S.No. : 02 CH1_EE_A_250619
Control Systems

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CLASS TEST 2019-2020

ELECTRICAL ENGINEERING

Date of Test : 25/06/2019

ANSWER KEY ➤ Control Systems

1.	(d)	7.	(b)	13.	(c)	19.	(b)	25.	(a)
2.	(b)	8.	(b)	14.	(b)	20.	(b)	26.	(d)
3.	(c)	9.	(b)	15.	(b)	21.	(d)	27.	(a)
4.	(d)	10.	(b)	16.	(b)	22.	(b)	28.	(a)
5.	(d)	11.	(b)	17.	(b)	23.	(d)	29.	(c)
6.	(b)	12.	(d)	18.	(d)	24.	(d)	30.	(a)

DETAILED EXPLANATIONS

1. (d)

$$(s + 3 + j4)\,(s + 3 - j4) = 0$$

$$(s + 3)^2 - (j4)^2 = 0;\ s^2 + 6s + 9 + 16 = 0;\ s^2 + 6s + 25 = 0$$

$$\omega_n = \sqrt{25}\ ;\quad \omega_n = 5 \text{ rad/sec};\quad 2\zeta\omega_n = 6$$

$$\zeta = \frac{6}{2 \times 5} = 0.6$$

2. (b)

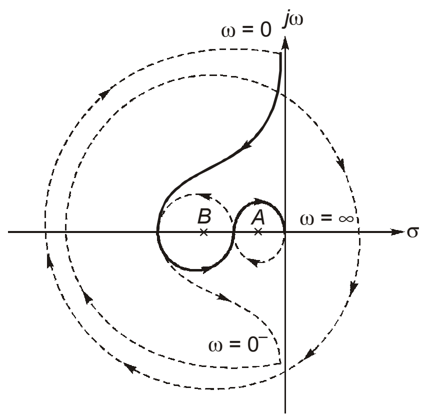
Since, all the poles of the open loop system lie on the LHS of s-plane hence, the open loop system is stable. However, the number of encirclement to the critical point is two means two closed loop poles are located on the RHS of s-plane. Therefore, the closed loop system is unstable.

3. (c)

For given Nyquist plot,

We can find that, type of the system = 3

and complete the Nyquist plot as shown in figure



For (–1, 0) point of position A,

$$N = P - Z$$

Here, $N = -2$ and $P = 0$

$\therefore$ $-2 = -Z,$

$\Rightarrow$ $Z = 2$

Therefore system is unstable at point A.

For (–1, 0) lying at position B,

$$N = P - Z$$

Here, $N = 0,\ P = 0$

$\therefore$ $Z = 0$; i.e. stable system.

Therefore, system is stable at point B.

4. (d)

Using Routh Table:

s^6	1	6	10	5
s^5	1	5	5	
s^4	1	5	5	
s^3	0	0		

The Routh table construction procedure breaks down here, since the s^3 row has all zeros. The auxiliary polynomial coefficients are given by the s^4 row. Therefore the auxiliary polynomial is,

$$A(s) = s^4 + 5s^2 + 5$$

$$\frac{dA(s)}{ds} = 4s^3 + 10s$$

Replacing the s^3 row in the Routh table with the coefficients of $\frac{dA(s)}{ds}$, we have

s^6	1	6	10	5
s^5	1	5	5	
s^4	1	5	5	
s^3	4	10		
s^2	$\frac{20-10}{4} = 2.5$	5		
s^1	$\frac{25-20}{2.5} = 2$			
s^0	5			

Examining the first column of this table we see that there are no sign changes. Hence, there is no root lying in the RHS of s-plane.

5. (d)

2nd order characteristics equation $s^2 + 2\,\xi\omega_n\, s + \omega_n^2 = 0$ have poles at $-\xi\omega_n \pm j\omega_d$

On comparing, we have

$$-1 \pm j\pi$$

$$\omega_d = \pi$$

$$\Rightarrow \quad t_p = \text{peak time} = \frac{\pi}{\omega_d} \text{ (first peak)}$$

$$= \frac{\pi}{\pi} = 1\,\text{sec.}$$

6. (b)

$$\dot{x}_1 = x_1 - 5x_2$$

$$\dot{x}_2 = (8 - g_1)\,x_1 - g_2 x_2$$

$$\therefore \quad A = \begin{bmatrix} 1 & -5 \\ 8-g_1 & -g_2 \end{bmatrix}$$

$$\text{Characteristic equation} = |(sI - A)| = 0$$

$$= \left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & -5 \\ 8-g_1 & -g_2 \end{bmatrix} \right| = 0$$

$$= \left| \begin{bmatrix} s-1 & 5 \\ -8+g_1 & s+g_2 \end{bmatrix} \right| = 0$$

$$= (s-1)\,(s+g_2) - 5\,(g_1 - 8) = 0$$

$$= s^2 + s(g_2 - 1) + (-g_2 - 5g_1 + 40) = 0$$

$$\Rightarrow \quad \omega_n = \sqrt{40 - 5g_1 - g_2} = \sqrt{2}$$

$$\Rightarrow \quad +5g_1 + g_2 = +38$$

$$\text{and} \quad \xi = \frac{1}{\sqrt{2}}$$

$$(g_2 - 1) = 2\frac{1}{\sqrt{2}} \times \sqrt{2}$$

$\Rightarrow$ $$g_2 = 3 \text{ and } g_1 = 7$$

7. (b)

Method 1:

$$H(s) = 1$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)}{1+G(s)}$$

$$\frac{s+4}{s^2+7s+13} = \frac{\frac{s+4}{s^2+6s+9}}{1+\frac{s+4}{s^2+6s+9}}$$

So, $$G(s) = \frac{s+4}{s^2+6s+9}$$

or, $$G(s) = \frac{4}{9}\cdot\frac{(1+s/4)}{\left(1+\frac{6}{9}s+\frac{1}{9}s^2\right)} \quad \text{(In time constant form)}$$

Thus, open loop DC gain of given system = $\frac{4}{9}$.

Method 2:

$$G(s) = \frac{\text{Num}}{\text{Den} - \text{Num}} = \text{O.L.T.F.} \quad (H(s) = 1)$$

$$= \frac{s+4}{(s^2+7s+13)-(s+4)} = \frac{s+4}{s^2+6s+9}$$

Put $s = 0$, $$G(0) = \frac{4}{9}$$

8. (b)

$$\text{Error} = 20\log(2\xi)$$

$$\text{Error} = 20\log(2\times 1.4)$$

$$\text{Error} = 8.94 \text{ dB}$$

9. (b)

$$G(s) = \frac{6}{(s^2+2s+6)}$$

Comparing with the standard form

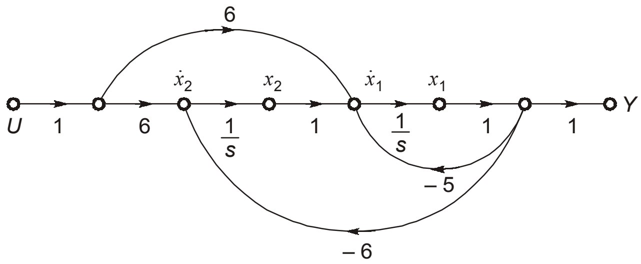
$$\omega_n = \sqrt{6} \text{ and } 2\xi\omega_n = 2$$

$\therefore$ $$\xi = 0.408$$

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100 = e^{\frac{-\pi\times 0.408}{\sqrt{1-0.408^2}}} \times 100$$

$$= 24.56\% \approx 24.6\%$$

10. (b)



$$\text{Here} \qquad \dot{x}_1 = 6U + x_2 - 5x_1$$

$$\dot{x}_2 = 6U - 6x_1$$

$$\text{and} \qquad Y = x_1$$

$$\therefore \qquad A = \begin{bmatrix} -5 & 1 \\ -6 & 0 \end{bmatrix}$$

11. (b)

$$\text{Impulse response} = \frac{d}{dt}[\text{step response}]$$

Now, as we know at $t = 0; \quad c(t) \neq 0$

Here, we can write $c(t) = (1 - e^{-3t} + 3e^{-t})\, u(t)$

Now, $t = 0; \quad c(t) = 0$

$$\text{Hence,} \qquad h(t) = \frac{d}{dt}[c(t)]$$

$$h(t) = \delta(t) - e^{-3t}\,\delta(t) + 3e^{-3t}\,u(t) + 3e^{-t} * \delta(t) - 3e^{-t}.\ u(t)$$

$$= \delta(t) - \delta(t) + 3\delta(t) + (3e^{-3t} - 3e^{-t})u(t)$$

$$h(t) = 3\delta(t) + (3e^{-3t} - 3e^{-t})\, u(t)$$

$$H(s) = 3 + \frac{3}{s+3} - \frac{3}{(s+1)} = \frac{3(s^2+4s+3) + 3(s+1) - 3(s+3)}{(s+3)(s+1)}$$

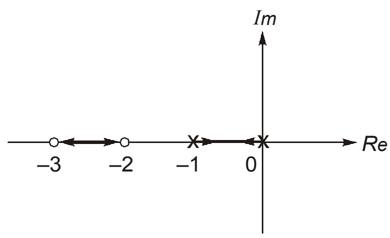
$$= \frac{3(s^2+4s+3) - 6}{(s+3)(s+1)} = \frac{3(s^2+4s+1)}{3\left(1+\frac{s}{3}\right)(1+s)}$$

Thus, $\text{Gain} = 1$

12. (d)

The transfer function

$$G(s)\,H(s) = \frac{K(s+2)(s+3)}{s(s+1)}$$



The break points can be calculated as

$$\frac{dK}{ds} = 0$$

$\therefore$ $$1 + G(s) = 0$$

or, $$1+\frac{K(s+2)(s+3)}{s(1+s)} = 0$$

$$K = \frac{-(s^2+s)}{s^2+5s+6}$$

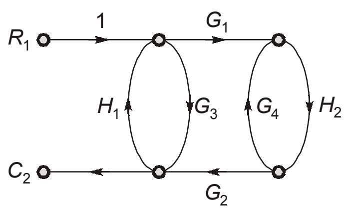
$$\frac{dK}{ds} = \frac{(s^2+5s+6)(2s+1)-(s^2+s)(2s+5)}{s^2+5s+6} = 0$$

$$= -2.37 \text{ and } -0.634$$

Here break in point is –2.37 and break away point is –0.634.

13. (c)

Assuming $C_1 = 0$ and $R_2 = 0$.



Here, forward path $P_1 = G_3$; $\Delta_1 = (1-G_4H_2)$

$P_2 = G_1G_2H_2$; $\Delta_2 = 1$

LOOPs

$L_1 = G_3H_1$, $L_2 = G_4H_2$, $L_3 = G_1G_2H_1H_2$

and $L_4 = G_3G_4H_1H_2$ (Non touching loops)

$\therefore$ $$\frac{C_2}{R_1} = \frac{G_3(1-G_4H_2)+G_1G_2H_2}{1-G_3H_1-G_4H_2-G_1G_2H_1H_2+G_3G_4H_1H_2}$$

$$= \frac{G_3+H_2(G_1G_2-G_3G_4)}{1-G_3H_1-G_4H_2+H_1H_2(G_3G_4-G_1G_2)}$$

14. (b)

$$G(s) = \frac{40}{s^2(s+18)}$$

e_{ss} due to parabolic input

$$e_{ss} = \frac{A}{K_a}$$

where $$K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} s^2 \times \frac{40}{s^2(s+18)} = \frac{40}{18}$$

$$e_{ss} = \frac{3\times 2}{40/18} = 2.7$$

15. (b)

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$$

$$s^2\, Y(s) + 3s\, Y(s) + 2\, Y(s) = X(s)$$

$$x(t) = 2\, u(t)$$

$$X(s) = \frac{2}{s}$$

$$\therefore \qquad (s^2 + 3s + 2)\, Y(s) = \frac{2}{s}$$

$$Y(s) = \frac{2}{s(s+2)\,(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1}$$

$$2 = A(s + 2)\,(s + 1) + B(s)\,(s + 1) + C\, s(s + 2)$$

$s = 0$; $\quad 2 = A2$

$A = 1$

$s = -1$; $\quad 2 = -C$

$C = -2$

$s = -2$; $\quad 2 = 2B$

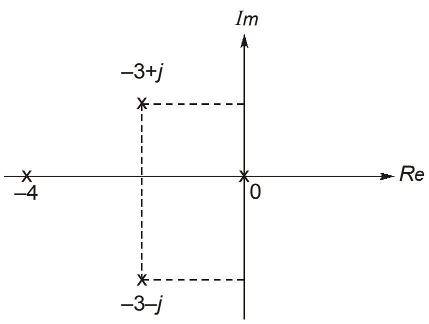
$B = 1$

$$Y(s) = \frac{1}{s} + \frac{1}{s+2} - \frac{2}{s+1}$$

$$y(t) = [1 + e^{-2t} - 2e^{-t}]\, u(t)$$

16. (b)

$$\phi_d = 180° - \phi$$



$$\phi = 90° + 45° + 161.565° = 296.56°$$

$$\therefore \qquad \phi_d = 180° - 296.5650 = \mp 116.56°$$

17. (b)

The transfer function can be

$$G(s)\, H(s) = \frac{K\left(1+\frac{s}{2}\right)}{s} = \frac{4\left(1+\frac{s}{2}\right)}{s}$$

or, $\quad K = 4$

With starting slope of –20 dB/dec, at $\omega = 2$ rad/sec

$-20 \log_{10}(2) + 20 \log_{10}(4) = 6$ dB

18. (d)

The loop equations considering the Laplace transform of the network is

$$s\,I_1(s) + I_1(s) - I_2(s) = V_i(s)$$

$$(s+1)\,I_1(s) - I_2(s) = V_i(s) \qquad ...(i)$$

and $I_2(s) - I_1(s) + I_2(s) + s\,I_2(s) = 0$

$$I_1(s) = (s+2)\,I_2(s) \qquad ...(ii)$$

Substituting equation (*ii*) in equation (*i*),

$$(s+1)(s+2)\,I_2(s) - I_2(s) = V_i(s)$$

$$I_2(s) = \frac{V_i(s)}{s^2+3s+1}$$

also, $$V_0(s) = s\,I_2(s)$$

$$V_0(s) = \frac{sV_i(s)}{s^2+3s+1}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{s}{s^2+3s+1}$$

19. (b)

Characteristic equation

$$1 + G(s)\,H(s) = 0$$

$$\Rightarrow \quad 1+\frac{11\beta}{s^3+4s^2+3s+1} = 0$$

or, $s^3 + 4s^2 + 3s + 1 + 11\beta = 0$

Routh array

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & (11\beta+1) \\ s^1 & \frac{12-(11\beta+1)}{4} & 0 \\ s^0 & (11\beta+1) & \end{array}$$

for stability,

$$\frac{12-(11\beta+1)}{4} \geq 0$$

or $$12 \geq (11\beta+1)$$

or $$\beta \leq 1$$

20. (b)

Comparing with standard transfer function

$$G(s) = \frac{K(1+sT)}{(1+\alpha sT)}$$

$$T = \frac{21}{97}$$

$$\alpha T = \frac{1}{33}$$

or, $$\alpha = \frac{97}{33 \times 21} = 0.1399$$

$\because$ $\alpha < 1$ lead compensator

$$\phi = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$= \sin^{-1}\left(\frac{1-0.1399}{1+0.1399}\right) = 48.97°$$

21. (d)

$$\text{Transfer function} = \frac{64}{s^2 + 14s + 64}$$

Where, $$\omega_n^2 = 64$$

$$2\xi\omega_n = 14$$

or, $$\xi = \frac{14}{2 \times 8} = 0.875 \text{ (underdamped)}$$

$$\tau_{sett} = \frac{4}{\xi\omega_n} = \frac{4}{7} = 0.571 \text{ sec}$$

22. (b)

Let $K = 4$ then,

$$G(s)H(s) = \frac{K}{(s+2)^2}$$

Characteristic equation $= 1 + G(s)\,H(s) = s^2 + 4s + (4 + K) = 0$

Routh array

$$\begin{array}{c|cc} s^2 & 1 & (4+K) \\ s^1 & 4 & 0 \\ s^0 & 4+K & \end{array}$$

For stability $K > -4$

now, calculating ω_{pc}

$$-180° = 2\tan^{-1}\omega_{pc}$$

or $$\omega_{pc} = \infty$$

Hence $$\text{GM} = 0 \text{ dB} = \infty$$

and $$|G(\omega)H(\omega)| = \frac{4}{\left(\sqrt{(j\omega_{gc})^2 + 2^2}\right)^2} = 1$$

$$\frac{4}{(\omega_{gc}^2 + 4)} = 1$$

or $$\omega_{gc}^2 = 0$$

$$\omega_{gc} = 0$$

$$\text{PM} = 180° - \tan^{-1}\frac{\omega_{gc}}{2} - \tan^{-1}\frac{\omega_{gc}}{2}$$

$$= 180°$$

23. (d)

1 pole at origin ($\omega = 0$)

$$\downarrow -20 \text{ dB/dec}$$

2 poles at $\omega = 2$ rad/s

$$\downarrow -60 \text{ dB/dec}$$

2 poles at $\omega = 4$ rad/s

$$\downarrow -100 \text{ dB/dec}$$

1 zero at $\omega = 10$ rad/s

$$\downarrow -80 \text{ dB/dec}$$

$\therefore$ The slope of the line between frequency of 4 rad/s and 10 rad/sec is –100 dB/decade.

24. (d)

Check for controllability -

$$Q_C = [B : AB]$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & -4 \end{bmatrix}$$

$\because$ $\rho(A) \neq \rho(Q_C)$

and $|Q_C| = 0$

$\therefore$ System is uncontrollable.

Check for observavly -

$$Q_o = [C^T : A^T C^T]$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} : \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$\because$ $\rho(A) \neq \rho(Q_o)$

Also $|Q_o| = 0$

$\therefore$ System is unobservable.

25. (a)

Given, $$G(s)H(s) = \frac{1}{s^2 (s-a)(s-b)(s-c)}$$

Put $s = j\omega$, $$G(j\omega)\,H(j\omega) = \frac{1}{(j\omega)^2 (j\omega - a)(j\omega - b)(j\omega - c)}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\omega^2 \sqrt{\omega^2 + a^2} \cdot \sqrt{\omega^2 + b^2} \cdot \sqrt{\omega^2 + c^2}}$$

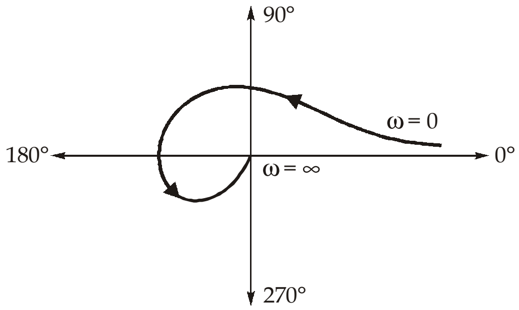
when, $\omega = 0$ Magnitude = ∞; Phase = 0°

$\omega = \infty$ Magnitude = 0; Phase = 270°

$$\angle G(j\omega)\,H(j\omega) = -\left[180° + \left(180° - \tan^{-1}\left(\frac{\omega}{a}\right)\right) + \left(180° - \tan^{-1}\left(\frac{\omega}{b}\right)\right) + \left(180° - \tan^{-1}\left(\frac{\omega}{c}\right)\right)\right]$$

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) + \tan^{-1}\left(\frac{\omega}{b}\right) + \tan^{-1}\left(\frac{\omega}{c}\right)$$

Note: Since a, b and c are positive values, for $\omega = 1$, phase of $G(j\omega)\,H(j\omega)$ will be a positive quantity. So, the polar plot will start into the first quadrant.



26. **(d)**

$$T(s) = \frac{C(s)}{R(s)} = \frac{50}{(s^2T + s)(1 + 0.5s) + 50}$$

Characteristic equation

$$Ts^3 + (1 + 2T)\,s^2 + 2s + 100 = 0$$

Using Routh array

$$\begin{array}{c|cc} s^3 & T & 2 \\ s^2 & (1+2T) & 100 \\ s^1 & \dfrac{2(1+2T) - 100T}{(1+2T)} & 0 \\ s^0 & 100 & \end{array}$$

$\therefore$ $\quad T > 0$ and $T > -\dfrac{1}{2}$

Also $\quad 2 - 96\,T > 0$

or, $\quad T < \dfrac{1}{48}$

The system becomes unstable for

$$T = \frac{1}{48}$$

27. **(a)**

The open loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

(a) Finite poles are at $s = 0, -1, -2, (P = 3)$

Finite zeros are nil $(z = 0)$

(b) Number of asymptotes $= P - Z = 3$

The centroid σ (or the meeting point of the asympotes) is at

$$\sigma = \frac{0-1-2}{3} = -1$$

The angles of the asymptotes are given by

$$\theta_K = \frac{(2K+1)\pi}{P-Z}, K = 0,1 \;...., (|P-Z|-1)$$

$$= \frac{(2K+1)\pi}{3}, K = 0,1,2 \; = 60°, 180°, 300° \; (-60°)$$

(c) The breakaway points are given by

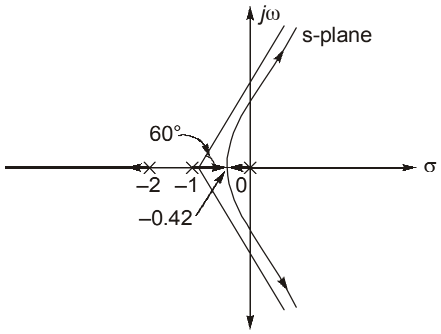
$$\frac{dK}{ds} = 0 \quad \text{or} \quad \frac{dK}{ds} = \frac{d}{ds}[-s(s+1)(s+2)] = 0$$

or $$-(3s^2 + 6s + 2) = 0$$

or $$s = \frac{-6 \pm \sqrt{36-24}}{6} = -0.42, -1.577 \text{ (invalid)}$$

Thus, $s = -0.42$ is only valid break away point

(d) The number of branches of the root loci is the greater of P and Z, viz, 3. Using all the above information, we plot the root loci,



Hence choice (a) is correct.

28. (a)

Zeros $s = -2$

Poles: $s = -1 + j2, -1 - j2$

The transfer function is

$$G(s) = \frac{K(s+2)}{(s+1-j2)(s+1+j2)}$$

where $$K = \frac{\text{multiplication of vector lengths drawn from all poles}}{\text{multiplication of vector lengths drawn from all zeros}}$$

$$= \frac{\sqrt{10}\times\sqrt{2}}{\sqrt{5}} = 2$$

$$G(s = j1) = \frac{2(j1+2)}{(j1+1-j2)(j1+1+j2)} = \frac{2(2+j1)}{(1-j1)(1+j3)}$$

$$= \frac{2\times 2.236\angle 26.6°}{1.4142\angle -45°\times 3.162\angle 71.57°} = 1\angle 0°$$

Hence choice (a) is correct.

29. (c)

$$G(s) = \frac{10}{s(s+1)^2}$$

Putting $s = j\omega$,

$$G(j\omega) = \frac{10}{j\omega(j\omega+1)^2} = \frac{10}{j\omega(-\omega^2+2j\omega+1)} = \frac{10}{-2\omega^2+j(\omega-\omega^3)}$$

This can be divided into the real and imaginary parts as shown below

$$G(j\omega) = \frac{10\left\{-2\omega^2 - j(\omega-\omega^3)\right\}}{\left\{-2\omega^2+j(\omega-\omega^3)\right\}\left\{-2\omega^2-j(\omega-\omega^3)\right\}}$$

$$= \frac{-20\omega^2}{4\omega^4+(\omega-\omega^3)^2} - \frac{10j(\omega-\omega^3)}{4\omega^4+(\omega-\omega^3)^2}$$

In the real axis at $\omega = \omega_0$,

the imaginary portion goes to zero.

Hence $$\frac{\omega_0-\omega_0^3}{4\omega_0^4+(\omega_0-\omega_0^3)^2} = 0$$

or $$\omega_0-\omega_0^3 = 0$$

or $$\omega_0(1-\omega_0^2) = 0$$

which gives the possible solution

$$\omega_0 = 0, 1$$

At $\omega_0 = 0$,

$$\text{Real part of } G(j\omega) = \frac{-20\omega_0^2}{4\omega_0^4+\left(\omega_0-\omega_0^3\right)^2};$$

$$\text{Re}\{G(j\omega)\} = \infty$$

Similarly at $\omega_0 = 1$,

$$\text{Re}\{G(j\omega)\} = \frac{-20}{4+0} = -5$$

Hence the real part and ω_0 are –5, 1 respectively.

30. (a)

The equations of performance for the system are.

$$B_1(\dot{X}_1 - \dot{X}_0) + K_1(X_1 - X_0) = K_2 X_0$$

or $$(sB_1 + K_1)X_1(s) - (sB_1 + K_1)\,X_0(s) = K_2 X_0(s)$$

$$\frac{X_0(s)}{X_1(s)} = \frac{sB_1 + K_1}{sB_1 + K_1 + K_2}$$

$$T = \frac{K_1\left(1 + \frac{B_1 s}{k_1}\right)}{(K_1 + K_2)\left(1 + \frac{sB_1}{K_1 + K_2}\right)}$$

Let $$\frac{K_1 + K_2}{K_1} = a$$

where $$a > 1$$

and $$\frac{B_1}{K_1 + K_2} = T;$$

Then, $$\frac{X_0(s)}{X_1(s)} = \frac{1}{a}\left(\frac{1 + aTs}{1 + Ts}\right)$$

Therefore zero is nearer to origin than pole i.e. Lead network.

■■■■