



MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Lucknow | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | E-mail: info@madeeasy.in | Ph: 011-45124612

ENGINEERING MECHANICS

MECHANICAL ENGINEERING

Date of Test : 12/07/2022

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (b) | 13. (b) | 19. (d) | 25. (b) |
| 2. (b) | 8. (b) | 14. (c) | 20. (a) | 26. (d) |
| 3. (c) | 9. (b) | 15. (c) | 21. (c) | 27. (a) |
| 4. (c) | 10. (a) | 16. (a) | 22. (b) | 28. (b) |
| 5. (d) | 11. (b) | 17. (c) | 23. (a) | 29. (a) |
| 6. (d) | 12. (a) | 18. (d) | 24. (c) | 30. (c) |

DETAILED EXPLANATIONS

1. (b)

As per given information,

$$h = 40 \text{ m}, u = 50 \text{ m/s}$$

Let the speed be 'v' when it strike to the ground

Apply law of conservation of energy

$$mgh + \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

$$m \times 10 \times 40 + \frac{1}{2} \times m \times (50)^2 = \frac{1}{2} \times m \times v^2$$

$$400 + 1250 = \frac{v^2}{2}$$

$$v = 57.44 \text{ m/s}$$

2. (b)

$$R_2 \cos 45^\circ = R_1$$

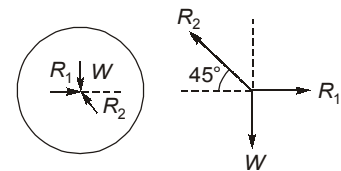
$$R_2 \sin 45^\circ = W$$

$$\Rightarrow R_2 = W\sqrt{2}$$

$$\therefore R_1 = W\sqrt{2} \times \frac{1}{\sqrt{2}} = W$$

$$W = 50 \text{ N}$$

$$\therefore R_1 = 50 \text{ N}$$



3. (c)

$$\text{Lagrangian, } L = T - V$$

$$= v^2 \dot{u}^2 + 2\dot{v}^2 - u^2 + v^2$$

$$= v^2(1 + \dot{u}^2) + (2\dot{v}^2 - u^2)$$

The equation of motion, using langrangian (L) for $q = u$,

$$\Rightarrow \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{u}} \right) - \frac{\partial L}{\partial u} = 0$$

$$\Rightarrow \frac{d}{dt} (2v^2 \dot{u}) - (-2u) = 0$$

$$\Rightarrow 2[v^2 \ddot{u} + 2v\dot{u}] + 2u = 0$$

$$\Rightarrow 2v^2 \ddot{u} + 4v\dot{u} + 2u = 0$$

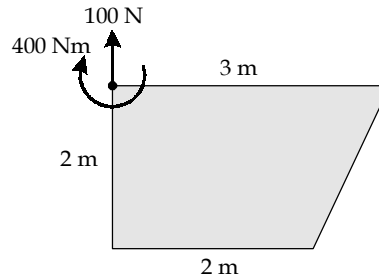
4. (c)

Force in member AH should be zero, as the AH is corner member with only two member connected to each other at 90° . Hence in both member AH and GH force is zero.

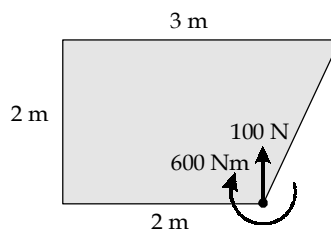
5. (d)

6. (d)

Force-couple system,



Equivalent force couple system,



7. (b)

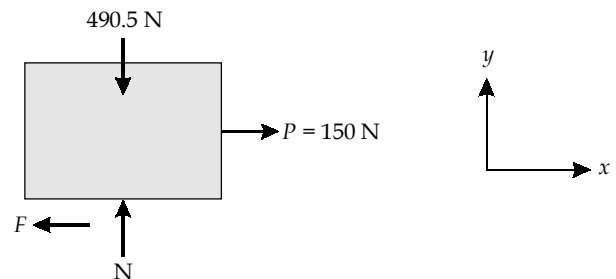
When body is at rest, for equilibrium,

$$N = 490.5 \text{ N}$$

$$\text{Applied force, } P = 150 \text{ N}$$

Maximum static friction force,

$$\begin{aligned} F_{\max} &= \mu_s N \\ &= 0.5 (490.5) \\ &= 245.25 \text{ N} \end{aligned}$$



Because $P < F_{\max}$, we conclude that the block is in static equilibrium and correct value of friction force is,

$$F = 150 \text{ N}$$

8. (b)

Given : Velocity \propto Distance i.e. $\frac{dV}{dt} = -kx$ [where, x : distance]

$$V = \frac{dx}{dt}$$

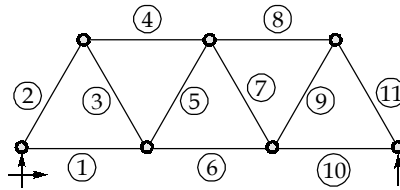
So, $\frac{d^2x}{dt^2} = -kx$

$$\frac{dx}{dt} = -kxt + c_1$$

$$x = -kx \frac{t^2}{2} + c_1 t + c_2$$

From above equation, we can see that the distance covered will be quadratic in time.

9. (b)

Total number of members, $i = 11$ Number of reactions, $r = 3$ Total number of joints $= 2 \times j = 2 \times 7 = 14$

$$i + r = 2j$$

$$11 + 3 = 2 \times 7$$

$$14 = 14$$

Therefore, the truss is stable and internally determinate.

10. (a)

As the rod reaches its lowest position, the center of mass is lowered by a distance l . Its gravitational potential energy is decreased by mgl .

Rotation occurs about the horizontal axis through the clamped end.

$$\text{Moment of inertia, } I = \frac{ml^2}{3}$$

Now, by work energy theorem;

Total work done = Change in kinetic energy

$$(\Delta W)_{mgl} = (KE)_f - (KE)_i$$

$$mgl = \frac{1}{2}I\omega^2 - 0$$

$$\frac{1}{2}I\omega^2 = (mgl)$$

$$\frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2 = (mgl)$$

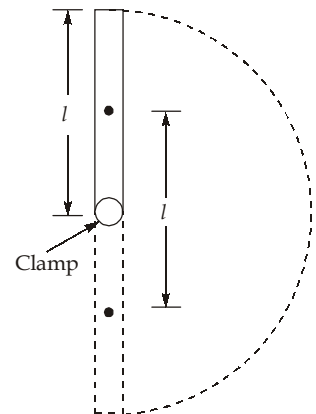
$$\omega^2 = \frac{6g}{l}$$

$$\omega = \sqrt{\frac{6g}{l}}$$

Linear speed of the free end at given instant, $v = l\omega$

$$V = l \times \sqrt{\frac{6g}{l}}$$

$$V = \sqrt{6gl}$$



11. (b)

As per given information,

$$m = 30 \text{ kg}; \quad r = 0.2 \text{ m}$$

$$\omega = 20 \text{ rad/s}; \quad T = 5 \text{ Nm}$$

$$F = 10 \text{ N}$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2} \times 30 \times 0.2^2 = 0.6 \text{ kg.m}^2$$

Let the disk rotate an angle of θ rad.

From work energy principle

$$T \cdot \theta + F \times r \cdot \theta = \frac{1}{2} \times I \times \omega^2 \quad [\because \text{Workdone} = \text{change in energy}]$$

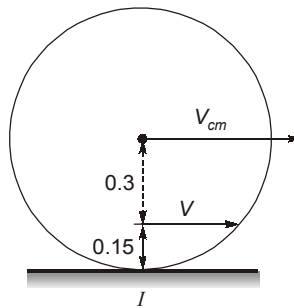
$$5 \cdot \theta + 10 \times 0.2 \times \theta = \frac{1}{2} \times 0.6 \times (20)^2$$

$$7 \cdot \theta = 120$$

$$\theta = 17.14 \text{ rad}$$

$$\text{Number of revolution} = \frac{\theta}{2\pi} = \frac{17.14}{2\pi} = 2.73 \text{ rev}$$

12. (a)



$$V_{cm} = 0.45 \omega$$

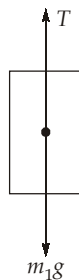
$$\omega = \frac{3}{0.45} \text{ rad/s}$$

$$V = 0.15 \omega = \frac{0.15 \times 3}{0.45}$$

$$V = 1 \text{ m/s}$$

13. (b)

Cylinder

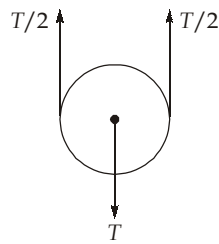


From Newton's first law,

$$m_1g - T = 0$$

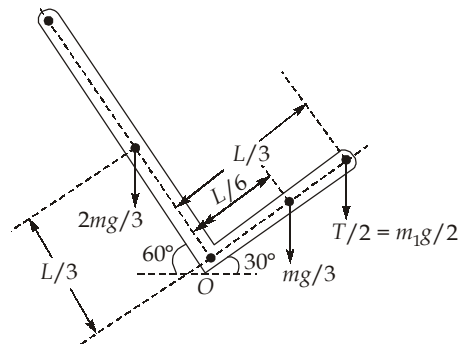
$$T = m_1g$$

Pulley



$$\frac{T}{2} = \frac{m_1 g}{2}$$

To cause loss of contact at A, reaction at A will be zero.



$$\Sigma M_o = 0$$

$$\frac{2mg}{3} \times \frac{L}{3} \cos 60^\circ - \frac{mg}{3} \times \frac{L}{6} \cos 30^\circ - \frac{T}{2} \times \frac{L}{3} \cos 30^\circ = 0$$

$$\Rightarrow \frac{2mg}{9} gL \cos 60^\circ = \frac{m}{18} gL \cos 30^\circ + \frac{m_1 g}{2} \times \frac{L}{3} \cos 30^\circ$$

$$\Rightarrow \frac{2m}{9} \cos 60^\circ = \frac{m}{18} \cos 30^\circ + \frac{m_1}{6} \cos 30^\circ$$

$$m_1 = 0.436 m$$

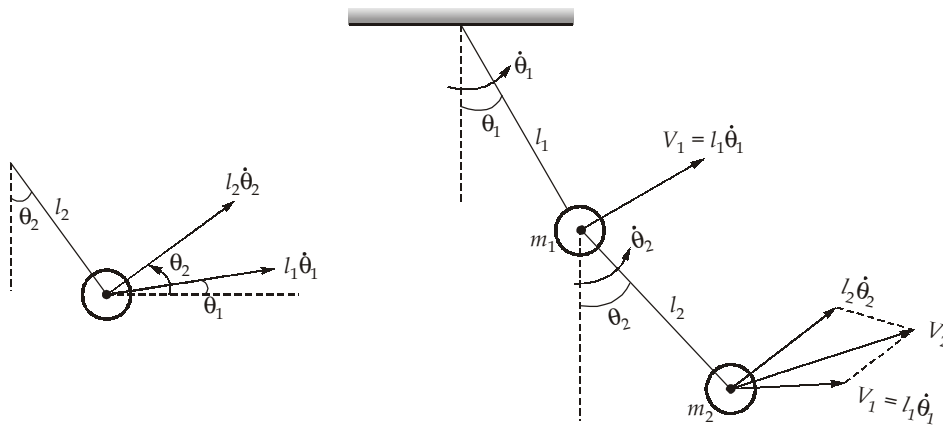
14. (c)

Let the general co-ordinates are $q_1 = \theta_1$ and $q_2 = \theta_2$, c [two degree of freedom so two general co-ordinates]

$$L = T - V$$

$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

where $v_1^2 = (l_1 \dot{\theta}_1)^2$ and V_2 is resultant of $l_1 \dot{\theta}_1$ and $l_2 \dot{\theta}_2$



By using parallelogram law,
 Resultant velocity V_2

$$\Rightarrow V_2^2 = l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1)$$

Here, $\theta_2 - \theta_1 \approx 0$

$$\Rightarrow \cos(\theta_2 - \theta_1) \approx 1$$

So, total kinetic energy, $T = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 [l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2]$... (i)

Potential energy change due to θ_1 and θ_2 .

$$\Rightarrow V = m_1 g l_1 (1 - \cos \theta_1) + m_2 g [l_1 (1 - \cos \theta) + l_2 (1 - \cos \theta_2)]$$

$$\Rightarrow V = m g l_1 \frac{\theta_1^2}{2} + \frac{m_2 g}{2} [l_1 \theta_1^2 + l_2 \theta_2^2]$$
 ... (ii)

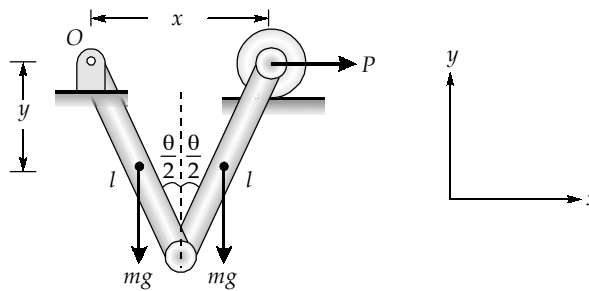
$$\left[\because 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \approx \frac{\theta^2}{2} \right]$$

From equation (i) and (ii)

Lagrangian, $L = T - V$

$$= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 [l_2^2 \dot{\theta}_2^2 + l_1^2 \dot{\theta}_1^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2] - \left[m_1 g l_1 \frac{\theta_1^2}{2} + \frac{m_2 g}{2} (l_1 \theta_1^2 + l_2 \theta_2^2) \right]$$

15. (c)



$$x = 2l \sin \frac{\theta}{2}$$

$$\partial x = l \cos \frac{\theta}{2} \partial \theta$$

$$y = -\frac{l}{2} \cos \frac{\theta}{2}$$

$$\partial y = +\frac{l}{4} \sin \frac{\theta}{2} \partial \theta$$

$$+P(\partial x) + (-2mg) \cdot \partial y = 0$$

$$P \left(l \cos \frac{\theta}{2} \partial \theta \right) - 2mg \left(\frac{l}{4} \sin \frac{\theta}{2} \partial \theta \right) = 0$$

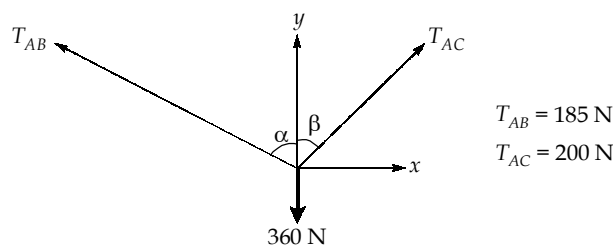
$$Pl \cos \frac{\theta}{2} \partial \theta = 2mg \times \frac{l}{4} \sin \frac{\theta}{2} \partial \theta$$

$$\tan \frac{\theta}{2} = \frac{2P}{mg}$$

$$\theta = 2 \tan^{-1} \left(\frac{2P}{mg} \right)$$

16. (a)

As per given condition,



$$T_{AB} \sin \alpha = T_{AC} \sin \beta$$

$$\sin \alpha = \frac{200}{185} \sin \beta$$

...(i)

$$T_{AB} \cos \alpha + T_{AC} \cos \beta = 360$$

$$\cos \alpha = \frac{360}{185} - \frac{200}{185} \cos \beta$$

...(ii)

From equation (i) and (ii), we get

$$1 = \left(\frac{200}{185} \right)^2 + \left(\frac{360}{185} \right)^2 - \left(2 \times \frac{360 \times 200}{185^2} \cos \beta \right)$$

$$2 \times \frac{360 \times 200}{185^2} \cos \beta = 3.955$$

$$\cos \beta = 0.9401$$

$$\beta = 19.9^\circ \approx 20^\circ$$

$$\sin \alpha = \frac{200}{185} \sin 20^\circ$$

$$\alpha = 21.7^\circ$$

17. (c)

Moment about the point 'c'

(Vector method), $M_c = \vec{r} \times \vec{F}$

$$\begin{aligned} \text{Force vector, } \vec{F} &= 500 \frac{\overline{AB}}{|\overline{AB}|} \\ &= 500 \left(\frac{2\hat{i} - 4\hat{j} + 3\hat{k}}{\sqrt{(2)^2 + (-4)^2 + (3)^2}} \right) \\ &= 92.847(2\hat{i} - 4\hat{j} + 3\hat{k}) \end{aligned}$$

Position vector, $r_{CA} = -2\hat{i} - 0\hat{j} + 0\hat{k}$

$$\begin{aligned} M_C &= \vec{r}_{CA} \times \vec{F} \\ &= (-2\hat{i})[92.847(2\hat{i} - 4\hat{j} + 3\hat{k})] \end{aligned}$$

$$M_C = 92.847 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 0 \\ 2 & -4 & 3 \end{vmatrix}$$

$$= 92.847(6\hat{j} + 8\hat{k})$$

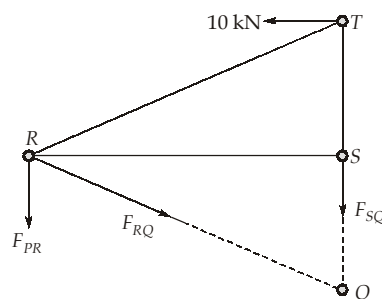
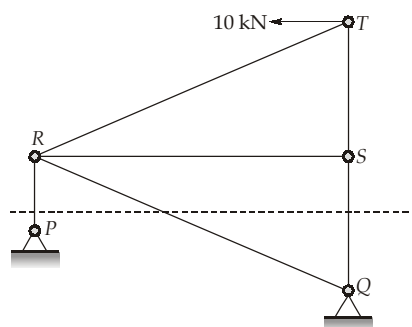
$$= 557.086\hat{j} + 742.776\hat{k}$$

Magnitude, $M_C = \sqrt{(557.086)^2 + (742.776)^2}$

$$M_C = 928.47 \text{ Nm}$$

18. (d)

From method of section,



Taking moment about Q,

$$10 \times 6 + F_{PR} \times 4 = 0$$

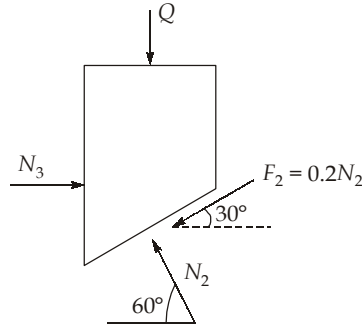
$$F_{PR} = -\frac{60}{4} = -15 \text{ kN}$$

$$= 15 \text{ kN (Compressive)}$$

19. (d)

From Newton's first law,

$$\Sigma F_y = 0$$



$$N_2 \sin 60^\circ - 0.2N_2 \sin 30^\circ - Q = 0$$

$$Q = 0.766 N_2$$

$$\Sigma F_x = 0$$

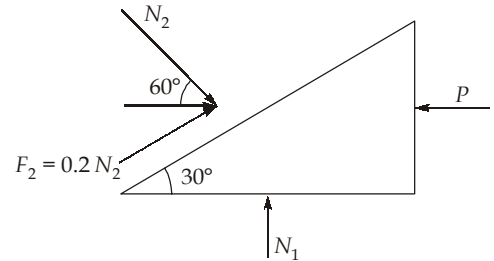
$$N_2 \cos 60^\circ + 0.2N_2 \cos 30^\circ - P = 0$$

$$P = 0.673 N_2$$

$$\frac{P}{Q} = \frac{0.673}{0.766}$$

$$P = 0.878Q \approx 0.9Q$$

$$\alpha = 0.9$$



20. (a)

There are three forces acting on the bar AB; pull Q at B, tension in string T and reaction at point A i.e. R_a .

For isosceles triangle ABC,

$$\beta = \gamma = \left(\frac{\pi - \alpha}{2} \right) = 90^\circ - \left(\frac{\alpha}{2} \right)$$

If there is no friction on pulley, tension in string BC will be P.

Taking moment about point A,

$$(P \cos \delta) \times (l \sin \alpha) + (P \sin \delta)(l \cos \alpha) = Ql \sin \alpha$$

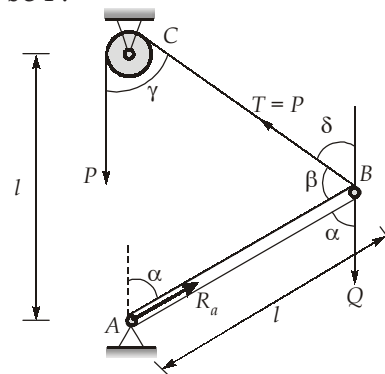
$$Pl \sin(\alpha + \delta) = Ql \sin \alpha$$

$$P \sin(180^\circ - \beta) = Q \sin \alpha$$

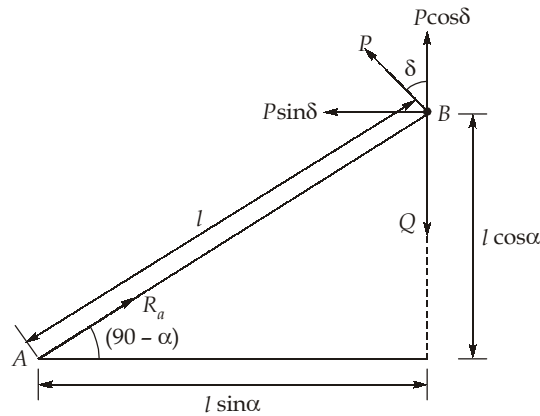
$$P \sin \left[180 - 90 + \frac{\alpha}{2} \right] = Q \sin \alpha$$

$$P \cos \frac{\alpha}{2} = 2Q \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}$$

$$\left(\cos \frac{\alpha}{2} \right) \left[P - 2Q \sin \frac{\alpha}{2} \right] = 0$$



or
$$\sin \frac{\alpha}{2} = \frac{P}{2Q}$$



$$\alpha = 2 \sin^{-1} \left(\frac{P}{2Q} \right) = 2 \sin^{-1} \left(\frac{900}{2 \times 2200} \right) = 23.6057^\circ$$

$$\alpha = 23.6057 \times \left(\frac{\pi}{180} \right) = 0.412 \text{ radian}$$

21. (c)

Given: $M = 2000 \text{ kg}$, $v_1 = 2 \text{ m/s}$, $v_2 = 0$, for drum $m = 50 \text{ kg}$, $k = 0.7 \text{ m}$, $R = 0.75 \text{ m}$, $h = 0.5 \text{ m}$

$$\Delta kE \text{ of mass} = \frac{1}{2} M (v_1^2 - v_2^2) = \frac{1}{2} \times 2000 \times (2^2 - 0^2) = 4000 \text{ J}$$

$$\Delta kE \text{ of drum} = \frac{1}{2} m k^2 (\omega_1^2 - \omega_2^2) = \frac{1}{2} \times 50 \times 0.7^2 \times \left[\left(\frac{2}{0.75} \right)^2 - 0^2 \right] = 87.11 \text{ J}$$

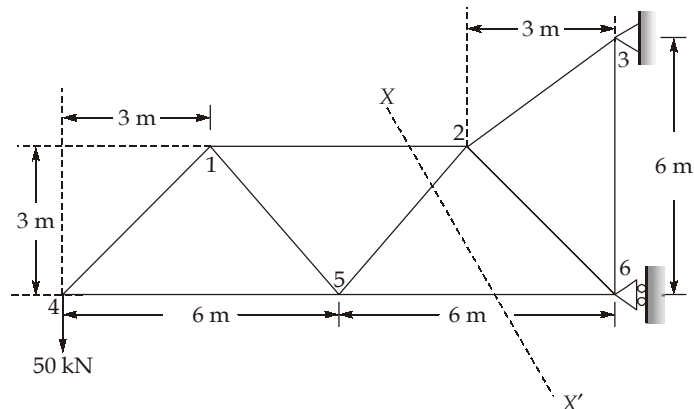
$$\Delta kE \text{ of the mass} = mgh = 2000 \times 9.81 \times 0.5 = 9810 \text{ J}$$

the total energy absorbed by the break is given by

$$\begin{aligned} E &= \Delta kE \text{ of mass} + \Delta kE \text{ of drum} + \Delta PE \text{ of mass} \\ &= 4000 + 87.11 + 9810 \\ E &= 13897.11 \text{ J} \approx 13897 \text{ J} \end{aligned}$$

22. (b)

Cutting section about, 1-2, 2-5 and 5-6:



Taking moment of left part about 5.

$$F_{1-2} = \frac{50 \times 6}{3} = 100 \text{ kN(T)}$$

Cutting section through 1-2, 1-5 and 4-5 and balancing vertical forces for left part only,

$$\Sigma F_V = 0$$

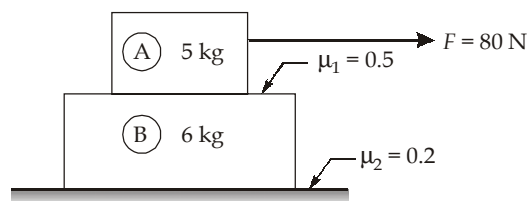
$$\Rightarrow F_{1-5} \times \cos 45^\circ + 50 = 0$$

$$F_{1-5} \cos 45^\circ = -50$$

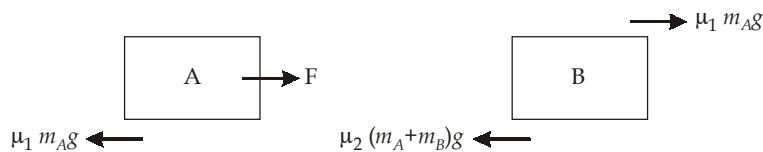
$$\frac{F_{1-5}}{\sqrt{2}} = -50 \Rightarrow F_{1-5} = 50\sqrt{2} \text{ kN(C)}$$

23. (a)

According to question:



FBD of (A) and (B)



Equation of motion for A,

$$F - \mu_1 m_A g = m_A a_A$$

$$80 - 0.5(5)(9.81) = (5)a_A$$

$$a_A = 11.095 \text{ m/s}^2$$

So,

$$V_A = U_A + a_A t = 0 + 11.095(0.1)$$

$$= 1.1095 \text{ m/s}$$

Now, equation of motion for B,

$$\mu_1 m_A g - \mu_2 (m_A + m_B) g = m_B a_B$$

$$0.5(5)(9.81) - 0.2(5 + 6)9.81 = 6(a_B)$$

$$a_B = 0.4905 \text{ m/s}^2$$

So,

$$V_B = U_B + a_B t$$

$$= 0 + 0.4905(0.1) = 0.04905 \text{ m/s}$$

$$\therefore \text{Relative velocity of A with respect to B} = V_A - V_B$$

$$= 1.1095 - 0.04905$$

$$= 1.06045 \text{ m/s}$$

24. (c)

$$I = 2000 \times 0.25^2 = 125 \text{ kg-m}^2$$

for retardation, $\omega = \omega_0 + \alpha t$

$$\omega = 0$$

$$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi \times 3000}{60}$$

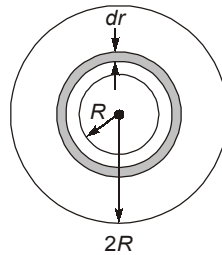
$$t = 10 \text{ min} = 600 \text{ sec}$$

$$\alpha = \frac{2\pi \times 3000}{60 \times 600} = 0.5236 \text{ rad/s}$$

So, average frictional torque,

$$I\alpha = 65.44 \text{ Nm}$$

25. (b)



∴ $I =$ Moment of inertia of disc

Total mass m is contained in area $3\pi R^2$

Let dm be the mass in area $2\pi r dr$

$$\therefore dm = \frac{m \times 2\pi r dr}{3\pi R^2}$$

$$I = \int_R^{2R} dm r^2$$

$$\therefore I = \frac{5}{2} m R^2$$

$$\text{K.E.} = (\text{K.E.})_{\text{translation}} + (\text{K.E.})_{\text{rotation}} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

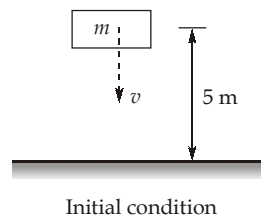
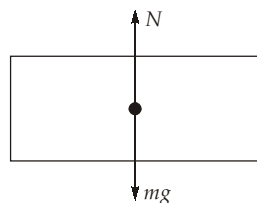
But $V = 2R\omega$

$$\therefore \text{K.E.} = \frac{1}{2} m v^2 + \frac{1}{2} \times \frac{5R^2}{2} \times \frac{V^2}{4R^2} = \frac{1}{2} m v^2 + \frac{5}{16} m v^2$$

$$\text{K.E.} = \frac{8m v^2 + 5m v^2}{16} = \frac{13}{16} m v^2 = \frac{13}{16} \times 5 \times (2)^2$$

$$\therefore \text{K.E.} = 16.25 \text{ J}$$

26. (d)



$$\begin{aligned} \text{Velocity when block reaches the ground} &= \sqrt{2gh} \\ &= \sqrt{2 \times 10 \times 5} = 10 \text{ m/s} \end{aligned}$$

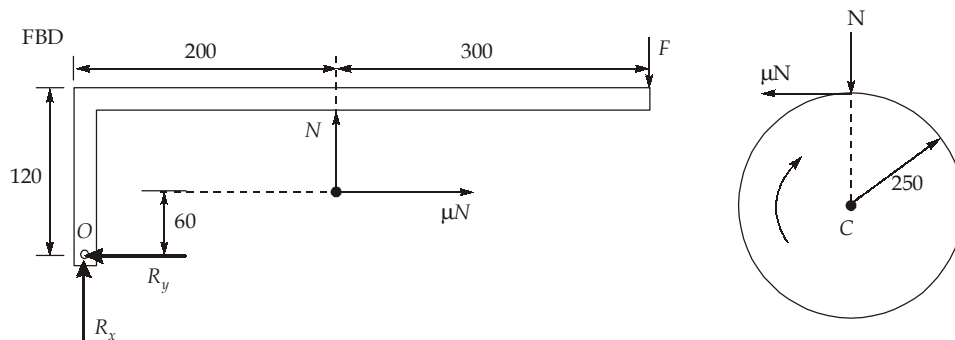
By momentum conservation:

$$\begin{aligned} (F) \times dt &= \text{Momentum just after striking the ground} - \text{momentum just before striking the ground} \\ (N - mg) \times dt &= m \times 0 - (-m \times 10) \\ (N - mg) &= \frac{m \times 10}{dt} \end{aligned}$$

$$N = \frac{10 \times 10}{(1/10)} + 10 \times 10$$

Force of interaction, $N = 1100 \text{ N}$

27. (a)



For the drum, about C

$$\begin{aligned} \mu N \times 0.250 &= 30 \\ N &= 400 \text{ N} \end{aligned}$$

For the link,

$$\Sigma T_{\text{net}_O} = 0: \mu N \times 60 + F \times 500 = N \times 200$$

$$F = \frac{N \times 200 - \mu N \times 60}{500}$$

$$F = \frac{400 \times 200 - 0.3 \times 400 \times 60}{500}$$

$$F = 145.6 \text{ N}$$

28. (b)

$$mg(\sin\theta + \mu \cos\theta) = 3mg(\sin\theta - \mu \cos\theta)$$

$$(\sin 45^\circ + \mu \cos 45^\circ) = 3(\sin 45^\circ - \mu \cos 45^\circ)$$

$$\mu = 0.5$$

29. (a)

Taking all as a system

$$F_{\text{net}} = (m_A + m_B + m_C) a_{\text{net}}$$

$$a_{\text{Net}} = \frac{m_C a_C + m_B \times 0 + m_A a_A}{m_A + m_B + m_C}$$

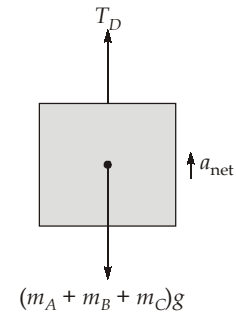
$$a_C = +3 \text{ m/s}^2$$

$$a_A = -5 \text{ m/s}^2$$

$$\Rightarrow a_{\text{net}} = -\frac{7}{6} \text{ m/s}^2$$

$$\Rightarrow T_D - 30 \times g = (m_A + m_B + m_C) a_{\text{net}} = -30 \times \frac{7}{6}$$

$$\Rightarrow T_D = 265 \text{ N}$$



30. (c)

Energy conservation between point (1) and (2).

$$mgH = mg(R + R \sin \theta) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \dots(1)$$

w = angular speed of ball about its own axis

α = angular acceleration of ball = $\frac{dw}{dt}$

w_1 = angular speed of ball about 'O' = $\frac{d\theta}{dt}$

$\alpha_1 = \frac{dw_1}{dt}$, $v = w_1(R - r)$

For pure rolling $w_1 R = w \times r$ $\left(\frac{dw_1}{dt} = \alpha_1 \right)$
 $\alpha_1 R = \alpha \times r$

differentiating equation (1) with respect to time t ,

$$0 = 0 + w_1 mgR \cos \theta + mv \cdot \frac{dv}{dt} + I w \alpha$$

$$\Rightarrow 0 = mgR \cos \theta \times w_1 + mw_1(R - r)^2 \alpha_1 + \frac{2}{5}mR^2 \times \frac{w_1 R}{r} \times \frac{\alpha_1 R}{r}$$

$$\Rightarrow -gR \cos \theta = \alpha_1 \left[(R - r)^2 + \frac{2}{5} \frac{R^4}{r^2} \right]$$

$$\Rightarrow \alpha_1 = \frac{-gR \cos \theta}{(R - r)^2 + \frac{2}{5} \frac{R^4}{r^2}}$$

