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CLASS TEST 2019-2020

ELECTRONICS ENGINEERING

Subject : Electromagnetic Theory

Date of test : 28/04/2019

Answer Key

1.	(c)	7.	(b)	13.	(c)	19.	(d)	25.	(a)
2.	(d)	8.	(a)	14.	(d)	20.	(d)	26.	(a)
3.	(b)	9.	(c)	15.	(c)	21.	(a)	27.	(a)
4.	(d)	10.	(a)	16.	(d)	22.	(b)	28.	(d)
5.	(b)	11.	(d)	17.	(c)	23.	(d)	29.	(a)
6.	(b)	12.	(c)	18.	(b)	24.	(c)	30.	(a)

Detailed Explanations

1. (c)

$$\theta_B = \tan^{-1}\sqrt{\varepsilon_r}$$

$$\Rightarrow \quad \sqrt{\varepsilon_r} = \tan 70°$$

$$\Rightarrow \quad \varepsilon_r = 7.55$$

2. (d)

$$I = \int \vec{J}\cdot\vec{ds} = \int_0^{16}\int_0^{2\pi} \frac{40}{\rho^2+4}\cdot\rho\, d\rho\, d\phi$$

$$\text{Let } p = \rho^2 + 4$$

$$dp = 2\rho\, d\rho$$

$$\frac{dp}{2} = \rho\, d\rho$$

$$= \frac{40(2\pi)}{2}\cdot\int_4^{260}\left(\frac{dp}{p}\right) = 40\pi\, ln\frac{260}{4}$$

$$I = 524.57 \text{ Amp}$$

3. (b)

$$\bar{F} = I(\bar{L}\times\bar{B})$$

$$= 10\,(4\, a_y \times 0.05\, a_x)$$

$$= -2\, a_z \text{ N}$$

4. (d)
Equation of straight line is

$$\frac{x-0}{2-0} = \frac{y-0}{1-0} = \frac{z-0}{3-0} = t$$

$$\Rightarrow \quad x = 2t,\ y = t,\ z = 3t$$

$$\Rightarrow \quad dx = 2dt,\ dy = dt,\ dz = 3dt$$

$$\text{Workdone} = \int_c \bar{F}\cdot d\bar{r} = \int_c 3x^2 dx + (2xz - y)dy + zdz$$

$$= \int_0^1 \left[3(2t)^2(2dt) + \big(2(2t)(3t) - t\big)dt + (3t)3dt\right]$$

$$= \int_0^1 \left[36t^2 + 8t\right]dt = 16 \text{ N-m}$$

5. (b)

$$\beta = \sqrt{6^2+8^2} = \sqrt{36+64} = 10$$

$$f = \frac{c}{\lambda} = \frac{c}{2\pi}\times\beta$$

$$= \frac{3\times10^8}{2\pi}\times10$$

$$= 0.477 \times 10^9 \text{ Hz}$$

6. (b)

Given
$$\vec{H} = (\hat{x} + j\hat{y})e^{j(\omega t-\beta z)} \text{ mA/m}$$

$$\vec{E} = 120\pi(-\hat{y} + j\hat{x})e^{j(\omega t-\beta z)} \text{ mV}$$

[$\vec{E}\times\vec{H} =$ direction of propagation]

$\vec{E}$ contains two orthogonal components with equal in magnitude and y component leads x component by 90°

∴ Wave is left circularly polarized.

7. (b)

Total flux passing through all 6 faces is 30 mC. Flux passing through one face is $\frac{30}{6}$ = 5 mC

8. (a)

$$Z_{in} = Z_0\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l} = 50\frac{150 + j50\tan72^\circ}{50 + j150\tan72^\circ} \simeq 18.2 - j14.3\ \Omega$$

9. (c)

$$\nabla\cdot\vec{D} = \rho$$

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\frac{r^2\theta}{\pi r^2}(1-\cos3r)\right) = \rho$$

$$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\frac{\theta}{\pi}(1-\cos3r)\right) = \rho$$

$$\frac{\theta}{r^2\pi}\left(\frac{\partial}{\partial r}(-\cos3r)\right) = \rho$$

$$\rho = \frac{3\theta}{r^2\pi}\sin3r$$

10. (a)

$$\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

$$\Rightarrow \quad f = \frac{1}{\delta^2\pi\mu\sigma} = \frac{1}{36\times10^{-6}\times\pi\times4\pi\times10^{-7}\times10^6} = 7036.2 \text{ Hz} \simeq 7 \text{ kHz}$$

11. (d)

From the $\vec{E}$ field, propagation vector is

$$\vec{k}_1 = 3a_x + \sqrt{3}a_z$$

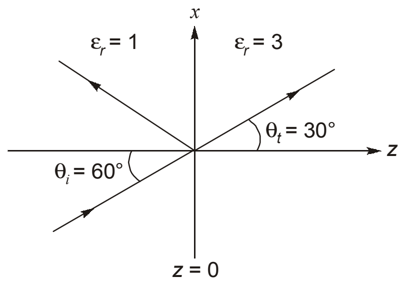
$$\Rightarrow \qquad k_1 = \sqrt{12} = \frac{\omega}{c}$$

$$\omega = 3\sqrt{12} \times 10^8 \text{ m/s}$$

A unit vector normal to the interface ($z = 0$) is a_z. The plane containing K and a_z is y = constant, Which is XZ plane, the plane of incidence. Since E_1 is normal to the plane, it is perpendicular polarization.

$$\tan\theta_i = \frac{k_{ix}}{k_{iz}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\theta_i = 60°$$



$$\sqrt{\varepsilon_r} \cdot \sin 60° = \sqrt{3} \cdot \sin\theta_t$$

$$1 \times \frac{\sqrt{3}}{2} = \sqrt{3} \cdot \sin\theta_t$$

$$\theta_t = 30°$$

$$\theta_i = 60°$$

$$\Gamma_1 = \frac{E_{ro}}{E_{io}} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$\eta_1 = \eta_0 = 377\ \Omega$$

$$\eta_2 = \frac{377}{\sqrt{3}}$$

$$\Gamma_1 = \frac{\frac{377}{\sqrt{3}} \cdot \frac{1}{2} - \frac{377\sqrt{3}}{2}}{\frac{377}{\sqrt{3}} \cdot \frac{1}{2} + \frac{377\sqrt{3}}{2}} = \frac{\frac{377}{2}\left(\frac{1}{\sqrt{3}} - \sqrt{3}\right)}{\frac{377}{2}\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)}$$

$$\Gamma_1 = \frac{1-3}{1+3} = \frac{-2}{4} = -0.5$$

$$\Gamma_1 = -0.5 = \frac{E_{ro}}{10}$$

$$E_{ro} = -5$$

$$\vec{E}_r = -5\cos(\omega t - 3x + \sqrt{3}z)\, a_y \text{ V/m}$$

12. (c)

Given, $\qquad E_t = -4\, E_r$

$$\Rightarrow \qquad \frac{E_t}{E_i} = -4\frac{E_r}{E_i}$$

$$\Rightarrow \qquad \tau = -4\,\Gamma$$

where $\qquad \tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}, \qquad \Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

$$\frac{2\eta_2}{\eta_2+\eta_1} = -4\frac{\eta_2-\eta_1}{\eta_2+\eta_1}$$

$$\Rightarrow \quad \frac{1}{1+\frac{\eta_1}{\eta_2}} = 2\cdot\frac{\frac{\eta_1}{\eta_2}-1}{\frac{\eta_1}{\eta_2}+1}$$

$$\Rightarrow \quad \frac{2\eta_1}{\eta_2} = 3$$

$$\Rightarrow \quad \frac{120\pi}{\frac{120\pi}{\sqrt{\varepsilon_r}}} = 1.5$$

$$\Rightarrow \quad \varepsilon_r = 2.25$$

13. (c)

$$P_{avg} = \frac{1}{2}\frac{|E|^2}{\eta} = \frac{1}{2}\frac{(100+400)}{120\pi}\ \text{W/m}^2$$

$$\text{Power} = P_{avg}\times \text{Area} = \frac{1}{2}\times\frac{500}{120\pi}\times\pi\times 36 = \frac{500}{240}\times 36 = \frac{50}{2}\times 3 = 75\ \text{W}$$

14. (d)

We know,

$$RC = \frac{\epsilon}{\sigma}$$

$$C = \frac{A\epsilon}{d} = \frac{\pi}{4}(b^2-a^2)\epsilon/t$$

$$R\times\frac{\pi\epsilon}{4t}(b^2-a^2) = \frac{\epsilon}{\sigma}$$

$$R = \frac{4t}{\sigma\pi(b^2-a^2)}$$

Option (d) is correct.

15. (c)

$$V = \frac{kq_1}{L_1}+\frac{kq_2}{L_2}+\frac{kq_3}{L_3}+\frac{kq_4}{L_4}$$

$q_1 = q$ $\qquad L_1 = L$

$q_2 = q$ $\qquad L_2 = L$

$q_3 = -q$ $\qquad L_3 = \sqrt{5}L$

$q_4 = -q$ $\qquad L_4 = \sqrt{5}L$

$$V = \frac{1q}{4\pi\epsilon_0}\left(\frac{1}{L}+\frac{1}{L}-\frac{1}{\sqrt{5}L}-\frac{1}{\sqrt{5}L}\right) = \frac{1}{4\pi\epsilon_0}\cdot\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$$

16. (d)
Given,

$$\vec{F}(\rho,\phi,z) = \rho\hat{a}_\rho + \rho\sin^2\phi\,\hat{a}_\phi - z\hat{a}_z$$

$$\nabla\cdot\vec{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}\rho^2 + \frac{\partial}{\partial\phi}\rho\sin^2\phi + \frac{\partial\rho}{\partial z}(-z)\right]$$

$$= \frac{1}{\rho}[2\rho + 2\rho\sin\phi\cos\phi - \rho] = \frac{1}{\rho}[\rho + \rho\sin 2\phi]$$

$$\nabla\cdot\vec{F} = 1 + \sin 2\phi$$

$$\nabla\cdot\vec{F}\Big|_{\phi=\frac{\pi}{4}} = 1 + 1 = 2$$

$$\nabla\cdot\vec{F}\Big|_{\phi=0} = 1$$

$$\therefore \quad \nabla\cdot\vec{F}\Big|_{\phi=\frac{\pi}{4}} = 2\nabla\cdot\vec{F}\Big|_{\phi=0}$$

17. (c)

$$E = \frac{\Delta V}{\Delta Z} = \frac{250-100}{5\times10^{-3}} = 3\times10^4 \text{ V/m}$$

$$\vec{E} = -\nabla V = -3\times10^4\hat{a}_z \text{ V/m}$$

$$\vec{D} = \epsilon_0\epsilon_r\vec{E} = \left(\frac{10^{-9}}{36\pi}\right)\times 2.2\times(-3\times10^4)\hat{a}_z$$

$$\vec{D} = -5.8356\times10^{-7}\hat{a}_z \text{ C/m}^2$$

Since $\vec{D}$ is constant between the disks, and $D_n = \rho_s$ at a conductor surface.

$$\therefore \quad \rho_s = \pm 5.8356\times10^{-7} \text{ C/m}^2$$

Positive sign on the upper plate and negative sign on the lower plate.

18. (b)

$$\vec{B} = \nabla\times\vec{A}$$

$$= -\frac{\partial A_z}{\partial\rho}\hat{a}_\phi = \frac{\rho}{2}\hat{a}_\phi$$

$$\text{Flux} = \iint_s \vec{B}\cdot d\vec{S} = \int_{\rho=1}^{2}\int_{z=0}^{5}\frac{\rho}{2}\hat{a}_\phi\cdot d\rho\,dz\,\hat{a}_\phi$$

$$= 5\int_{\rho=1}^{2} \frac{\rho}{2} d\rho = \frac{5}{2}\frac{\rho^2}{2}\Big]_1^2$$

$$= \frac{5}{4}[3] = \frac{15}{4} = 3.75 \text{ Wb}$$

19. (d)

$$\vec{E}(y,t) = 120\pi \times 2\cos(\omega t - \beta y + \pi)\,\hat{a}_z - 120\pi \times 2\cos(\omega t - \beta y)\,\hat{a}_x \text{ V/m}$$

$$= 240\pi[\cos(\omega t - \beta y + \pi)\,\hat{a}_z - \cos(\omega t - \beta y)\,\hat{a}_x] \text{ V/m}$$

wave contains two components and phase difference is π. Therefore wave is linearly polarized.

20. (d)

Force on ϕ due to line charge

$$F_1 = \frac{\phi \cdot \rho_L}{2\pi\epsilon_0 4}\hat{a}_y$$

Force on ϕ due to Q

$$F_2 = \frac{\phi \cdot Q}{4\pi\,\epsilon_0 4^2}(-\hat{a}_y)$$

$$F_1 + F_2 = 0 \Rightarrow \frac{\rho_L}{4} = \frac{Q}{2\times 16}$$

$$\Rightarrow \quad Q = 8\rho_L = 8 \times 2 = 16 \text{ mC}$$

21. (a)

$$\beta = \frac{\pi}{4}\text{rad/m}$$

$$\eta = 150\,\Omega \quad \Rightarrow \quad 150 = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$\Rightarrow \quad \sqrt{\epsilon_r} = \frac{120\pi}{150} = 2.51$$

now,

$$\omega = \beta V = \frac{\pi}{4}\times\frac{3\times 10^8}{2.51} = 93.75 \times 10^6 \text{ rad/sec}$$

22. (b)

$$Y_L = 0.01 + j0.02\,\mho$$

$$Z_L = \frac{1}{Y_L} = 20 - j40\,\Omega$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{20 - j40 - 300}{20 - j40 + 300} = 0.877\angle{-164.7^\circ}$$

$$\Gamma_L(l) = \Gamma_L e^{-j2\beta l} = \Gamma_L e^{\frac{-j2\pi}{\lambda}\cdot 0.1\lambda}$$

$$= \Gamma_L e^{-j36^\circ} = 0.877\angle{-200.7^\circ}$$

23. (d)

$$\vec{H}_i = 4\sin(\omega t - \beta x)\,\hat{y} \text{ mA/m}$$

$$\vec{E}_i = 120\pi \times 4\sin(\omega t - \beta x)\,\hat{a}_{E_i} \text{ mV/m}$$

Where, $\hat{a}_{E_i} = \hat{y} \times \hat{x} = -\hat{z}$

$\Rightarrow$ $\vec{E}_i = 480\pi\sin(\omega t - \beta x)(-\hat{z}) \text{ mV/m}$

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{1}{3} - 1}{\frac{1}{3} + 1} = -\frac{2}{4} = -\frac{1}{2}$$

$\Rightarrow$ $\vec{E}_r = -\frac{1}{2}480\pi\sin(\omega t + \beta x)(-\hat{z}) \text{ mV/m} = 240\pi\sin(\omega t + \beta x)\hat{z} \text{ mV/m}$

24. (c)

$$\oint_L \vec{A}\cdot\vec{dl} = \Psi_m$$

$$\oint_L \vec{A}\cdot\vec{dl} = \int_S (\nabla \times \vec{A})\cdot\vec{ds}$$ summation of all the lines

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 2\rho\sin\phi & 4z\rho\cos\phi & 25z \end{vmatrix}_{\phi = \frac{\pi}{2}} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 2\rho & 0 & 25z \end{vmatrix} = 0$$

So, $\Psi_m = 0$

25. (a)

With the line charge along the x-axis, the x co-ordinates of the two points may be ignored

$$\rho_A = \sqrt{9+16} = 5 \text{ m}$$

$$\rho_B = \sqrt{25+144} = 13 \text{ m}$$

$\therefore$ $$V_{AB} = -\int_{\rho_B}^{\rho_A} \frac{\rho_l}{2\pi \epsilon_0 \rho} d\rho = \frac{-\rho_l}{2\pi\varepsilon_0} ln\frac{\rho_A}{\rho_B} = \frac{-400 \times 10^{-12}}{8.854 \times 10^{-12} \times 2\pi} \times ln\left(\frac{5}{13}\right)$$

$$= 6.87 \text{ V}$$

26. (a)

$\vec{E}$ at (3, –2, 1) due to 10 μC/m^2 is

$$\frac{\rho_s}{2\epsilon_0}\hat{n} = \frac{10}{2\epsilon_0}(\hat{x}) = \frac{10\hat{x}}{2\epsilon_0}$$

$\vec{E}$ due to 20 μC/m^2 is

$$\frac{\rho_s}{2\epsilon_0}\hat{n} = \frac{20}{2\epsilon_0}(-\hat{z}) = -\frac{20}{2\epsilon_0}\hat{z}$$

Total, $$\vec{E} = \frac{5\hat{x}}{\epsilon_0} - \frac{10}{\epsilon_0}\hat{z} = \frac{5}{\epsilon_0}(\hat{x} - 2\hat{z}) \ \mu\text{V/m}$$

27. (a)

For a distortionless transmission line,

$$Z_0 = \sqrt{\frac{R}{G}} \quad \text{...(i)}$$

and
$$\alpha = \sqrt{RG} \quad \text{...(ii)}$$

From equations (i) and (ii), $\alpha = \frac{R}{Z_0} = \frac{0.5}{100} = 0.005 \text{ Np/m}$

28. (d)

$$\frac{P_t}{P_i} = \left(1 - |\Gamma|^2\right)$$

Where,
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{120\pi}{3} - \frac{120\pi}{2}}{\frac{120\pi}{3} + \frac{120\pi}{2}} = -\frac{1}{5}$$

$\therefore$
$$\frac{P_t}{P_i} = \left(1 - \frac{1}{25}\right) = \frac{24}{25} = 0.96$$

29. (a)

$\vec{E}$ at (3, –2, 1) due to 10 μC/m^2 is

$$\frac{\rho_s}{2\epsilon_0}\hat{n} = \frac{10}{2\epsilon_0}(\hat{x}) = \frac{10\hat{x}}{2\epsilon_0}$$

$\vec{E}$ due to 20 μC/m^2 is

$$\frac{\rho_s}{2\,\epsilon_0}\hat{n} = \frac{20}{2\,\epsilon_0}(-\hat{z}) = -\frac{20}{2\,\epsilon_0}\hat{z}$$

Total,
$$\vec{E} = \frac{5\hat{x}}{\epsilon_0} - \frac{10}{\epsilon_0}\hat{z} = \frac{5}{\epsilon_0}(\hat{x} - 2\hat{z})\ \mu\text{V/m}$$

30. (a)

For short circuited transmission line,

$$Z_{in} = jZ_0 \tan\beta l$$

where,
$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{c/f} = \frac{2\pi}{\frac{3\times10^{10}}{3\times10^{10}}} \text{ rad/cm}$$

$$\beta = 2\pi \text{ rad/cm}$$

and
$$\beta l = 4\pi \text{ rad}$$

$\therefore$
$$Z_{in} = jZ_0\tan(4\pi)$$
$$Z_{in} = 0$$

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