S.No. : 03 **LS1_EC_A_030619**

Electromagnetic Theory

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | **Noida** | **Bhopal** | **Hyderabad** | **Jaipur** | **Lucknow** | **Indore** | **Pune** | **Bhubaneswar** | **Kolkata** | **Patna**

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

Electromagnetic Theory

Date of Test : 03/06/2019

Answer Key

1.	(b)	7.	(a)	13.	(c)	19.	(b)	25.	(a)
2.	(a)	8.	(c)	14.	(c)	20.	(c)	26.	(d)
3.	(a)	9.	(c)	15.	(b)	21.	(a)	27.	(b)
4.	(a)	10.	(b)	16.	(b)	22.	(b)	28.	(a)
5.	(d)	11.	(a)	17.	(a)	23.	(d)	29.	(d)
6.	(b)	12.	(c)	18.	(b)	24.	(a)	30.	(d)

DETAILED EXPLANATIONS

1. (b)

$$\nabla \times \vec{A} = \vec{B}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A}) = \nabla \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\Rightarrow \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

4. (a)

Because of image formation.

5. (d)

$$I = \int \vec{J} \cdot \vec{ds} = \int_0^{16} \int_0^{2\pi} \frac{40}{\rho^2 + 4} \cdot \rho \, d\rho \, d\phi$$

$$\text{Let } p = \rho^2 + 4$$

$$dp = 2\rho \, d\rho$$

$$\frac{dp}{2} = \rho \, d\rho$$

$$= \frac{40(2\pi)}{2} \cdot \int_4^{260} \left(\frac{dp}{p}\right) = 40\pi \, ln \frac{260}{4}$$

$$I = 524.57 \text{ Amp}$$

6. (b)

$$J_C = \sigma E$$

$$|J_d| = \omega \varepsilon E$$

$$\therefore \quad \left|\frac{J_C}{J_d}\right| = \frac{\sigma}{\omega \varepsilon} = \frac{2 \times 10^{-3}}{2\pi \times 10^3 \times 81 \times 8.85 \times 10^{-2}} = 0.44 \times 10^{-3}$$

9. (c)

Radiation resistance given by

$$R_r = 80\pi^2 \left(\frac{dl}{d\lambda}\right)^2$$

$$l = \frac{\lambda}{16}, \qquad \frac{l}{\lambda} = \frac{1}{16}$$

$$R_r = 80\pi^2 \left(\frac{1}{16}\right)^2 = 3.084 \ \Omega$$

10. (b)

Both are nonmagnetic medium. Brewester angle is given by,

$$\tan\theta_B = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}} = \sqrt{\frac{2.6\varepsilon_0}{\varepsilon_0}} = 1.612 = 58.18°$$

11. (a)

$$\text{Impedance of a dipole antenna} = Z_L = (73 + 42.5j)\ \Omega$$

$$\text{Reflection coefficient, } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{73 + j42.5 - 75}{73 + j42.5 + 75} = 0.276 \angle 76.67°$$

$$\text{Standing wave ratio, } S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.276}{1-0.276} = 1.763$$

12. (c)

For a TE_{m0} mode:

$$\lambda_c = \frac{2a}{m}$$

$\Rightarrow$ if $m = 2 \rightarrow TE_{20}$ $\quad \lambda_c = a$

$$\lambda_c = \frac{2b}{n}$$

$\Rightarrow$ if $n = 1 \rightarrow TE_{01}$ $\quad \lambda_c = 2b = a$

13. (c)

$$V = \frac{kq_1}{L_1} + \frac{kq_2}{L_2} + \frac{kq_3}{L_3} + \frac{kq_4}{L_4}$$

$q_1 = q,$ $\quad L_1 = L$

$q_2 = q,$ $\quad L_2 = L$

$q_3 = -q,$ $\quad L_3 = \sqrt{5}L$

$q_4 = -q,$ $\quad L_4 = \sqrt{5}L$

$$V = \frac{1q}{4\pi\varepsilon_0}\left(\frac{1}{L} + \frac{1}{L} - \frac{1}{\sqrt{5}L} - \frac{1}{\sqrt{5}L}\right)$$

$$V = \frac{kq}{L}\left(2 - \frac{2}{\sqrt{5}}\right) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{L}\left(1 - \frac{1}{\sqrt{5}}\right)$$

14. (c)

Since characteristics impedance is real; i.e. is a lossless line

$\therefore$ $\quad R = G = 0$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \text{... (i)}$$

$$B = \omega\sqrt{LC} \quad \text{... (ii)}$$

$Z_0 . \beta = \omega L,$

$$L = \frac{Z_0\beta}{\omega} = \frac{70 \times 8}{2\pi \times 400 \times 10^6} = 222.8 \text{ nH/m}$$

From 1,

$$C = \frac{L}{Z_0^2} = \frac{222.8 \times 10^{-9}}{70^2} = 45.4 \text{ pF/m}$$

15. (b)

Since $u = \frac{\omega}{\beta}$

$$\therefore \quad \beta = \frac{\omega}{u}$$

$$\because \quad Bl = \frac{\omega l}{u} = \frac{2\pi \times 10^6 \times 30}{0.6 \times 3 \times 10^8} = \frac{\pi}{3} = 60°$$

Bl is an electrical length of the line input impedance

$$Z_{in} = Z_0\left[\frac{Z_i + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= 50\left[\frac{40 + 60 + j50\tan 60°}{50 + j(40+60)\tan 60°}\right] = (51.89 - 69.25j)\,\Omega$$

16. (b)

$$\text{Reflection co-efficient, } \tau_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{80 - j40 - 70}{80 - j40 + 70} = \left(\frac{10 - j40}{150 - j40}\right) = 0.265j\angle -61.03°$$

$$\therefore \quad \theta_r = -61.03°$$

First voltage minimum is at,

$$d_{min} = \frac{\lambda}{4\pi}(\pi + \theta_L) = \frac{\lambda}{720}(180° - 61.03°)$$

$$= 0.165\,\lambda$$

First voltage maxima from load end is at

$$d_{max} = d_{min} \pm \frac{\lambda}{4} = 0.165\lambda + \frac{\lambda}{4} = 0.415\lambda$$

17. (a)

Since $E_z \neq 0$

$\therefore$ This is TM mode

$$E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \quad \sin\left(\frac{n\pi y}{b}\right)e^{-j\beta Z}$$

$$\therefore \quad \frac{m\pi}{a} = 40\pi \quad \Rightarrow a = 5\,; \quad \therefore\; m = 2$$

$$\therefore \quad \frac{n\pi}{b} = 50\pi \quad \Rightarrow b = 2\,; \quad \therefore\; n = 1$$

$\therefore$ TM_{21} mode

$$\text{Cutoff frequency, } f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_c = \frac{3 \times 10^8}{2}\sqrt{\left(\frac{2}{0.05}\right)^2 + \left(\frac{1}{0.02}\right)^2} = \frac{3 \times 10^8}{2}\sqrt{40^2 + 50^2} = 9.6 \text{ GHz}$$

18. (b)

Power density at a distance is given by

$$W = \frac{P_t \cdot G_t}{4\pi r^2} = \frac{20 \times 10^3 \times 10^{0.5}}{4\pi \times (10^4)^2} = 5.03 \times 10^{-5} \text{ Watt/m}^2$$

Also,
$$W = \frac{|E_s^2|}{2\eta}; \quad \eta = \text{impedance of free space} = 377\,\pi$$

$$\therefore \quad E_s = \sqrt{W 2\eta} = 0.194 \text{ V/m}$$

19. (b)

$$\text{Efficiency, } \eta = \frac{P_{rad}}{P_{in}}$$

$$\therefore \quad P_{\text{rad}} = \eta P_{\text{in}} = 0.9 \times 0.4 = 0.36\text{W}$$

$$\text{Directivity, } D = 4 \times \frac{U_{\max}}{P_{rad}} = \frac{4\pi \times 0.5}{0.36} = 17.45$$

20. (c)

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^6} = 200$$

$\because$ $l << \lambda$

$\therefore$ It is a Hertzian dipole antenna

$$\therefore \quad \text{Radiation resistance, } R_{\text{rad}} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 = 80\pi^2 \left(\frac{1}{200}\right)^2 = 19.7 \text{ m}\Omega$$

$$P_{\text{rad}} = \frac{1}{2} I_0^2 \text{ Re}$$

$$\therefore \quad I_0^2 = \frac{2P_{\text{rad}}}{R_{\text{rad}}} = \frac{2 \times 4}{19.7 \times 10^{-3}} = 405.28$$

$$I_o = 20.13\text{A}$$

21. (a)

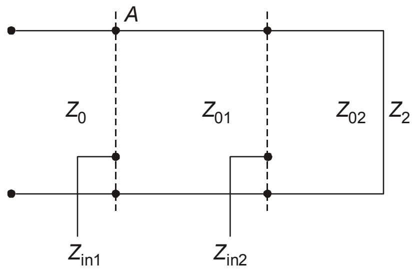
From $\lambda/4$ transform

$$Z_0^2 = Z_{\text{in}} \times Z_L$$

$\therefore$ Hence,
$$Z_{\text{in}2} = \frac{Z_{02}^2}{Z_L} = \frac{30^2}{75} = 12\,\Omega$$

For no reflection to left side of A,
$$Z_{\text{in}1} = Z_0 = 50\ \Omega$$

$$\therefore \quad Z_{\text{in}1} = \frac{Z_{01}^2}{Z_{in2}}$$



$$\therefore \quad Z_{01}^2 = Z_{\text{in}1} \times Z_{\text{in}2} = 50 \times 12 = 600\ \Omega$$

$$Z_{01} = \sqrt{600} = 24.5\,\Omega$$

22. (b)

For good conductor $\frac{\sigma}{\omega c} >> 1$

$$\therefore \quad \text{For wet marshy oil, } \frac{\sigma}{\omega c} = \frac{10^{-2} \times 36\pi}{2\pi \times 8 \times 10^6 \times 15 \times 10^{-9}} = 1.5 \text{ non conducted}$$

For Germanium, $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{0.025 \times 36\pi}{2\pi \times 8 \times 10^6 \times 15 \times 10^{-9}} = 3.575$ (lossy)

For sea water, $\dfrac{\sigma}{\omega\varepsilon} = \dfrac{25 \times 36\pi}{2\pi \times 8 \times 10^6 \times 81 \times 10^{-7}} = 694.4$ (conducting)

$\therefore$ Hence only 3

23. (d)

$\omega = 10^9$ rad/s, $\quad \beta = 8$

$$u = \frac{\omega}{\beta} = \frac{c}{\sqrt{\varepsilon l}}$$

$$\therefore \quad \sqrt{\varepsilon l} = \frac{c \times \beta}{\omega} = \frac{3 \times 10^8 \times 8}{10^9} = 2.4$$

$$\varepsilon l = 5.76$$

$$\eta = \frac{120\pi}{\sqrt{\varepsilon l}} = 50\,\pi\Omega$$

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

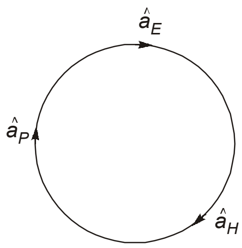
$$\bar{E}_1 = 50\cos(\omega t - 8x)a\hat{y}$$

$\because$ Wave is propagating in $a\hat{x}$ direction

$$a\hat{H} = a\hat{P} + a\hat{E} = a\hat{x} \times a\hat{y} = a\hat{z}$$

$$\frac{E_{01}}{H_{01}} = \eta$$

$$H_{01} = \frac{E}{\eta} = \frac{50}{50\pi} = \frac{1}{\pi}\,A/m$$



$$\therefore \quad \bar{H}_1 = \frac{1}{\pi}\cos(10^9 t - 8x)a\hat{z}$$

Similarly, $\bar{E}_2 = \bar{E}_{02}\sin(\omega t - \beta x)a\hat{z}$

$$a\hat{H} = a\hat{P} + a\hat{E} = a\hat{x} \times a\hat{z} = -a\hat{j}$$

$$\frac{E_{02}}{H_{02}} = \eta$$

$$H_{02} = \frac{E_{02}}{\eta} = \frac{40}{50\pi} = \frac{4}{5\pi}\,A/m$$

$$\therefore \quad \bar{H}_2 = \frac{0.8}{\pi}\sin(10^9 t - 8x)a\hat{y}$$

$$\therefore \quad \bar{H} = \bar{H}_1 + \bar{H}_2 = -\frac{0.8}{\pi}\sin(10^9 t - 8x)a\hat{y} + \frac{1}{\pi}\cos(10^9 t - 8x)a\hat{x} \text{ A/m}$$

24. (a)

Time average power, $\bar{P} = \dfrac{E_0^2}{2\eta}a\hat{k}$

$a\hat{k}$ - unit vector along the direction of wave propagation.

$$a\hat{k} = 0.866a\hat{y} + 0.5a\hat{z}$$

$$\bar{P} = \frac{100^2}{2\eta_0}a\hat{k} = 13.26(0.866a\hat{y} + 0.5a\hat{z}) = 11.48a\hat{y} + 6.63a\hat{z}$$

25. (a)

$$\eta_2 = \eta_0\sqrt{\frac{u_2}{\varepsilon_2}} = \eta_0\sqrt{\frac{8}{2}} = 2\eta_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1}{3}$$

$$\Gamma H = -T = -\frac{1}{3}$$

$$-\Gamma = \frac{Hr\theta}{Hi0}$$

$\therefore$ $$Hr\theta = -\Gamma\ Hi0$$

$$Hr = -\frac{1}{3}\times 10\cos(10^8 t + \beta_1 z) = -\frac{10}{3}\cos(10^8 t + \beta z)\ \text{mA/m}$$

When wave reflected, it travels in opposite direction. Hence now it travels in negative directions of z-axis.

26. (d)

For a rectangular waveguide $a > b$ TE_{10} is a dominant mode.

$$\text{cut-off frequency } f_{c10} = \frac{u}{2a} \qquad \left(u = \frac{c}{\sqrt{\mu_r\varepsilon_r}}\right)$$

$\therefore$ $$f_{c10} = \frac{c}{\sqrt{\varepsilon_r}2a} = \frac{3\times10^8}{\sqrt{4}\times2\times1.5\times10^{-2}} = 5\text{ GHz}$$

Intrinsic wave impedance for TE mode

$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_{c10}}{f}\right)^2}} = \frac{120\pi}{\sqrt{\varepsilon l}\sqrt{1-\left(\frac{5}{75}\right)^2}} = 252.89\ \Omega$$

27. (b)

$a = 1$ cm, $b = 3$ cm

Cut-off frequency for TE_{12} mode

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{1}\right)^2+\left(\frac{2}{3}\right)^2}\times100$$

$$= 18.03\text{ GHz}$$

$f = 1.2\times f_c = 21.6$ GHz

28. (a)

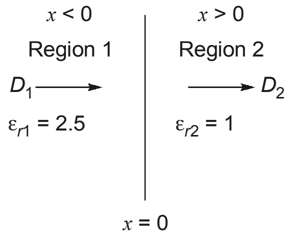
$$I = \int \bar{J}.d\bar{s} \qquad ds = \rho\, d\phi\, dz\hat{a\rho}$$

$$= \int_0^{2\pi}\int_1^5 10z\sin^2\phi\,\rho\, d\phi\, dz\Big|_{\rho=2} = 10\times2\int_1^5 z\,dz\int_0^{2\pi}\sin^2\phi\, d\phi$$

$$= 10\times2\left[\frac{z^2}{2}\right]_1^5\times\pi = 240\pi = 754\text{ A}$$

29. (d)

According to boundary condition



1. Normal component of electric field density for source free region is continuous,

$\therefore \quad \bar{D}_{n1} = \bar{D}_{n2}$

$\therefore \quad D_{n1} = 12a\hat{x}$

$\therefore \quad D_{n2} = D_{n1} = 12a\hat{x}$

2. Tangential component of electric field intensity is continuous

$\therefore \quad \bar{E}_{t1} = \bar{E}_{t2}$

$$\frac{\bar{D}_{t1}}{\varepsilon_{r1}} = \frac{\bar{D}_{t2}}{\varepsilon_{r2}}$$

$$\therefore \quad \bar{D}_{n2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}\bar{D}_{t1} = 2.5(-10a\hat{y} + 4a\hat{z}) = -4a\hat{y} + 1.6a\hat{z}$$

$$\therefore \quad \bar{D}_2 = \bar{D}_{t2} + \bar{D}_{t2} = 12a\hat{x} - 4a\hat{y} + 1.6a\hat{z}$$

30. (d)

Power density is given by $\rho_{avg} = \frac{1}{2}\eta_0 H_0^2 a\hat{x}$

Power passing through surfaces given by

$$P_t = \int \bar{P}_{avg} d\bar{s} = \int \bar{P}_{avg} s\, a\hat{n} \qquad [s = 10 \times 10 \times 10^{-4}\ \text{m}^4]$$

$\bar{d}s = \delta a\hat{n}, \qquad a\hat{n} = \frac{\nabla_f}{|\nabla_f|}$

$$f = x + y - 1$$

$$a\hat{n} = \frac{a\hat{x} + a\hat{y}}{\sqrt{2}}$$

$$\therefore \quad P_t = \int \frac{1}{2}\eta_0 H_0^2 a\hat{x} \times s \times \left(\frac{a\hat{x} \times a\hat{y}}{\sqrt{2}}\right)$$

$$= \frac{1}{2} \times 120\pi \times (0.2)^2 \times 10^{-2} \times \frac{1}{\sqrt{2}} = 53.31\ \text{mW}$$

■■■■