}^2\, x\, dx$

$$\int_{\sqrt{3}}^{1/\sqrt{3}} \frac{-du}{u^2} = \left[\frac{u^{-2+1}}{-2+1}\right]_{\sqrt{3}}^{1/\sqrt{3}} = \left[u^{-1}\right]_{\sqrt{3}}^{1/\sqrt{3}} = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

19. (c)

$$\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(\frac{3 + a^x + b^x + c^x - 3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1 + \frac{a^x + b^x + c^x - 3}{3}\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1 + \frac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{3}\right)^{1/x}$$

We know that:

$$\lim_{x\to 0}(1+\lambda x)^{1/x} = e^{\lambda}$$

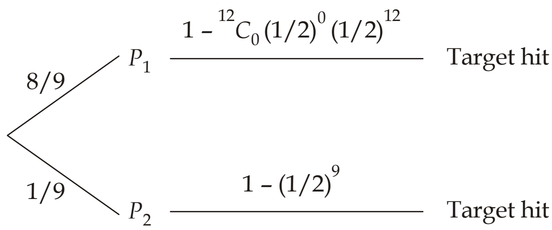
$$= e^{\lim_{x\to 0}\frac{(a^x-1)}{3x}+\frac{(b^x-1)}{3x}+\frac{(c^x-1)}{3x}} = e^{\lim_{x\to 0}\frac{1}{3}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)}$$

$$= e^{1/3\ (\log a + \log b + \log c)} \qquad \left[\because \lim_{x\to 0}\frac{a^x-1}{x} = \log a\right]$$

$$= e^{1/3 \log (abc)} = e^{\log (abc)^{1/3}} = (abc)^{1/3}$$

$$= \sqrt[3]{abc}$$

20. (c)



$$P(\text{Target hit}) = \frac{8}{9}\left(1-\frac{1}{2^{12}}\right)+\frac{1}{9}\left(1-\frac{1}{2^9}\right)$$

So option (c) is correct answer.

21. (c)

$$AX = \lambda X$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

$$3a - 6b = -18 \qquad \text{... (i)}$$

$$3c - 6d = 36 \qquad \text{... (ii)}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$3a - 3b = -9 \qquad \text{... (iii)}$$

$$3c - 3d = 9 \qquad \text{... (iv)}$$

From equation (i) and (iii), $a = 0$ and $b = 3$.

From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

22. (b)

$$y = 7x^2 + 12x$$

Using Lagrange's mean value theorem:

At $\quad x = 1, y = 7 + 12 = 19$

$\quad x = 3, y = 63 + 36 = 99$

$$f'(x) = \frac{f(b)-f(a)}{b-a}$$
$$= \frac{99-19}{3-1} = 40$$

So option (b) is correct answer.

23. (a)

Since, $\sum_{x=0}^{4} P(x) = 1$

$$c + 2c + 2c + c^2 + 5c^2 = 1$$
$$6c^2 + 5c - 1 = 0$$
$$c = \frac{1}{6}, -1$$

Since $P(x) \geq 0$, the possible value of

$$c = \frac{1}{6}$$

x	0	1	2	3	4
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{36}$	$\frac{5}{36}$
$xP(x)$	0	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{3}{36}$	$\frac{20}{36}$

$$\text{Mean} = \sum_{x=0}^{4} xP(x) = 0 + \frac{2}{6} + \frac{4}{6} + \frac{3}{36} + \frac{20}{36} = \frac{59}{36} = 1.638$$

$$\text{Variance} = \sigma^2 = E(x^2) - [E(x)]^2$$

$$= \left[0\left(\frac{1}{6}\right) + 1\left(\frac{2}{6}\right) + 4\left(\frac{2}{6}\right) + 9\left(\frac{1}{36}\right) + 16\left(\frac{5}{36}\right) - \left(\frac{59}{36}\right)^2\right] = 1.45$$

24. (b)

$$f(x) = (x^2 - 4)^2$$
$$f'(x) = 2(x^2 - 4) \times 2x$$
$$= 4x(x^2 - 4) = 0$$
$$x = 0,\ x = 2 \text{ and } x = -2 \text{ are the stationary points}$$
$$f''(x) = 4[x(2x) + (x^2 - 4) \times 1]$$
$$= 4[(2x^2 + (x^2 - 4)] = 4[3x^2 - 4]$$
$$= 12x^2 - 16$$
$$f''(0) = -16 < 0 \quad \text{(So maxima at } x = 0)$$
$$f''(2) = (12)2^2 - 16 = 32 > 0 \quad \text{(So minima at } x = 2)$$
$$f''(-2) = 12(-2)^2 - 16 = 32 > 0 \quad \text{(So minima at } x = -2)$$

$\therefore$ There is only one maxima and only two minima for this function.

25. (b)

- $ABA^{-1} = B$ given,

$\Rightarrow$ $AB = BA$ since matrix multiplication is not commutative. So false even if A is invertible.

- A is idempotent, so $A^2 = A$, since A is non-singular, so it is invertible i.e. A^{-1} exist.

$$I = A^{-1} . A = A^{-1} . A^2 = IA = A$$

So A must be identity matrix. So true.

- If coefficient matrix A is invertible for $Ax = b$ then $x = A^{-1}$ unique solution exist. So false
- If B is zero matrix, then also $AB = B$ = zero matrix. So false

26. (d)

The characteristic equation is $|A - \lambda I| = 0$

i.e.,
$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 8-\lambda \end{vmatrix} = 0$$

$$(4 - \lambda)(8 - \lambda) - 10 = 0$$
$$\lambda^2 - 12\lambda + 20 = 0$$
$$(\lambda - 10)(\lambda - 2) = 0$$
$$\lambda = 10, 2$$

Corresponding to $\lambda = 10$, we have

$$[A - \lambda I]X = \begin{bmatrix} -6 & 6 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

which gives
$$\left.\begin{matrix} -6a + 6b = 0 \\ 2a - 2b = 0 \end{matrix}\right\} a = b$$

i.e., eigen vector can be the answer and is present in one of the option (d). Similarly $\lambda = 2$ also have eigen vectors i.e. not mentioned in any options.

27. (b)

$$x + \frac{50}{x} > 15$$

$\Rightarrow$
$$x^2 - 15x + 50 > 0$$
$$x^2 - 5x - 10x + 50 > 0$$
$$(x - 5)(x - 10) > 0$$

Cases :

(i) $x > 5$ and $x > 10 \;\Rightarrow\; x > 10$

(ii) $x < 5$ and $x < 10 \;\Rightarrow\; x < 5$

So, $x < 5 = \{1, 2, 3, 4\}$

$x > 10 = \{11, 12, 13, \ldots\ldots, 20\}$

So, total favourable cases : 4 + 10 = 14

$$\text{The required probability} = \frac{14}{20}$$

28. (c)

$$P = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$$

$$A = PDP^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A^4 = A^2 \times A^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$$

$$A^5 = A^4 \times A = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}\begin{bmatrix} 2 & -12 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 32 & -192 \\ 0 & -32 \end{bmatrix}$$

29. (c)

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix} = AA^t$$

There are three cases for the rank of A.

Case I : rank $(A) = 0$

$\Rightarrow A$ is null. So, $B = AA^1$ also has to be null and hence rank (B) is also equal to 0. Therefore in this case rank (A) = rank (B).

Case II : rank $(A) = 2$

So, A has to be non-singular, i.e., $|A| \neq 0$. Therefore, $|B| = |A|^2$ is also $\neq 0$. So, rank $(B) = 2$. Therefore, in this case also rank (A) = rank (B).

Therefore, in all three cases rank (A) = rank (B). So, rank of A is N, then the rank of matrix B is also N.

30. (a)

Augmented matrix:

$$[A\,|\,B] = \left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 2 & 3 & 5 & 9 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_3 \leftarrow R_3 - R_2$

$R_2 \leftarrow 4R_2 - R_1$

$$\left[\begin{array}{ccc|c} 8 & 3 & -2 & 8 \\ 0 & 9 & 22 & 28 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$$

If $\lambda = 5$ and $\mu \neq 9$, then system has no solution because Rank$[A \mid B] \neq$ Rank [4].

■■■■