CLASS TEST

Sl. 02 SK_ABCD_10072022[G]

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ELECTROMAGNETIC FIELDS

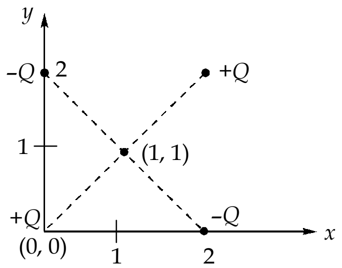
ELECTRICAL ENGINEERING

Date of Test : 10/07/2022

ANSWER KEY

1. (a)	7. (b)	13. (a)	19. (d)	25. (b)
2. (d)	8. (d)	14. (c)	20. (a)	26. (b)
3. (c)	9. (d)	15. (a)	21. (c)	27. (d)
4. (a)	10. (a)	16. (c)	22. (b)	28. (d)
5. (c)	11. (a)	17. (a)	23. (d)	29. (c)
6. (d)	12. (a)	18. (a)	24. (a)	30. (c)

1. (a)



The two positive charges Q are diagonally opposite in position and at the same distance from the point (1, 1, 0) fields produced by them are equal and opposite and so their resultant field is zero. Similarly for negative charges.

2. (d)

From Biot savart law,

$$\vec{H} = \int_0^{2\pi} \frac{IRd\phi\hat{a}_\phi \times (-\hat{a}_\rho)}{4\pi R^2}$$

$$= \left(\frac{I}{4\pi}\int_0^{2\pi}\frac{Rd\phi}{R^2}\right)\hat{a}_z$$

$$\vec{H} = \frac{I}{2R}\hat{a}_z$$

3. (c)

If the divergence of a given vector is zero, then it is said to be solenoidal.

$$\nabla.\vec{A} = 0$$

By Divergence theorem,

$$\int_V (\nabla.\vec{A})\mathrm{dv} = \oint_s \vec{A}.\vec{ds}$$

So, for a solenoidal field,

$$\nabla.\vec{A} = 0 \text{ and } \oint_s \vec{A}.\vec{ds} = 0$$

4. (a)

As both the coils are same axis and carrying currents in opposite directions, the field components produced by both the coils are in opposite direction and they cancel out each other. So, the net field at the point on the axis midway between the coils is zero.

5. (c)

Continuity equation, $\nabla.\vec{J} = \dfrac{-\partial\rho_v}{\partial t}$

for static fields, $\dfrac{\partial\rho_v}{\partial t} = 0$

So, for static fields, $\nabla.\vec{J} = 0$

6. (d)

$$\text{Net flux} = \iiint \rho_v \cdot dv = \int_0^{2\pi}\int_0^{\pi}\int_1^{2} \frac{10}{r^2} r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$$= 10 \times 1 \times 2 \times 2\pi = 40\pi \text{ mC}$$

7. (b)

$$\nabla \times \vec{A} = \vec{B}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A})$$

$$= \nabla \times \left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

8. (d)

$$\vec{R}_{21} = (\vec{R}_1 - \vec{R}_2)$$

$$= (1, -2, 3) - (2, -1, 0)$$

$$= (-1, -1, 3)$$

$$= -\hat{i} - \hat{j} + 3\hat{k}$$

$$F_{21} = \frac{Q_1 Q_2}{4\pi \in_0 |\vec{R}_{21}|^2}\hat{R}_{21}$$

$$= \frac{25 \times 20 \times 10^{-12} \times 9 \times 10^9}{\left(\sqrt{1+1+9}\right)^2}\left(\frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1+1+9}}\right)$$

$$= 0.123(-\hat{i} - \hat{j} + 3\hat{k})\text{N}$$

9. (d)

$$\nabla\left(\ln\sqrt{x^2+y^2+z^2}\right) = \frac{1}{2|r|^2}\left[2x\,\hat{a}_x + 2y\,\hat{a}_y + 2z\,\hat{a}_z\right]$$

$$= \frac{x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z}{|r|^2} = \frac{\vec{r}}{|r|^2}$$

10. (a)

$$E = -N\frac{d\phi}{dt} = -100(3t^2 - 2) \times 10^{-3}$$

$$= -100(12 - 2) \times 10^{-3} = -1 \text{ V}$$

11. (a)

$$\vec{P} = x^3y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\vec{a}_z$$

For solenoidal, $\nabla \cdot \vec{P} = 0$

$$\Rightarrow \quad \nabla \cdot \vec{P} = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$$

$$= 3x^2y - 2x^2y - x^2y$$

$$= 0$$

$\Rightarrow$ $\vec{P}$ is solenoidal

For irrotational, $\nabla \times \vec{P} = 0$

$$\Rightarrow \quad \nabla \times \vec{P} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$$

$$= \vec{a}_x(-x^2z) + \vec{a}_y(2xyz) + \vec{a}_z(-2xy^2 - x^3)$$

$$\neq 0$$

$\Rightarrow$ $\vec{P}$ is not irrotational.

12. (a)

$$\vec{H} = -\nabla.V_m$$

(This relation is only define in a region where J = 0) i.e., current free region.

$$= -\left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)(x^2y + y^2x + z)$$

$$= -(2xy + y^2)\hat{a}_x - (x^2 + 2xy)\hat{a}_y - \hat{a}_z$$

$$\vec{H} \text{ at } (1, 0, 1) = -\hat{a}_y - \hat{a}_z$$

$$|\vec{B}| = \mu_0\mu_r|\vec{H}|$$

$$= 4\pi \times 10^{-7} \times 1 \times \sqrt{1^2 + 1^2}$$

$$|\vec{B}| = 1.77 \times 10^{-6} \text{ T}$$

13. (a)

Vector from the line to the point P

$$\vec{r} = -2u_x + 3u_y$$

$$\vec{E} = \frac{\lambda}{2\pi \in_0 r}\hat{r}$$

$$\vec{E} = \frac{0.1 \times 10^{-6}}{2\pi \in_0 \sqrt{4+9}}\left(\frac{-2\hat{u}_x + 3\hat{u}_y}{\sqrt{13}}\right)$$

$$= -276.92\hat{u}_x + 415.38\hat{u}_y$$

14. **(c)**

$$E = \int (v \times B) \cdot dl$$

$$v \times B = -0.15 \sin 10^3 t\, u_x \text{ V/m}$$

$$E = \int_0^{0.25} -0.15 \sin 10^3 t\, dx$$

$$E = -0.15 \sin 10^3 t [x]_0^{0.25}$$

$$E = -0.0375 \sin 10^3\, t \text{ V}$$

15. **(a)**

$$E = \frac{\rho_s}{2\epsilon_0} \hat{a}_n$$

$$E = E_1 + E_2 + E_3$$

$$= \left(\frac{10}{2\epsilon_0}\hat{a}_x + \frac{(-20)}{2\epsilon_0}\hat{a}_y + \frac{30}{2\epsilon_0}(-\hat{a}_z)\right) \times 10^{-6}$$

$$= \frac{10}{2\epsilon_0}\left[\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z\right] \times 10^{-6}$$

$$|E| = \frac{10}{2\epsilon_0}\sqrt{1+2^2+3^2} \times 10^{-6}$$

$$= \frac{10}{2\epsilon_0}\sqrt{14} = \frac{18.71}{\epsilon_0} \times 10^{-6} = 2.11 \times 10^6 \text{ V/m}$$

16. **(c)**

Force, $\vec{F} = I(\vec{L} \times \vec{B})$

$$= 10(2\hat{a}_z \times 0.02(\hat{a}_y - \hat{a}_x))$$

$$= 10(0.04(-\hat{a}_x) - 0.04\hat{a}_y)$$

$$= -0.4\hat{a}_x - 0.4\hat{a}_y$$

Force acting per unit length,

$$\frac{\vec{F}}{L} = \frac{-0.4\hat{a}_x - 0.4\hat{a}_y}{2}$$

$$= -0.2\hat{a}_x - 0.2\hat{a}_y$$

$$\frac{\vec{F}}{L} = -0.2(\hat{a}_x + \hat{a}_y)$$

17. (a)

$$\mu_1 = 2\,\mu_0;$$
$$\mu_2 = 5\,\mu_0$$
$$B_2 = 10\hat{a}_\rho + 15\hat{a}_\phi - 20\hat{a}_z \text{ mWb/m}^2$$
$$B_{1n} = B_{2n} = 15\hat{a}_\phi$$
$$H_{1t} = H_{2t}$$
$$B_{1t} = \frac{\mu_1}{\mu_2}B_{2t} = \frac{2}{5}\left(10\hat{a}_\rho - 20\hat{a}_z\right)$$
$$B_{1t} = (4\hat{a}_\rho - 8\hat{a}_z)\text{mWb/m}^2$$
$$W_{m1} = \frac{1}{2}B_1 \cdot H_1 = \frac{B_1^2}{2\mu_1}$$
$$= \frac{(4^2 + 15^2 + 8^2)\times 10^{-6}}{2\times 2\times 4\pi\times 10^{-7}}$$
$$= \frac{305}{16\pi}\times 10 = 60.68 \text{ J/m}^3$$

18. (a)

$$M = X_m H$$
$$= X_m\frac{B}{\mu} = \frac{(\mu_r - 1)}{\mu_r\mu_0}B = \left(1 - \frac{1}{4.5}\right)\times\frac{4y}{\mu_0}\times 10^{-3}\hat{a}_z$$
$$= \frac{28y}{9\mu_0}\times 10^{-3}\hat{a}_z$$
$$J = \nabla\times M$$
$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 0 & \dfrac{28y}{9\mu_0} \end{vmatrix} = \frac{28}{9\mu_0}\times 10^{-3}\hat{a}_z = 2475.7 \text{ A/m}^2$$

19. (d)

$$C = C_1 + C_2$$
$$= 100 + 50$$
$$= 150\ \mu\text{F}$$
$$\text{Total energy stored} = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(150\times 10^{-6})\times 10^6$$
$$= 75 \text{ Joules}$$

20. (a)

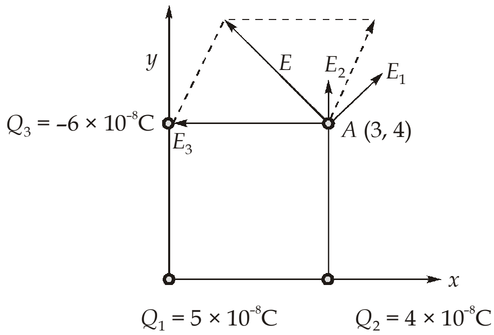
$Q_1 = +5\times 10^{-8}$C, $Q_2 = +4\times 10^{-8}$ C and $Q_3 = -6\times 10^{-8}$ C

The potential at point A

$$V_A = \frac{Q_1}{4\pi\varepsilon_0 r_1} + \frac{Q_2}{4\pi\varepsilon_0 r_2} + \frac{Q_3}{4\pi\varepsilon_0 r_3}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times 10^{-8}}{5} + \frac{4\times 10^{-8}}{4} - \frac{6\times 10^{-8}}{3}\right]$$

$$= 0$$



21. (c)

$$H = H_x + H_y$$

$$H = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$H_x = \frac{5}{4\pi\times 2}(\cos 0° - \cos 90°)(-\hat{a}_x\times\hat{a}_z) = \frac{5}{8\pi}\hat{a}_y \text{A/m}$$

$$H_y = \frac{5}{4\pi\times 2}(\cos 0° - \cos 90°)(\hat{a}_y\times\hat{a}_z) = \frac{5}{8\pi}\hat{a}_x$$

$$\vec{H} = \frac{5}{8\pi}(\hat{a}_x + \hat{a}_y) \text{ A/m}$$

22. (b)

$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

$$\rho = \sqrt{5^2+5^2} = \sqrt{50}\,\text{m}$$

$$= \frac{2}{4\pi\sqrt{5^2+5^2}}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$\cos\alpha_2 = \frac{10}{\sqrt{50+100}} = \frac{10}{\sqrt{150}}$$

$$\cos\alpha_1 = \cos 90° = 0$$

$$\hat{a}_\phi = \hat{a}_l\times\hat{a}_\rho = \hat{a}_z\times\left(\frac{5\hat{a}_x+5\hat{a}_y}{5\sqrt{2}}\right) = \left(\frac{-\hat{a}_x+\hat{a}_y}{\sqrt{2}}\right)$$

$$\vec{H} = \frac{2}{4\pi \times 5\sqrt{2}} \times \left(\frac{10}{\sqrt{150}} - 0\right) \times \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$$

$$= \frac{1}{20\pi}\left(-\hat{a}_x + \hat{a}_y\right) \times \frac{10}{5\sqrt{6}}$$

$$= \frac{1}{10\pi\sqrt{6}}\left(-\hat{a}_x + \hat{a}_y\right)\text{A/m}$$

23. (d)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{d\rho}\right) + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = 0$$

$$= \frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{d\rho}\right) = 0$$

$$\frac{dV}{d\rho} = \frac{A}{\rho}$$

$$V = A \ln \rho + B$$

$$V(\rho = 4) = A \ln 4 + B = 0$$

$$V(\rho = 12) = A \ln 12 + B = V_0$$

$$V_0 = A \ln 12 - A \ln 4 = A \ln 3$$

$$E = -\nabla V = -\left[\frac{dV}{d\rho}\hat{a}_\rho\right] = -\frac{A}{\rho}\hat{a}_\rho$$

$$E(\rho = 8) = -6\,\hat{a}_\rho \text{ kV/m}$$

$$A = 48$$

$$V_0 = 48 \ln 3 \text{ V}$$

24. (a)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\frac{\rho dV}{d\rho} + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = \frac{-\rho_v}{\epsilon} = \frac{-\rho_v}{\epsilon_0\epsilon_r}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{dp}\right) = \frac{-\rho_v}{\epsilon} = \frac{-10}{\rho} \times \frac{10^{-12}}{3.6 \times 1} \times 36\pi \times 10^9$$

$$\nabla^2 V = \frac{-\rho_v}{\epsilon}$$

$$\frac{d}{d\rho}\frac{\rho dV}{d\rho} = -0.1\,\pi$$

$$\frac{\rho dV}{d\rho} = -0.1\,\pi\rho + A$$

$$V = -0.1\,\pi\rho + A \ln \rho + B$$

$$-0.1\,\pi \times 2 + A \ln 2 + B = 0$$

$$A \ln 2 + B = 0.2\,\pi$$

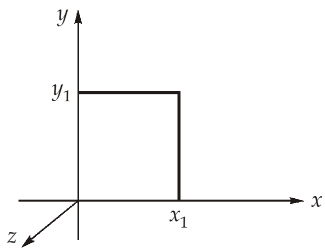
$$-0.1\,\pi \times 5 + A \ln 5 + B = 60$$

$$A \ln 5 + B = 60 + 0.5\,\pi$$
$$A \ln 2.5 = (60 + 0.3\,\pi)$$
$$A = \left(\frac{60+0.3\pi}{\ln 2.5}\right) = 66.51$$
$$E = -\nabla V$$
$$= -\left(-0.1\pi + \frac{A}{\rho}\right)\hat{a}_\rho$$

At $\rho = 1$,
$$E = -\left(-0.1\pi + \frac{66.51}{1}\right)\hat{a}_\rho = -66.19\hat{a}_\rho \text{ V/m}$$

25. (b)

$$x_1 = y_1 = 1 \text{ m}$$
$$B_0 = \sin \pi x \sin \pi y \; T$$
$$B = B_0 \cos \omega_0 t$$



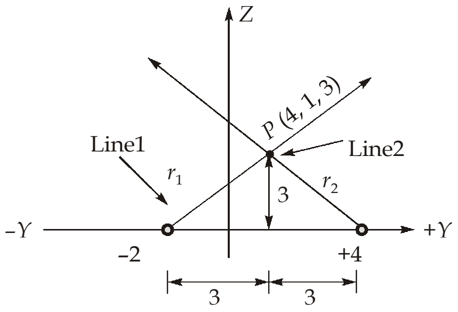
$$E(t) = \iint_s \frac{-dB}{dt}\cdot \hat{n}\, dS$$
$$= \iint_s \omega_0 B_0 \sin\omega t\, \hat{n}\, dS$$
$$E_{max} = \omega_0 \int_{y=0}^{1}\int_{x=0}^{1} \sin \pi x \sin \pi y \, dx\, dy$$
$$E_{max} = 1000 \times 2\pi \times \frac{4}{\pi^2} = \frac{8000}{\pi} \text{V/turns}$$
$$E_{rms} = \frac{1}{\sqrt{2}} \times \frac{8000}{\pi} \times 10 = 18 \text{ kV}$$

26. (b)

$$C = 4\pi \in_0 \left(\frac{ab}{a-b}\right)$$
$$A_a = 4\pi a^2$$
$$A_b = 4\pi b^2$$
$$a = \sqrt{\frac{A_a}{4\pi}}\,;\; b - \sqrt{\frac{A_b}{4\pi}}$$
$$ab = \frac{\sqrt{A_a A_b}}{4\pi}\,;$$
$$a - b = \frac{\sqrt{A_a} - \sqrt{A_b}}{\sqrt{4\pi}}$$
$$C = 4\pi \in_0 \left[\frac{\sqrt{A_a A_b}}{4\pi} \times \frac{\sqrt{4\pi}}{\left(\sqrt{A_a} - \sqrt{A_b}\right)}\right]$$
$$= \sqrt{4\pi} \in_0 \frac{\sqrt{A_a A_b}}{\sqrt{A_a} - \sqrt{A_b}}$$

27. (d)

Let r_1 and r_2 be the directed line segments from the lines 1 and 2 respectively to the point P(in Y-Z plane).



Then, $$r_1 = 3\hat{a}_y + 3\hat{a}_z$$

and $$r_2 = -3\hat{a}_y + 3\hat{a}_z$$

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 r_1}\left(\frac{3\hat{a}_y + 3\hat{a}_z}{r_1}\right) \quad \text{. . . (i)}$$

where, $$r_1^2 = 3^2 + 3^2 = r_2^2 \text{ (in magnitude)} = 18 = r^2$$

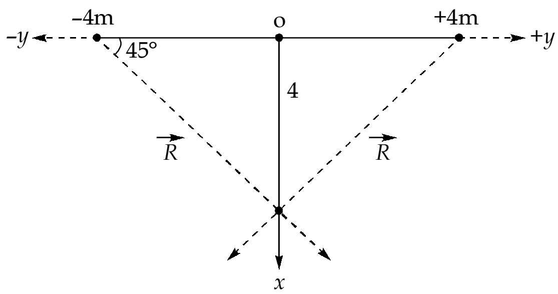
$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 r_2}\left(\frac{-3\hat{a}_y + 3\hat{a}_z}{r_2}\right) \quad \text{. . . (ii)}$$

Adding (i) and (ii), we obtain the resultant field

$$E = E_1 + E_2 = \frac{\lambda}{2\pi\varepsilon_0 r^2}(2\times 3\hat{a}_z) \qquad \text{(replacing } r_1 \text{ and } r_2 \text{ by } r)$$

$$= \frac{5\times 10^{-9}}{2\pi\varepsilon_0(18)}(6\hat{a}_z) = 30\hat{a}_z \text{ V/m}$$

28. (d)



The y components of the fields produced by two lines of charge cancel out and only x components will exist in effect.

The resultant field is,

$$\vec{E} = \pm 2\frac{\lambda}{2\pi\varepsilon_0|\vec{R}|}\cdot\frac{\vec{R}}{|\vec{R}|}$$

where, $$\vec{R} = 4\hat{a}_x \quad \text{and } |\vec{R}| = \sqrt{4^2+4^2} = 4\sqrt{2} \text{ m}$$

$$\vec{E} = \pm 2 \times \frac{4\times 10^{-9}}{2\pi\times 8.854\times 10^{-12}} \times \frac{4\hat{a}_x}{(4\sqrt{2})^2}$$

$$\vec{E} = \pm 18\hat{a}_x \text{ V/m}$$

29. (c)

The flux Φ_1, at $i_1 = 5$ A is

$$\Phi = B \times A$$
$$= 1 \times (30 \times 10^{-4})$$
$$\Phi_1 = 30 \times 10^{-4} \text{ Wb}$$
$$\Phi_2 \text{ at } i_2 = 10 \text{ A is,}$$
$$\Phi_2 = 1.5 \times 30 \times 10^{-4}$$

Increase of flux when current is increased from 5 to 10 A

$$= 0.5 \times 30 \times 10^{-4} \text{ Wb}$$

$$\frac{d\phi}{dt}(\text{average}) = \frac{0.5\times 30\times 10^{-4}}{5}$$

$$\frac{d\phi}{dt}(\text{average}) = 3 \times 10^{-4} \text{ Wb/A}$$

$\therefore$ Mean value of inductance,

$$L = N\frac{d\phi}{di}$$
$$L = 2000 \times 3 \times 10^{-4}$$
$$L = 0.6 \text{ Henry.}$$

30. (c)

Given,

Voltage distribution across capacitance is in the ratio 2 : 3 : 4 and applied voltage is 135 V.

Then, $V_{C1} = 30$ V

$$V_{C2} = 45 \text{ V}$$
$$V_{C3} = 60 \text{ V}$$

Hence, $C_1 = \dfrac{4500}{30} = 150\ \mu\text{F}$

$$C_2 = \frac{4500}{45} = 100\ \mu\text{F}$$

$$C_3 = \frac{4500}{60} = 75\ \mu\text{F}$$

$\therefore$ $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$C_{eq} = 33.33\ \mu\text{F}$$

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