CLASS TEST

S.No. : 01 PT_EE+EC_100722

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Lucknow | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

SIGNALS & SYSTEMS

EC + EE

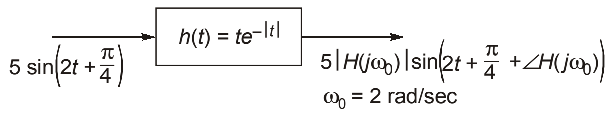
Date of Test : 21/07/2022

ANSWER KEY

1.	(d)	7.	(b)	13.	(c)	19.	(d)	25.	(b)
2.	(d)	8.	(b)	14.	(a)	20.	(c)	26.	(b)
3.	(a)	9.	(c)	15.	(b)	21.	(c)	27.	(b)
4.	(c)	10.	(a)	16.	(a)	22.	(d)	28.	(a)
5.	(c)	11.	(c)	17.	(a)	23.	(d)	29.	(c)
6.	(c)	12.	(d)	18.	(b)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1. (d)



Impulse response, $h(t) = te^{-|t|}$

$$H(j\omega) = j\frac{d}{d\omega}\left(\frac{2}{1+\omega^2}\right) = \frac{-4j\omega}{(1+\omega^2)^2}$$

$$|H(j\omega_0)| = \left|\frac{-4j(2)}{(1+4)^2}\right| = \frac{8}{25} \qquad (\omega_0 = 2 \text{ rad/sec})$$

$$\angle H(j\omega_0) = -90°$$

$$\text{output, } y(t) = 5\times\frac{8}{25}\sin\left(2t+\frac{\pi}{4}-\frac{\pi}{2}\right) = \frac{8}{5}\sin\left(2t-\frac{\pi}{4}\right)$$

$$= \frac{8}{5}\left(\frac{\sin 2t}{\sqrt{2}} - \frac{\cos 2t}{\sqrt{2}}\right)$$

$$= \frac{8}{5\sqrt{2}}(\sin 2t - \cos 2t)$$

$$= 1.13\,(\sin 2t - \cos 2t)$$

2. (d)

$\because$

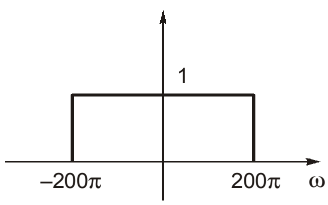
$$x(t) = \frac{1}{5}tu(t) - \frac{1}{5}(t-5)u(t-5)$$

$$= \frac{1}{5s^2} - \frac{e^{-5s}}{5s^2}$$

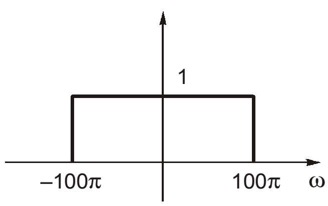
$$= \frac{1}{s^2}\left[\frac{1-e^{-s5}}{5}\right]$$

3. (a)

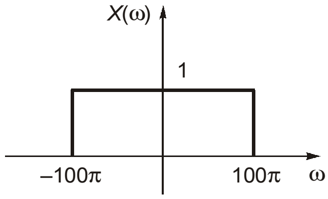
$$\frac{\sin(200\pi t)}{\pi t} \longleftrightarrow$$



$$\frac{\sin(100\pi t)}{\pi t} \longleftrightarrow$$



$$x(t) = \frac{\sin(200\pi t)}{\pi t} * \left(\frac{\sin(100\pi t)}{\pi t}\right) \longleftrightarrow$$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-100\pi}^{100\pi} (1)\, d\omega = \frac{200\pi}{2\pi} = 100$$

4. (c)

$$h(n) = 3\left(\frac{1}{2}\right)^n u(n) - 2\left(-\frac{1}{3}\right)^n u(n)$$

Taking 'z' transform, we get

$$H(z) = \frac{3}{\left(1-\frac{1}{2}z^{-1}\right)} - \frac{2}{\left(1+\frac{1}{3}z^{-1}\right)}$$

$$\frac{Y(z)}{X(z)} = \frac{3+z^{-1}-2+z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}} = \frac{1+2z^{-1}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}$$

$$Y(z) - \frac{1}{6}z^{-1}Y(z) - \frac{1}{6}z^{-2}Y(z) = X(z) + 2z^{-1}X(z)$$

Taking inverse 'z' transform

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n) + 2x(n-1)$$

$$y(n) = \frac{1}{6}y(n-1) + \frac{1}{6}y(n-2) + x(n) + 2x(n-1)$$

5. (c)

Given, $H(\omega) = -2j\omega$

From the definition of inverse fourier transform,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega$$

differentiate both sides,

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(\omega)\, e^{j\omega t} d\omega$$

$$-2\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{-2j\omega}_{H(\omega)}\, X(\omega)e^{j\omega t} d\omega$$

$\therefore$ Passing $x(t)$ through $H(\omega)$ is equivalent to perform $-2\frac{dx(t)}{dt}$

$$\therefore \quad y(t) = -2\frac{dx(t)}{dt}$$

given, $x(t) = e^{jt}$

$$\therefore \qquad y(t) = -2\frac{d}{dt}[e^{jt}]$$

$$y(t) = -2je^{jt}$$

6. (c)

Given, $x(n) \xleftrightarrow{z} X(z)$

by the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$= \sum_{n=-\infty}^{\infty}\sum_{k=0}^{\infty} a^k \delta[n-5k]z^{-n}$$

The term $\delta[n - 5k]$ is equal 1 if $n = 5k$ and equal to zero otherwise.

$$\therefore \qquad X(z) = \sum_{k=0}^{\infty} a^k z^{-5k} \qquad [\because n = 5k]$$

$$= \frac{1}{1-az^{-5}}$$

or $$X(z) = \frac{z^5}{z^5 - a}$$

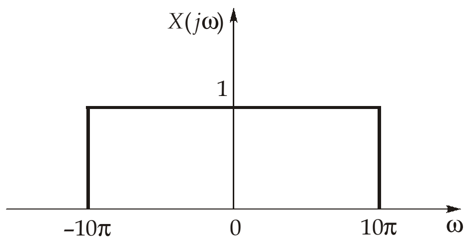
7. (b)

Given, $$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or



$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore \qquad T_s < \frac{1}{10}$$

8. (b)

The output of the given LTI system is,

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]e^{j\omega(n-k)} + \sum_{k=-\infty}^{+\infty} h[k]e^{j2\omega(n-k)}$$

$$= e^{j\omega n}\sum_{k=-\infty}^{+\infty} h[k]e^{-j\omega k} + e^{j2\omega n}\sum_{k=-\infty}^{+\infty} h[k]e^{-j2\omega k}$$

$$= e^{j\omega n}H(e^{j\omega}) + e^{j2\omega n}H(e^{j2\omega})$$

Since the input cannot be extracted from the above expression, the sum of the complex exponential is not an eigen function.

9. (c)

$$(1+\cos 300\pi t)^2 \rightarrow f_{1\,max} = 300\text{ Hz}$$

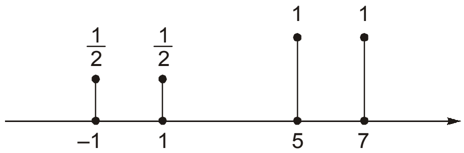
$$(\sin 4000\pi t)^2 \rightarrow f_{2\,max} = 4000\text{ Hz}$$

$$f_{max} = f_{1\,max} + f_{2\,max} = 4300\text{ Hz}$$

$$f_s = 2f_{max} = 8.6\text{ kHz}$$

10. (a)

$x[n+1] =$



$$x[2n+1] = 0$$

$\Rightarrow$ $$n\,x[2n+1] = y[n] = 0$$

$\Rightarrow$ $$\sum_{n=-\infty}^{\infty} y[n] = 0$$

11. (c)

Given that,

Let, $$y_1(t) = 2\pi X(-\omega)\big|_{\omega=t}$$

We have, $$y_1(t) = 2\pi\int_{u=-\infty}^{\infty} x(u)e^{jut}du$$

Similarly, let $y_2(t)$ be the output due to passing $x(t)$ through 'F' twice.

$$y_2(t) = 2\pi\int_{v=-\infty}^{\infty} 2\pi\int_{u=-\infty}^{\infty} x(u)e^{juv}du\,e^{jtv}dv$$

$$= (2\pi)^2\int_{u=-\infty}^{\infty} x(u)\int_{v=-\infty}^{\infty} e^{j(t+u)v}dv\,du$$

$$= (2\pi)^2\int_{u=-\infty}^{\infty} x(u)(2\pi)\delta(t+u)du$$

$$= (2\pi)^3 X(-t)$$

Finally, let $y_3(t)$ be the output due to passing $x(t)$ through F three times

$$y_3(t) = 2\pi \int_{u=-\infty}^{\infty} (2\pi)^3 x(-u) e^{jtu} du$$

$$= (2\pi)^4 \int_{-\infty}^{\infty} e^{-jtu} x(u) du = (2\pi)^4 X(t)$$

12. (d)

Given, $x_1(n) = x(2n)$

From the definition of z-transform,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\therefore \quad X_1(z) = \sum_{n=-\infty}^{\infty} x(2n) z^{-n}$$

$$K = 2n \Rightarrow n = \frac{K}{2}$$

at $n = -\infty \Rightarrow K = -\infty$

$n = +\infty \Rightarrow K = +\infty$

$$= \sum_{K=-\infty}^{\infty} x(K) z^{-\frac{K}{2}}$$

$$= \sum_{K=-\infty}^{\infty} \left[\frac{x(K) + (-1)^K x(K)}{2}\right] z^{-\frac{K}{2}}; \quad K \text{ even}$$

$$= \frac{1}{2} \sum_{K=-\infty}^{\infty} x(K) z^{-\frac{K}{2}} + \frac{1}{2} \sum_{K=-\infty}^{\infty} x(K) \left(-z^{\frac{1}{2}}\right)^{-K}$$

From the definition of z-transform

$$X_1(z) = \frac{1}{2}\left[X(\sqrt{z}) + X(-\sqrt{z})\right]$$

13. (c)

The fourier transform of $x(t)$ can be written

$$X_1(j\omega) = |X_1(j\omega)| e^{j\angle X_1(j\omega)}$$

Let, $$X_{1a}(j\omega) = \begin{cases} 1; & |\omega| < 3\pi \\ 0; & \text{otherwise} \end{cases}$$

Note that, given $X_1(j\omega)$ is "$3j\omega$" times $X_{1a}(j\omega)$

$$\therefore \quad X_1(j\omega) = \begin{cases} 3j\omega; & |\omega| < 3\pi \\ 0; & \text{otherwise} \end{cases}$$

Since, at $\omega = 3\pi,\ |X_1(j\omega)| = 9\pi$ and $\angle X_1(j\omega) = \frac{\pi}{2}$

$\omega = -3\pi,\ |X_1(j\omega)| = 9\pi$ and $\angle X_1(j\omega) = -\frac{\pi}{2}$

By taking inverse fourier transform,

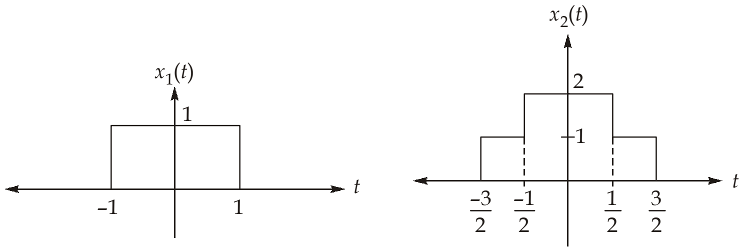
Thus, $x_1(t) = 3\frac{d}{dt}x_{1a}(t)$ [By using differential property]

also we can express $x_{1a}(t) = \frac{\sin 3\pi t}{\pi t}$

Thus,
$$x_1(t) = 3\frac{d}{dt}\left[\frac{\sin 3\pi t}{\pi t}\right]$$
$$= \frac{3}{\pi}\times\frac{1}{t^2}[3\pi t\cos 3\pi t - \sin 3\pi t]$$

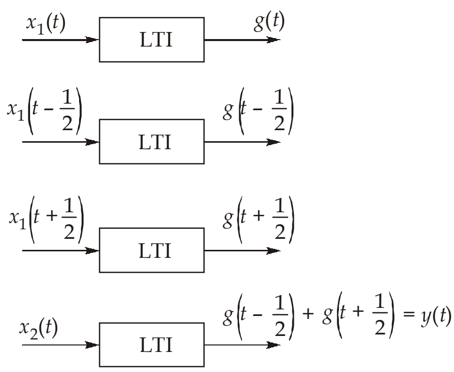
$\therefore$
$$x_1(t) = \frac{3}{\pi t^2}[3\pi t\cos 3\pi t - \sin 3\pi t]$$

14. (a)



$$x_2(t) = x_1\left(t-\frac{1}{2}\right) + x_1\left(t+\frac{1}{2}\right)$$

Since the system is LTI



$$y(0) = g\left(-\frac{1}{2}\right) + g\left(\frac{1}{2}\right) = 3.25 + 3.25 = 6.50$$

15. (b)

Let $x(t)$ be considered as combination of three signals

i.e. $$x(t) = x_1(t) \cdot x_2(t) + x_3(t)$$

$$\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}$$

and $$\frac{\sin at}{\pi t} \overset{F}{\longleftrightarrow} X(j\omega) = \begin{cases} 1 & |\omega| \le a \\ 0 & |\omega| > a \end{cases}$$

$\Rightarrow$ maximum frequency component in $\frac{\sin at}{\pi t}$ is $\frac{a}{2\pi}$(Hz)

f_{max} of signal sinc$(80t)$ $\quad f_1 = \frac{80\pi}{2\pi} = 40$ Hz

f_{max} of signal sinc$(120t)$ $\quad f_2 = \frac{120\pi}{2\pi} = 60$ Hz

f_{max} of signal $\frac{1}{2}\text{sinc}(50t)$ $f_3 = \frac{50\pi}{2\pi} = 25$ Hz

$x_1(t)$ and $x_2(t)$ are multiplied in time domain, thus their frequencies will add up since they will be convoluted.

f_{max} of sinc$(80t)$ · sinc$(120t)$ is

$$f_1 + f_2 = 40 + 60 = 100 \text{ Hz}$$

$\therefore f_{max}$ of $x(t)$ is $\quad f_m = \text{maximum}[f_1 + f_2, f_3]$

then, $f_s = 2f_m$

$$f_s = 2 \times 100 = 200 \text{ Hz}$$

16. (a)

Given, $$X(z) = \frac{10 - 8z^{-1}}{2 - 5z^{-1} + 2z^{-2}} = \frac{2}{(2 - z^{-1})} + \frac{4}{(1 - 2z^{-1})}$$

$$X(z) = \frac{2z}{2z - 1} + \frac{4z}{z - 2}$$

$$X(z) = \frac{z}{\left(z - \frac{1}{2}\right)} + \frac{4z}{(z - 2)}$$

Since, ROC includes unit circle,

$\therefore$ ROC of $X(z)$ is $\frac{1}{2} < |z| < 2$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

$\therefore$ $$x(1) = \frac{1}{2} = 0.5$$

17. (a)

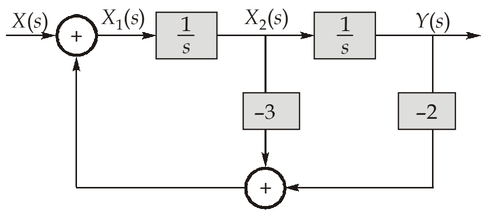
$$F_2(s) = F_1(s) \cdot e^{-s\tau}$$

$$\therefore \quad G(s) = e^{-s\tau} \cdot \frac{F_1(s) \cdot F_1^*(s)}{|F_1(s)|^2} = e^{-s\tau} \cdot \frac{|F_1(s)|^2}{|F_1(s)|^2}$$

$$= e^{-s\tau}$$

$$\therefore \quad g(t) = \delta(t - \tau)$$

18. (b)



now, $$Y(s) = \frac{X_2(s)}{s},$$

$$X_2(s) = \frac{X_1(s)}{s}$$

now, $$X_1(s) = X(s) - 2Y(s) - 3X_2(s)$$

$$s^2Y(s) = X(s) - 3sY(s) - 2Y(s)$$

$$(s^2 + 3s + 2)Y(s) = X(s)$$

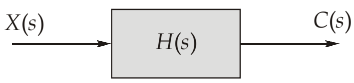
$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + 3s + 2} = \frac{1}{(s+1)(s+2)}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s+1} - \frac{1}{s+2}$$

19. (d)

Let, the given transfer function,

$$H(s) = \frac{b(s+a)}{(s+b)}$$



where, $$X(s) = \frac{1}{s} \qquad (\because \text{ unit step response})$$

$$C(s) = X(s)H(s) = \frac{b(s+a)}{s(s+b)}$$

Given, $$c(0) = 2$$

$$\Rightarrow \quad \underset{s \to \infty}{Lt}\; s \cdot \frac{b(s+a)}{s(s+b)} = 2$$

$$b = 2$$

and $$c(\infty) = 8$$

$$\Rightarrow \quad \underset{s \to 0}{Lt}\; s \cdot \frac{b(s+a)}{s(s+b)} = 8$$

$$a = 8$$

now, $$\frac{a}{b} = \frac{8}{2} = 4$$

20. **(c)**

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \qquad \cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

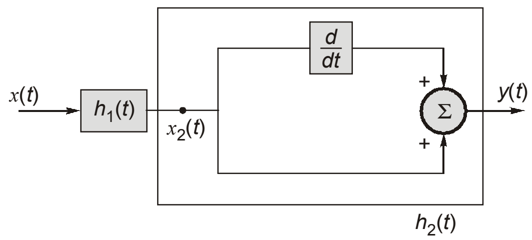
$$\begin{aligned} x(t) &= \cos(\pi t)u(t) - \cos \pi t\, u(t-1) \\ &= \cos(\pi t)u(t) - \cos \pi(t - 1 + 1)\, u(t-1) \\ &= \cos(\pi t)u(t) - \cos[\pi(t-1) + \pi]\, u(t-1) \\ &= \cos \pi\, t u(t) - [\cos \pi(t-1)(-1) - 0]\, u(t-1) \\ &= \cos \pi\, t u(t) + \cos \pi(t-1)\, u(t-1) \end{aligned}$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2+\pi^2} + \frac{s e^{-s}}{s^2+\pi^2} \qquad [\because x(t-t_0) = X(s)\cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1+e^{-s}]}{s^2+\pi^2}$$

21. **(c)**



Let
$$x_2(t) = \delta(t)$$

$$h_2(t) = \left(\delta(t) + \frac{d}{dt}\delta(t)\right)$$

$$h_1(t) = e^{-t}u(t)$$

$$h(t) = e^{-t}u(t) * \left(\delta(t) + \frac{d}{dt}\delta(t)\right)$$

$$h(t) = e^{-t}u(t) * \delta(t) + e^{-t}u(t) * \frac{d}{dt}\delta(t)$$

$$= e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right) * \delta(t)$$

$$= e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right)$$

$$= e^{-t}u(t) - e^{-t}u(t) + e^{-t}\delta(t)$$

$$h(t) = \delta(t) \qquad \because \; e^{-t}\delta(t) = e^{0}\delta(t) = \delta(t)$$

22. (d)

Given discrete-time signal

$$x[n] = n2^n \sin\left(\frac{\pi}{2}n\right)u[n]$$

we know that,

$$Z\left[\sin\left(\frac{\pi}{2}n\right)u[n]\right] = \frac{z\sin\left(\frac{\pi}{2}\right)}{z^2 - 2z\cos\left(\frac{\pi}{2}\right)+1} = \frac{z}{z^2+1}$$

Using the multiplication by exponential property, we have

$$\begin{bmatrix} x[n] \longleftrightarrow X(z) \\ a^n x[n] \longleftrightarrow X\left(\frac{z}{a}\right) \end{bmatrix}$$

$$Z\left[2^n \sin\left(\frac{\pi}{2}n\right)u[n]\right] = Z\left[\sin\left(\frac{\pi}{2}n\right)u[n]\right]\Bigg|_{z\to\left(\frac{z}{2}\right)}$$

$$= \frac{z}{z^2+1}\Bigg|_{z\to\frac{z}{2}} = \frac{2z}{z^2+4}$$

Using differentiation in z-domain property

$$\begin{bmatrix} x[n] \longleftrightarrow X(z) \\ nx[n] \longleftrightarrow -z\frac{d}{dz}X(z) \end{bmatrix}$$

$$Z\left[n2^n \sin\left(\frac{\pi}{2}n\right)u[n]\right] = -z\frac{d}{dz}\left\{Z\left[n2^n \sin\left(\frac{\pi}{2}n\right)u[n]\right]\right\}$$

$$= -z\frac{d}{dz}\left(\frac{2z}{z^2+4}\right)$$

$$= -z\left[\frac{(z^2+4)(2) - 2z(2z)}{(z^2+4)^2}\right] = -z\left[\frac{-2z^2+8}{(z^2+4)^2}\right]$$

$$Z\left[n2^n \sin\left(\frac{\pi}{2}n\right)u[n]\right] = \frac{2z(z^2-4)}{(z^2+4)^2}$$

23. (d)

We know that

$$FT[e^{-t}u(t)] = \frac{1}{1+j\omega}$$

Using duality property

$$x(t) \xleftrightarrow{FT} X(\omega)$$

$$X(t) \xleftrightarrow{FT} 2\pi x(-\omega)$$

we have $$FT\left[\frac{1}{1+jt}\right] \xleftrightarrow{FT} 2\pi e^{-(-\omega)}u(-\omega)$$

$$\xleftrightarrow{FT} 2\pi e^{\omega}u(-\omega)$$

Using the time reversal property,

i.e. $x(-t) = X(-\omega)$, we have

$$FT\left[\frac{1}{1-jt}\right] \xleftrightarrow{FT} 2\pi e^{-\omega}\, u(\omega)$$

$$\therefore \qquad e^{-\omega}u(\omega) \xleftrightarrow{IFT} \frac{1}{2\pi} FT\left[\frac{1}{1-jt}\right]$$

$$\therefore \qquad FT^{-1}\left[e^{-\omega}u(\omega)\right] = \frac{1}{2\pi(1-jt)}$$

24. (d)

Given, $x(t) = 2 + \cos(50\pi t)$

Frequency of signal $\omega_{sig} = 50\pi$

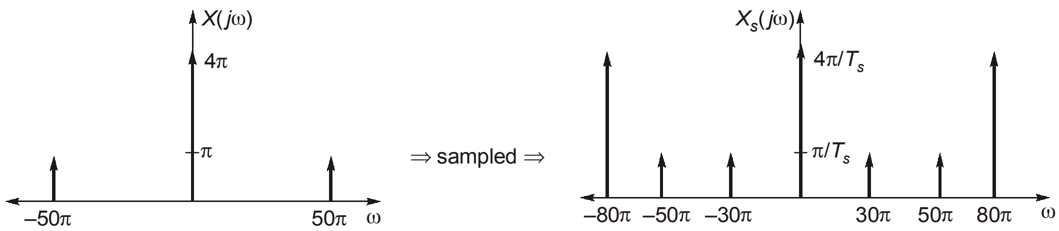
$T_s = 0.025$ sec

$\therefore$ sampling frequency $\omega_s = \dfrac{2\pi}{T_s} = 80\,\pi$ rad/sec

then, $X(j\omega) = 4\,\pi\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

Let the sampled signal be represented as $X_s(j\omega)$, where $X_s(j\omega)$ is given as

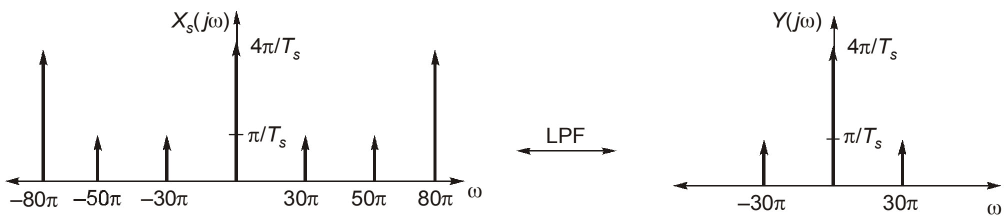
$$X_s(j\omega) = \frac{1}{T_s}\sum_{m=-\infty}^{\infty} X(j(\omega - n\omega_s))$$



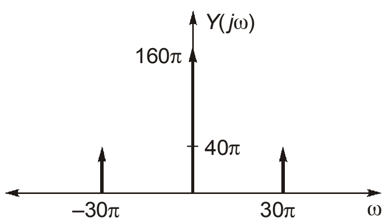
$$X_s(j\omega) = 40\sum_{m=-\infty}^{\infty}[4\pi\delta(\omega - 80\pi)] + \pi\delta(\omega - 50\pi - 80\pi n) - \pi\delta(\omega + 50\pi - 80\pi n)]$$

now, the sampled input $X_s(j\omega)$ is passed through a low passed filter having cut-off frequency at $\omega = 40\pi$.

Therefore the output $Y(j\omega)$ will contain only the components which are less than $\omega = 40\,\pi$.



Now by putting $T_s = 0.025$, we will get



25. (b)

Given, $$X(s) = \ln\left[1+\frac{\omega^2}{s^2}\right]$$

Let $$x(t) = L^{-1}[X(s)] = L^{-1}\left[ln\left(1+\frac{\omega^2}{s^2}\right)\right]$$

$\therefore$ $$L[x(t)] = \ln\left[1+\frac{\omega^2}{s^2}\right] = \ln\left[\frac{s^2+\omega^2}{s^2}\right]$$

$$= \ln[s^2 + \omega^2] - ln\ s^2$$

$$L[t\,x(t)] = \frac{-d}{dS}\left[\ln(s^2+\omega^2)-\ln s^2\right]$$

$$= \frac{-1}{s^2+\omega^2}\cdot 2s + \frac{1}{s^2}2s = \frac{2}{s} - \frac{2s}{s^2+\omega^2}$$

$\therefore$ $$t\,x(t) = L^{-1}\left[\frac{2}{s}-\frac{2s}{s^2+\omega^2}\right]$$

$$= (2 - 2\cos\omega t)u(t) = 2(1 - \cos\omega t)u(t)$$

$\therefore$ $$x(t) = \frac{2(1-\cos\omega t)}{t}u(t)$$

26. (b)

$$Y(z) = X(z)\cdot H(z)$$

for unit step input, $$X(z) = \frac{1}{1-z^{-1}} \qquad (\because x[n] = \text{unit step})$$

$\therefore$ $$Y(z) = \frac{z^3}{(z-1)\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$\frac{Y(z)}{z} = \frac{C_1}{z-1} + \frac{C_2}{z-\frac{1}{2}} + \frac{C_3}{z-\frac{1}{4}}$$

$$C_1 = \left.\frac{z^2}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{2}\right)}\right|_{z=1} = \frac{8}{3}$$

$$C_2 = \left.\frac{z^2}{(z-1)\left(z-\frac{1}{4}\right)}\right|_{z=\frac{1}{2}} = -2$$

$$C_3 = \left.\frac{z^2}{(z-1)\left(z-\frac{1}{2}\right)}\right|_{z=\frac{1}{4}} = \frac{1}{3}$$

$$\therefore \qquad Y(z) = \frac{8}{3}\frac{z}{z-1} - 2\frac{z}{z-\frac{1}{2}} + \frac{1}{3}\frac{z}{z-\frac{1}{4}}$$

Taking inverse z-transform,

$$y[n] = \frac{1}{3}\left[8 - 6\left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n\right]u[n]$$

27. (b)

$x(t)$ has hidden odd symmetry.

So,

$$a_n = 0$$

$$b_n = \frac{2}{T}\int_{<T>} x(t)\sin(2\pi nt)\,dt = 2\int_0^1 t\sin(2\pi nt)\,dt = -\frac{1}{\pi n}$$

$$\Rightarrow \qquad b_1 = -\frac{1}{\pi}$$

Power in first harmonic is,

$$P_1 = \frac{|b_1|^2}{2} = \frac{\left(\frac{1}{\pi}\right)^2}{2} = \frac{1}{\pi^2 \times 2} = 50.66 \times 10^{-3}$$

28. (a)

$$x(t) \longleftrightarrow X(\omega)$$

$$e^{-jt}x(t) \longleftrightarrow X(\omega+1)$$

$$e^{j\frac{t}{2}}x\left(-\frac{t}{2}\right) \longleftrightarrow \frac{1}{\left|-\frac{1}{2}\right|}X\left(\frac{\omega}{\left(-\frac{1}{2}\right)}+1\right)$$

$$e^{j\frac{t}{2}}x\left(-\frac{t}{2}\right) \longleftrightarrow 2X(-2\omega+1)$$

$$\frac{1}{2}e^{j\frac{t}{2}}x\left(-\frac{t}{2}\right) \longleftrightarrow X(-2\omega+1)$$

29. (c)

- Linear in $x[n]$
- Upper limit of summation operator is a function of n $\Rightarrow$ Time variant
- $n^2 > n \Rightarrow$ non causal
- When $x[n]$ is bounded, $y[n]$ is also bounded $\Rightarrow$ Stable

30. (c)

$$x(t) = \text{rect}\left(\frac{t+1}{2}\right) - \text{rect}\left(\frac{t-1}{2}\right)$$

$$\text{rect}(t) \longleftrightarrow \frac{2\sin(\omega/2)}{\omega}$$

$$\text{rect}\left(\frac{t}{2}\right) \longleftrightarrow \frac{4\sin(2\omega/2)}{2\omega} = \frac{2\sin\omega}{\omega}$$

$$\text{rect}\left(\frac{t+1}{2}\right) \longleftrightarrow \frac{2\sin(\omega)}{\omega}\cdot e^{j\omega}$$

$$\text{rect}\left(\frac{t-1}{2}\right) \longleftrightarrow \frac{2\sin(\omega)}{\omega}\cdot e^{-j\omega}$$

$\Rightarrow$
$$X(\omega) = \frac{2\sin\omega}{\omega}(e^{j\omega} - e^{-j\omega}) = \frac{j4\sin\omega\sin\omega}{\omega} = \frac{j2(1-\cos(2\omega))}{\omega}$$

■■■■