CLASS TEST

S.No.: 01JP_EC+EE_130522

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EC | EE

Control Systems

Duration : 1:00 hr. **Maximum Marks : 50**

Read the following instructions carefully

1. This question paper contains **30** objective questions. **Q.1-10** carry one mark each and **Q.11-30** carry two marks each.
2. Answer all the questions.
3. Questions must be answered on Objective Response Sheet (**ORS**) by darkening the appropriate bubble (marked **A**, **B**, **C**, **D**) using HB pencil against the question number. Each question has only one correct answer. In case you wish to change an answer, erase the old answer completely using a good soft eraser.
4. There will be **NEGATIVE** marking. For each wrong answer **1/3rd** of the full marks of the question will be deducted. More than one answer marked against a question will be deemed as an incorrect response and will be negatively marked.
5. Write your name & Roll No. at the specified locations on the right half of the **ORS**.
6. No charts or tables will be provided in the examination hall.
7. Choose the **Closest** numerical answer among the choices given.
8. If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that questions.
9. If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

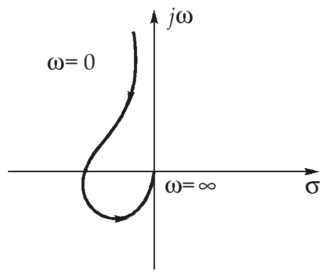
Q.No. 1 to Q.No. 10 carry 1 mark each

Q.1 The transfer function of three blocks connected in cascade is given by $\dfrac{(s+1)}{s(s+2)(s+3)}$. If block 1 has transfer function of $\dfrac{1}{s(s+2)}$ and block 2 has transfer function of $\dfrac{(s+2)}{(s+3)}$ then the transfer function of the 3rd block is

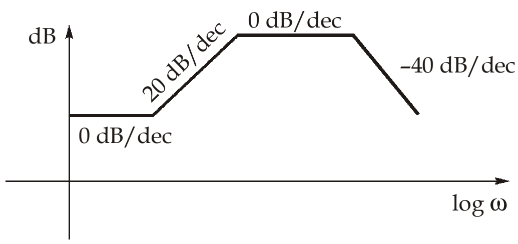
(a) $(s+1)(s+2)$ (b) $\dfrac{(s+1)}{(s+2)}$

(c) $\dfrac{(s+1)}{s(s+3)}$ (d) $\dfrac{(s+1)^2}{(s+2)^2}$

Q.2 The polar plot and Bode magnitude plot of two systems are given in the figure below,



System I



System II

(a) Both the systems are type 1 system.
(b) System I is type -3 and system II is type 1 system.
(c) System I is type 1 and system II is type 0 system.
(d) System I is type 3 and system II is type 0 system.

Q.3 The first two row of Routh's array of a third order system are given as

s^3	4	4
s^2	3	3

Which of the following option is correct for the given system?

(a) The characteristic equation has one root in the RHS of s-plane.
(b) The characteristic equation has two roots on the $j\omega$ axis.
(c) The characteristic equation has two roots in LHS of s-plane.
(d) The characteristic equation has one root at RHS of s-plane and two roots in $j\omega$ axis.

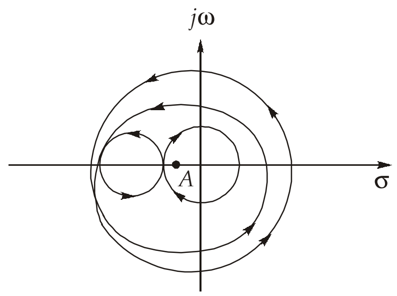
Q.4 The open loop transfer function of a system is given by

$$G(s)H(s) = \frac{(s-2)(s^2-2s+2)}{(s+2)(s^2+2s+2)}$$

The above system is a

(a) Stable system
(b) Minimum phase system
(c) Non-minimum phase system
(d) All pass system

Q.5 The total number of encirclement of the critical point 'A' marked in the below Nyquist plot is_____.



(a) 0 (b) 1
(c) 2 (d) 3

Q.6 A unity feedback system is defined by an open loop transfer function,

$$G(s) = \frac{K}{s(s+10)}$$

The system gain K for which the damping ratio is 0.75 will be _________.

(a) 25.42 (b) 38.65
(c) 44.44 (d) 50.25

Q.7 The steady state response of a unity negative feedback control system for a step input whose transfer function is given by $T(s) = \dfrac{10(s+2)}{(s^2+4s+8)}$ will be ___________.

(a) 0.5 (b) 1
(c) 1.5 (d) 2.5

Q.8 If the roots of a second order characteristic equation are given by $s_{1,2} = -3 \pm j2$, then the values of damping ratio ξ and the damped natural frequency ω_d are respectively

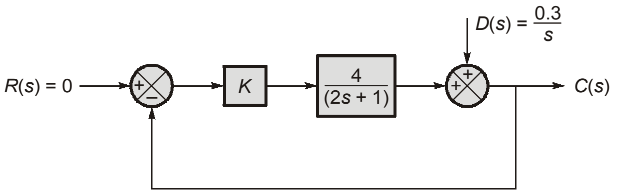
(a) $\left(\dfrac{1}{\sqrt{13}} \text{ and } \sqrt{13} \text{ rad/sec}\right)$

(b) $\left(\dfrac{3}{\sqrt{13}} \text{ and } 2 \text{ rad/sec}\right)$

(c) $\left(\dfrac{3}{\sqrt{13}} \text{ and } \sqrt{13} \text{ rad/sec}\right)$

(d) $\left(\dfrac{1}{\sqrt{13}} \text{ and } 2 \text{ rad/sec}\right)$

Q.9 Consider the system shown below :



The steady state offset due to $D(s)$ will be zero for

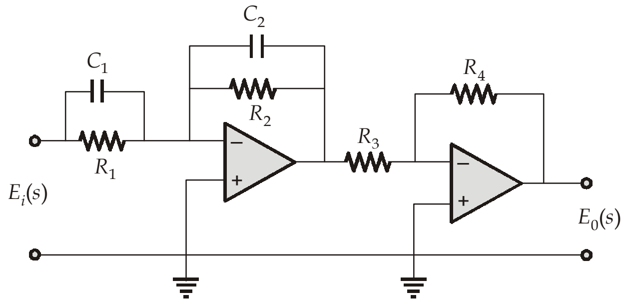
(a) $K = 0$ (b) $K = 0.3$
(c) $K = 2$ (d) $K = \infty$

Q.10 Consider a system having forward path and feedback path transfer functions as $[4/s(s+0.2)]$ and $(1+2s)$ respectively. The characteristic polynomial of the system is

(a) $s^2 + 8.2s + 4$
(b) $s^3 + 4.8s^2 + 0.2s + 1$
(c) $4s^3 + 16.8s^2 + 0.8s + 4$
(d) $s^2 + 0.2s + 4$

Q. No. 11 to Q. No. 30 carry 2 marks each

Q.11 The following circuit represents a



(assume $T_1 > T_2$)

(a) lead network
(b) lag network
(c) lag lead network
(d) lead lag network

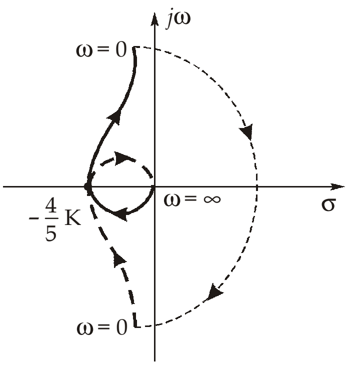
Q.12 The open loop transfer function of a unity negative feedback control system is given by

$$G(s) = \frac{2s^2 + as + 50}{(s+1)^2(s+2)}$$

The possible value of 'a' to make the given system stable is

(a) 1 (b) 2
(c) 3 (d) 4

Q.13 The Nyquist plot of a stable system is given by



For $K > \dfrac{5}{4}$, the total number of RHS poles of characteristic equation will be

(a) 0 (b) 1
(c) 2 (d) 3

Q.14 A unity negative feedback control system has its open loop transfer function given by

$$G(s) = \frac{4s+1}{4s^2}.$$

The unit step response of the system is

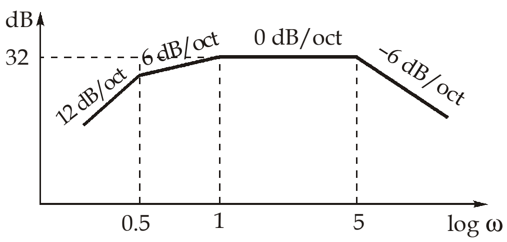
(a) $\left[1+\frac{t}{2}\right]u(t)$

(b) $[1 - 2e^{-0.5t}]u(t)$

(c) $[1 + (0.5t - 1)e^{-0.5t}]\,u(t)$

(d) $[te^{-0.5t}]u(t)$

Q.15 Consider the Bode magnitude plot shown in the figure below:



The open loop transfer function of the system is

(a) $\dfrac{79.6s^2}{(2s+1)(s+1)(0.2s+1)}$

(b) $\dfrac{2.5s^2}{(s+0.5)(s+1)(s+5)}$

(c) $\dfrac{79.6s^2}{(s+0.5)(s+1)(s+5)}$

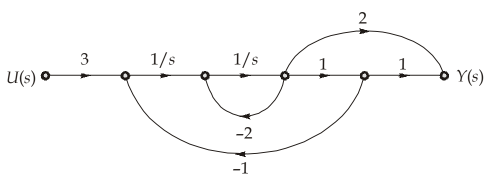
(d) $\dfrac{2.5s^2}{(2s+1)(s+1)(1+0.2s)}$

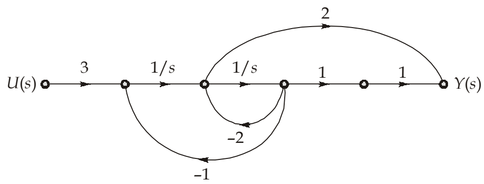
Q.16 The state variable representation of a system is given by

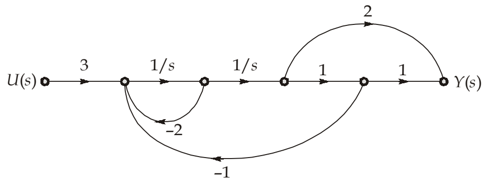
$$\dot{x} = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 3 \end{bmatrix} u$$

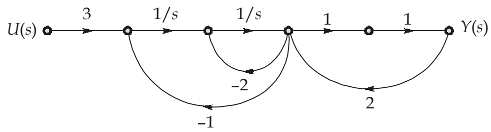
$$y = [1 \quad 2]x$$

The signal flow graph of the given system is

(a)


(b)


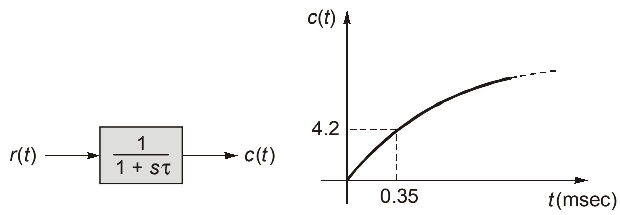
(c)


(d)


Q.17 For a system with open loop transfer function $G(s)H(s) = \dfrac{32}{s(s+\sqrt{6})^3}$, the gain margin and phase margin at $\omega = \sqrt{2}$ rad/sec will be respectively

(a) 0 dB and 90°
(b) ∞ and 90°
(c) 0 dB and 0°
(d) ∞ and 0°

Q.18 The response of a system shown in figure (a), to an input of $r(t) = 5u(t)$, is shown in figure (b). The time constant 'τ' of the system is equal to



(a) (b)

(a) 0.35 msec
(b) 0.29 msec
(c) 0.19 msec
(d) 0.083 msec

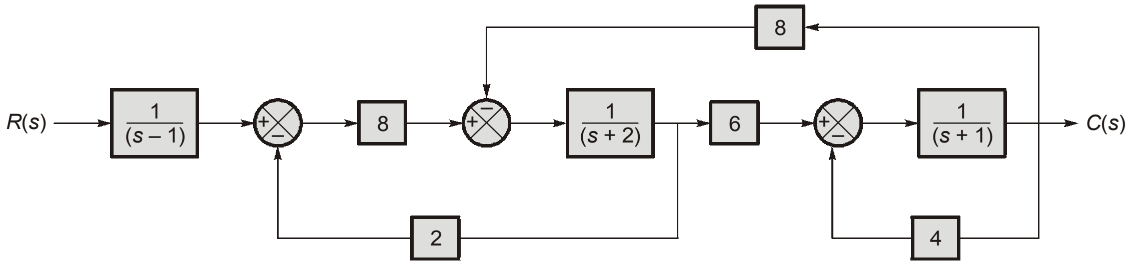
Q.19 A unity negative feedback control system has the closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{K+p}{s^2+qs+p}$$

The steady state error of the system for unit ramp input is given by

(a) 0
(b) $\dfrac{q-K}{p}$
(c) $\dfrac{p-q}{K}$
(d) ∞

Q.20 Consider the block diagram shown below:



The given system is said to be

(a) a non minimum phase system
(b) an unstable system
(c) a stable system
(d) a marginally stable system

Q.21 The characteristic equation of a unity feedback control system is given by,

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$$

The given system is said to be

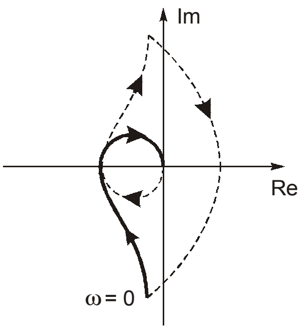
(a) stable
(b) marginally stable
(c) conditionally stable
(d) unstable

Q.22 The open loop transfer function of a negative feedback control system is given by,

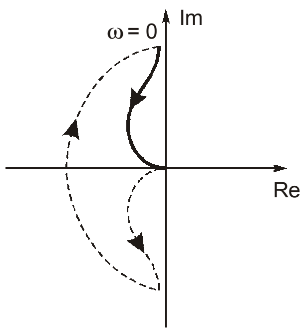
$$G(s)\,H(s) = \frac{1}{s(10s-1)}$$

The Nyquist plot of the system can be represented as

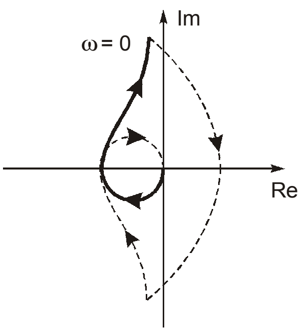
(a)



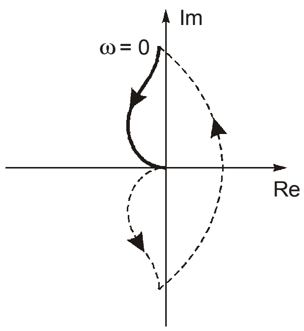
(b)



(c)



(d)



Q.23 The open transfer function of a unity negative feedback control system is given by

$$G(s) = \frac{32}{s(s+\sqrt{6})^3}$$

If $\omega = \sqrt{2}$ rad/sec satisfies the condition for gain cross-over frequency, then the gain margin and the phase margin of the system are respectively

(a) GM = 0 dB and PM = 180°
(b) GM = 0 dB and PM = 0°
(c) GM = ∞ and PM = 0°
(d) GM = ∞ and PM = 180°

Q.24 A unity negative feedback control system has uncompensated open loop transfer function,

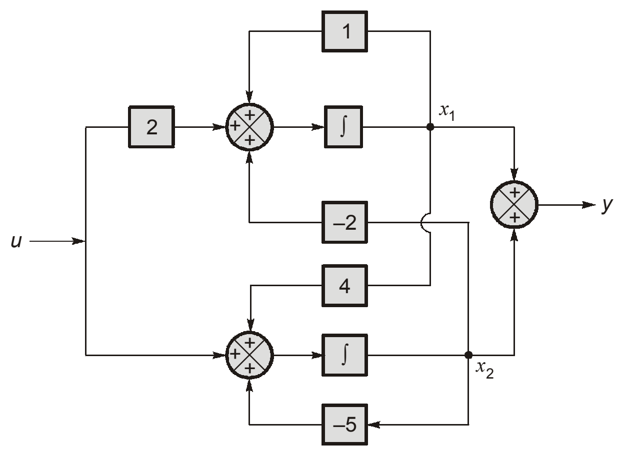
$$G(s) = \frac{25}{s(s+1)(s+5)}$$

If it is required that the steady state error of the system for unit ramp excitation should be less than 0.05, then which one of the following compensators should be connected in cascade with $G(s)$, to achieve the desired error performance?

(a) $G_c(s) = \frac{(s+10)}{(s+4)}$ (b) $G_c(s) = \frac{(s+4)}{(s+10)}$

(c) $G_c(s) = \frac{(s+20)}{(s+4)}$ (d) $G_c(s) = \frac{(s+4)}{(s+20)}$

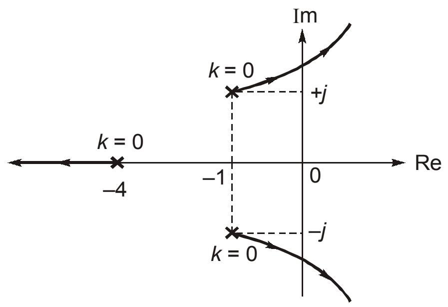
Q.25 The state block diagram of a control system is shown in the figure below:



The system is said to be

(a) controllable as well observable
(b) controllable but not observable
(c) observable but not controllable
(d) neither controllable nor observable

Q.26 Consider the root locus diagram shown below.



The root locus intersects the imaginary axis at

(a) $s = \pm j\sqrt{2}$
(b) $s = \pm j\sqrt{4}$
(c) $s = \pm j\sqrt{8}$
(d) $s = \pm j\sqrt{10}$

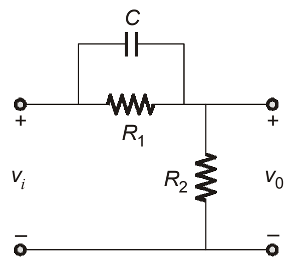
Q.27 An open loop transfer function of a unity negative feedback control system is given by,

$$G(s) = \frac{K}{s(s+a)(s+b)}$$

If the gain margin of the system is 32 dB for K = 2, then the value of K required to get the gain margin of 25 dB will be

(a) 1.48 (b) 4.48
(c) 7.42 (d) 10.11

Q.28 Consider the compensator network shown in the figure below:



The maximum phase difference between the input and output voltages of the network is

(a) $\sin^{-1}\left(\frac{R_1}{2R_2}\right)$ (b) $\sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$

(c) $\sin^{-1}\left(\frac{R_1}{R_2+2R_1}\right)$ (d) $\sin^{-1}\left(\frac{R_1}{2R_2\,C}\right)$

Q.29 Consider the state model of a system given as,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 3 & 1\\ 2 & 4 & 1\\ -5 & -2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}5\\ 0\\ 0\end{bmatrix}u$$

$$y = [1 \quad 0 \quad 0]\,x$$

The number of closed loop poles lie on the right half of s-plane is ________.

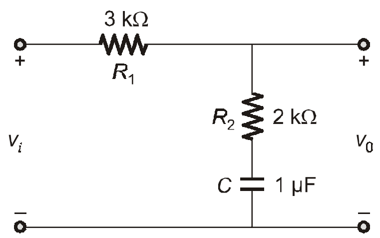
(a) 0 (b) 1

(c) 2 (d) 3

Q.30 The transfer function of a lag compensator is given as,

$$G(s) = k\frac{s+\frac{1}{\tau}}{s+\frac{1}{\alpha\tau}}$$

If this lag compensator is designed by the RC circuit shown below, then the value of α will be______.



(a) 2.5 (b) 3.2

(c) 3.5 (d) 4.4

■■■■

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CONTROL SYSTEMS

EC | EE

Date of Test : 13/05/2022

ANSWER KEY

1.	(b)	7.	(d)	13.	(c)	19.	(d)	25.	(a)
2.	(d)	8.	(b)	14.	(c)	20.	(b)	26.	(d)
3.	(b)	9.	(d)	15.	(a)	21.	(b)	27.	(b)
4.	(d)	10.	(a)	16.	(b)	22.	(b)	28.	(b)
5.	(b)	11.	(a)	17.	(c)	23.	(b)	29.	(b)
6.	(c)	12.	(d)	18.	(c)	24.	(c)	30.	(a)

Detailed Explanations

1. **(b)**

As the three blocks are connected in cascade the overall transfer function is given by the multiplication of individual blocks.

$$\therefore \quad x_1 \times x_2 \times x_3 = \frac{(s+1)}{s(s+2)(s+3)}$$

$$\frac{1}{s(s+2)} \times \frac{(s+2)}{(s+3)} \times x_3 = \frac{(s+1)}{s(s+2)(s+3)}$$

$$x_3 = \frac{(s+1)}{(s+2)}$$

2. **(d)**

At ω = 0, the plot for system I, started from –270° hence it represents a type 3 system.

At ω = 0, the plot for system II has slope of 0 dB/dec and therefore it is a type 0 system.

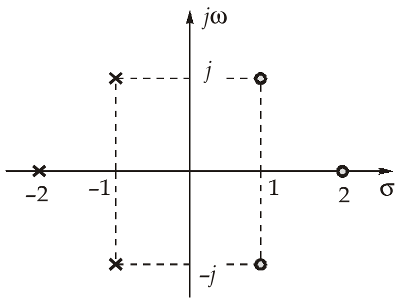
3. **(b)**

The Routh's table can be formed as

s^3	4	4
s^2	$3(s^2)$	$3(0)$
s^1	$(0)6$	(0)
s^0	$\frac{18-0}{6} = 3$	

as there is no sign change in the first column of Routh array thus, there will be no pole lie on the RHS of s-plane. Also the row of zero occurs that indicates the complex conjugate poles exists on $j\omega$ axis.

4. **(d)**



5. **(b)**

Here, the encirclement to the critical point is 1 (in clock wise direction) and 2 (in counter clockwise direction).

$$\therefore \quad N = -1 + 2 = 1$$

6. (c)

Closed loop transfer function $\frac{C(s)}{R(s)} = \frac{K}{s^2 + 10s + K}$

Here, $2\xi\omega_n = 10$

$\xi\omega_n = 5$

Therefore, $\omega_n = \frac{5}{0.75}$

$\because$ $K = \omega_n^2 = \left(\frac{5}{0.75}\right)^2 = 44.44$

7. (d)

$$T(s) = \frac{C(s)}{R(s)} = \frac{10(s+2)}{(s^2+4s+8)}$$

$\therefore$ The step response $C(s)$ is,

$$C(s) = \frac{10(s+2)}{(s^2+4s+8)} \times R(s) = \frac{10(s+2)}{s(s^2+4s+8)}$$

and the steady state response is,

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} sC(s) = \lim_{s\to 0} s\frac{10(s+2)}{s(s^2+4s+8)} = \frac{10}{4} = 2.5$$

8. (b)

The roots of the characteristic equation are given by,

$$s_{1,\,2} = -\xi\omega_n \pm \omega_n\sqrt{1-\xi^2}$$

Here, $-\xi\omega_n = -3$

$\Rightarrow$ $\omega_n = \frac{3}{\xi}$ ...(i)

and $\omega_n\sqrt{1-\xi^2} = 2 = \omega_d$ ...(ii)

By putting the value of ω_n in equation (ii), we get,

$$\frac{3}{\xi}\sqrt{1-\xi^2} = 2$$

$$\frac{9}{\xi^2}\times(1-\xi^2) = 4$$

or $9(1 - \xi^2) = 4\xi^2$

$9 - 9\xi^2 - 4\xi^2 = 0$

or $13\xi^2 = 9$

or $\xi = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$

9. (d)

The steady state offset is given by,

$$e_{\text{off}} = \lim_{s\to 0} sC(s)\Big|_{R(s)=0}$$

When $R(s) = 0$,

$$C(s) = D(s) - C(s)\left[\frac{4K}{(2s+1)}\right]$$

$$C(s) = \frac{D(s)}{1+\dfrac{4K}{(2s+1)}}$$

$\therefore$

$$e_{\text{off}} = \lim_{s\to 0} \frac{s\cdot\dfrac{0.3}{s}(2s+1)}{(2s+1)+4K}$$

or

$$e_{\text{off}} = \frac{0.3}{4K+1}$$

For $e_{\text{off}} = 0$, $K = \infty$

10. (a)

The characteristic equation is given by,

$$1 + G(s)\,H(s) = 0$$

Here,

$$G(s) = \frac{4}{s(s+0.2)} \text{ and } H(s) = (1+2s)$$

$\therefore$

$$1+\frac{4(1+2s)}{s(s+0.2)} = 0$$

$$s^2 + 0.2s + 8s + 4 = 0$$

$$s^2 + 8.2s + 4 = 0$$

11. (a)

The transfer function of the above circuit is,

$$\frac{E_0(s)}{E_i(s)} = \frac{R_2R_4(R_1C_1s+1)}{R_1R_3(R_2C_2s+1)} = \frac{R_4C_1}{R_3C_2}\left(\frac{s+\dfrac{1}{R_1C_1}}{s+\dfrac{1}{R_2C_2}}\right)$$

$$= K\alpha\left(\frac{1+sT}{1+\alpha sT}\right) = K_c\left(\frac{s+\dfrac{1}{T}}{s+\dfrac{1}{\alpha T}}\right)$$

Here, $T = R_1C_1$ and $\alpha T = R_2C_2$

If $R_1C_1 > R_2C_2$ then $\alpha < 1$.

Thus, it represents a phase lead network.

12. (d)

The CE of the given system is,

$$1 + G(s) = 0$$

$$1+\frac{2s^2+as+50}{(s+1)^2(s+2)} = 0$$

$(s^2 + 2s + 1)(s + 2) + 2s^2 + as + 50 = 0$

$s^3 + 2s^2 + s + 2s^2 + 4s + 2 + 2s^2 + as + 50 = 0$

$s^3 + 6s^2 + (5 + a)s + 52 = 0$

For system to be stable

$$(5 + a)6 > 52$$
$$30 + 6a > 52$$
$$6a > (52 - 30)$$
$$a > \frac{22}{6}$$
$$a > 3.667$$

13. **(c)**

As the system is said to be stable,

Therefore, no open loop pole in the RHS.

$\therefore \quad P = 0$

The intersection point $\left(-\frac{4}{5}K, 0\right)$ and $K > \frac{5}{4}$.

$$\therefore \quad \frac{4}{5}K > 1$$

or
$$-\frac{4}{5}K < -1$$

That means the Nyquist plot encircles the critical point two times in the clockwise direction

Hence, $\quad N = -2$

$\Rightarrow \quad N = P - Z$

$\Rightarrow \quad -2 = -Z \quad$ or $Z = 2$

14. **(c)**

The closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{4s+1}{4s^2+4s+1} = \frac{1}{4} \times \frac{4s+1}{\left(s+\frac{1}{2}\right)^2}$$

For unit step input, $R(s) = \frac{1}{s}$

$$C(s) = \frac{1}{4} \times \frac{4s+1}{s\left(s+\frac{1}{2}\right)^2} \quad \ldots \text{(i)}$$

15. **(a)**

1st line has slope of 12 dB/oct = 40 dB/dec, thus there is s^2 term in the numerator.

At ω = 0.5 rad/sec, slope changes from +12 dB/oct to +6 dB/oct. Therefore, the term $\left(1 + \frac{s}{0.5}\right)$ should be added to the denominator.

At ω = 1 rad/sec, slope changes from +6 dB/oct to 0 dB/oct, thus, a term $(1 + s)$ should be added to the denominator.

At ω = 5 rad/sec, again the slope changes from 0 dB/oct to –6 dB/oct, thus, the term $\left(1+\frac{s}{5}\right)$ should be added to the denominator.

∴ The transfer function can be written as

$$T(s) = \frac{K(s^2)}{\left(1+\frac{s}{0.5}\right)\left(1+\frac{s}{5}\right)(1+s)}$$

Determining the K value, we get,

$$32 = 20\log K + 40\log\omega - 20\log\frac{\omega}{0.5} - 20\log\frac{\omega}{5} - 20\log\omega$$

$$32\big|_{\omega=5\text{ rad/sec}} = 20\log K + 40\log 5 - 20\log\frac{5}{0.5} - 20\log 1 - 20\log 5$$

$$= 20\log K + 27.95 - 20 - 0 - 13.97$$

$$32 = 20\log K - 6.02$$

$$\log K = \frac{38.02}{20} = 1.901$$

$$K = 79.615$$

∴ The overall transfer function is, $T(s) = \dfrac{79.6s^2}{(2s+1)(s+1)(0.2s+1)}$

16. (b)

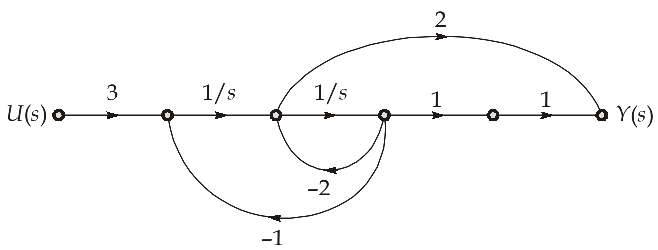
The state equation can be written as.

$$\dot{x}_1 = -2x_1 + x_2$$

$$\dot{x}_2 = -x_1 + 3u$$

and $y = x_1 + 2x_2$

∴ The signal flow graph corresponding to the state equations is



17. (c)

For the given system put $s = j\omega$

we get, $$G(j\omega)H(j\omega) = \frac{32}{j\omega\left(j\omega+\sqrt{6}\right)^3}$$

$$|G(j\omega)H(j\omega)| = \frac{32}{\omega\left(\sqrt{\omega^2+6}\right)^3}$$

at $\omega = \sqrt{2}$ rad/sec,

$$|G(\sqrt{2})H(\sqrt{2})| = \frac{32}{\sqrt{2}(\sqrt{2+6})^3} = 1$$

$$\because \quad |G(j\omega)H(j\omega)|_{\omega=\sqrt{2}\frac{\text{rad}}{\text{sec}}} = 1$$

Thus, the gain cross over frequency = $\sqrt{2}$ rad/sec

Also,
$$-180 = -90 - 3\tan^{-1}\frac{\omega_{pc}}{\sqrt{6}}$$

$$\tan^{-1}\frac{\omega_{pc}}{\sqrt{6}} = \frac{-90}{-3}$$

$$\Rightarrow \quad \frac{\omega_{pc}}{\sqrt{6}} = \tan 30°$$

$$\therefore \quad \omega_{pc} = \sqrt{2}\ \text{rad/sec}$$

$$\because \quad \omega_{gc} = \omega_{pc}$$

$$\text{GM} = 0\ \text{dB and PM} = 0°$$

The system represents a marginally stable system.

18. (c)

$$\frac{C(s)}{R(s)} = \frac{1}{1+s\tau}$$

The response of the system,

$$C(s) = \frac{1}{1+s\tau} \times R(s)$$

$$C(s) = \frac{1}{1+s\tau} \times \frac{5}{s}$$

Taking inverse Laplace transform, we get,

$$c(t) = 5(1 - e^{-t/\tau})\, u(t)$$

Now, $c(t) = 4.2$ at $t = 0.35$ msec

By putting these values, we get,

$$4.2 = 5(1 - e^{-0.35/\tau})$$

$$0.16 = e^{-0.35/\tau}$$

or $\tau = 0.19$ msec

19. (d)

Given,
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K+p}{s^2+qs+p}$$

For $H(s) = 1$,
$$\frac{G(s)}{1+G(s)} = \frac{K+p}{s^2+qs+p}$$

or
$$G(s)\,[s^2 + qs + p] = (K + p) + G(K + p)$$

or
$$G(s) = \frac{K+p}{s^2+qs+p-K-p} = \frac{K+p}{s^2+qs-K}$$

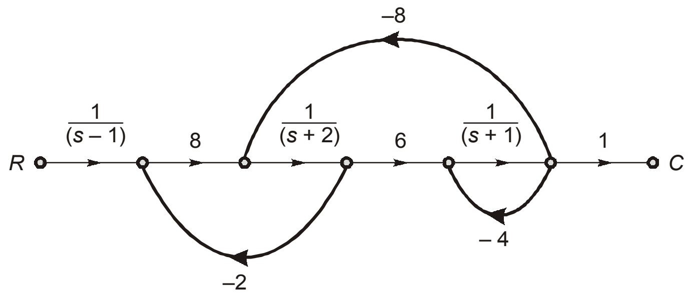
This is a type-0 system.

For a type-0 system, for unit ramp input,

$$e_{ss} = \infty$$

20. (b)

The signal flow graph of the given system can be drawn as,



The forward path,
$$F_1 = \frac{48}{(s-1)(s+2)(s+1)}$$

The feedback loops,
$$L_1 = -\frac{16}{s+2}$$
$$L_2 = -\frac{4}{(s+1)}$$
$$L_3 = -\frac{48}{(s+2)(s+1)}$$

The non-touching loop pair,
$$L_{1,\,2} = \frac{-(16)\times(-4)}{(s+2)(s+1)}$$

∴ The closed loop transfer function using Mason's gain formula is,

$$T(s) = \frac{\dfrac{48}{(s-1)(s+2)(s+1)}}{1+\dfrac{16}{s+2}+\dfrac{4}{s+1}+\dfrac{48}{(s+1)(s+2)}+\dfrac{64}{(s+1)(s+2)}}$$

$$= \frac{\dfrac{48}{(s-1)}}{(s+1)(s+2)+16(s+1)+4(s+2)+112}$$

$$T(s) = \frac{48}{(s-1)[s^2+3s+2+16s+16+4s+8+112]}$$

$$= \frac{48}{s^3+23s^2+138s-s^2-23s-138}$$

$$= \frac{48}{s^3+22s^2+115s-138}$$

∴ The characteristic equation is,
$$s^3 + 22s^2 + 115s - 138 = 0$$

Using Routh's criterion,

$$\begin{array}{c|cc} s^3 & 1 & 115 \\ s^2 & 22 & -138 \\ s^1 & 121.27 & 0 \\ s^0 & -138 & \end{array}$$

$\because$ There is a sign change in the first column of Routh's tabular form, the given system is unstable.

21. **(b)**

Using the Routh's tabular form

$$\begin{array}{c|cccc} s^6 & 1 & 8 & 20 & 16 \\ s^5 & 2 & 12 & 16 & 0 \\ s^4 & 2(s^4) & 12(s^2) & 16(s^0) & \\ s^3 & 8 & 24 & 0 & \\ s^2 & 6 & 16 & 0 & \\ s^1 & \frac{16}{6} & 0 & 0 & \\ s^0 & 16 & & & \end{array}$$

Since there is no sign change in the first column of the Routh array, the system does not have any pole in the RHS of s-plane. However the row of zeros occur which gives the auxiliary equation

$$A(s) \Rightarrow 2s^4 + 12s^2 + 16 = 0$$
$$\Rightarrow s^4 + 6s^2 + 8 = 0$$

and the roots are given by,

$$s = \pm j\sqrt{2}, \pm j2$$

Hence the system is said to be marginally stable.

22. **(b)**

The open loop transfer function is given as,

$$G(s)\,H(s) = \frac{1}{s(10s-1)}$$

Put, $s = j\omega$

$$G(j\omega)\,H(j\omega) = \frac{1}{j\omega(10j\omega-1)}$$

$$|G(j\omega)\,H(j\omega)| = \frac{1}{\omega\sqrt{100\omega^2+1}}$$

$$\angle G(j\omega)\,H(j\omega) = -90° - 180° + \tan^{-1}(10\omega) = -270° + \tan^{-1}(10\omega)$$

For $\omega = 0$,

$$|G(0)\,H(0)| = \infty$$

and $$\angle G(0)\,H(0) = \left(-270° + \tan^{-1}(10\omega)\right)\Big|_{\omega=0} = -270°$$

For $\omega = \infty$, $$|G(\infty)\,H(\infty)| = 0$$

and $$\angle G(\infty)\,H(\infty) = \left(-270° + \tan^{-1}(10\omega)\right)\Big|_{\omega=\infty} = -180°$$

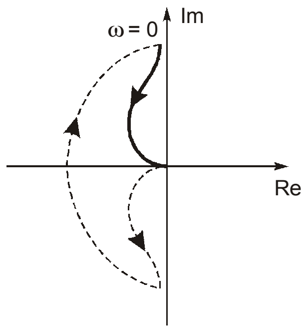
There is an open loop pole at origin. To map this pole,

$$s = re^{j\theta}\Big|_{r\to 0,\ \theta\to -\frac{\pi}{2}\text{ to }\frac{\pi}{2}}$$

$$G(s)\,H(s) = \frac{-1}{s(1-10s)} = \lim_{r\to 0}\frac{e^{j\pi}}{re^{j\theta}(1-10re^{j\theta})}\Bigg|_{\theta\to -\frac{\pi}{2}\text{ to }\frac{\pi}{2}}$$

$$= \lim_{r\to 0}\frac{1}{r}e^{j(\pi-\theta)}\Big|_{\theta\to -\frac{\pi}{2}\text{ to }\frac{\pi}{2}} = \infty e^{j\theta}\Big|_{\theta\to\frac{3\pi}{2}\text{ to }\frac{\pi}{2}}$$

Thus, the Nyquist plot can be drawn as,



23. **(b)**

The gain cross-over frequency ω_{gc} can be calculated as,

$$|G(j\omega)|_{\omega=\omega_{gc}} = 1$$

Here,
$$G(j\omega) = \frac{32}{j\omega(j\omega+\sqrt{6})^3}$$

$$|G(j\omega)| = \frac{32}{\omega(\sqrt{\omega^2+6})^3} = 1$$

At $\omega = \sqrt{2}$ rad/sec

$|G(j\omega)|\ \omega = \sqrt{2}$ rad/sec

$$\frac{32}{\sqrt{2}\times\sqrt{8}\times\sqrt{8}\times\sqrt{8}} = 1$$

Thus, $\omega = \sqrt{2}$ rad/sec is the gain cross-over frequency. Now, the phase cross-over frequency is calculated as

$$\angle G(j\omega)\,H(j\omega) = -180°$$

Here,
$$\angle G(j\omega) = -90° - 3\tan^{-1}\frac{\omega}{\sqrt{6}}$$

or
$$\frac{\tan^{-1}\omega}{\sqrt{6}} = 30°$$

or
$$\frac{\omega}{\sqrt{6}} = \tan 30° = \frac{1}{\sqrt{3}}$$

or
$$\omega_{pc} = \sqrt{2}\text{ rad/sec}$$

$$\omega_{gc} = \omega_{pc} = \sqrt{2}\text{ rad/sec}$$

The given system represents a marginally stable system having GM = 0 dB and PM = 0°.

24. **(c)**

Given that, $G(s) = \dfrac{25}{s(s+1)(s+5)}$

Let the compensator, $G_c(s) = \dfrac{(s+\omega_z)}{(s+\omega_p)}$

The open loop transfer function of the compensated system can be given as,

$$L(s) = G(s)\,G_c(s) = \frac{25(s+\omega_z)}{s(s+1)(s+5)(s+\omega_p)}$$

The velocity error constant of the compensated system will be,

$$K_v = \lim_{s\to 0} sL(s) = \frac{25}{5}\left(\frac{\omega_z}{\omega_p}\right) = 5\left(\frac{\omega_z}{\omega_p}\right)$$

Given that, $e_{ss} = \dfrac{1}{K_v} < 0.05$

So, $K_v > \dfrac{1}{0.05} = 20$

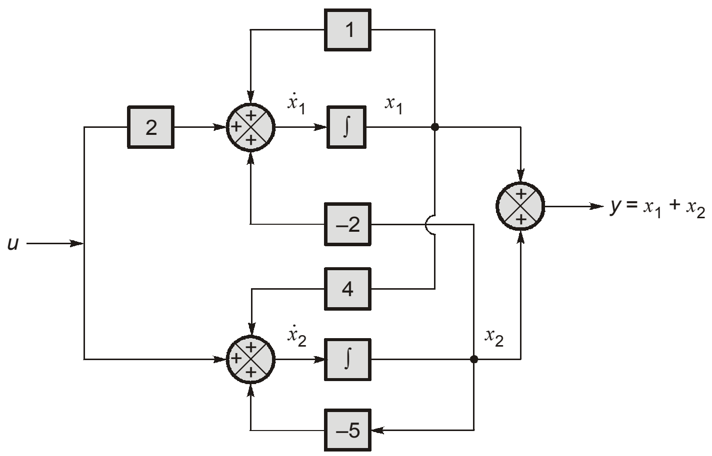
$$5\left(\frac{\omega_z}{\omega_p}\right) > 20$$

$$\frac{\omega_z}{\omega_p} > 4$$

Only option (c) satisfies this.

25. **(a)**

Redrawing the given block diagram, we get,



As per the block diagram, state equations are,

$$\dot{x}_1 = x_1 - 2x_2 + 2u$$

$$\dot{x}_2 = 4x_1 - 5x_2 + u$$

and $y = x_1 + x_2$

∴ State model,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1 & -2\\ 4 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 1\end{bmatrix}u$$

$$y = [1 \quad 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Check for controllability:

$$Q_c = [B \ : \ AB]$$

$$= \begin{bmatrix} 2 & : & \begin{pmatrix} 1 & -2 \\ 4 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} \\ 1 & : & \end{bmatrix} = \begin{bmatrix} 2 & : & \begin{pmatrix} 2-2 \\ 8-5 \end{pmatrix} \\ 1 & : & \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

$$|Q_C| \neq 0 \quad \Rightarrow \quad \text{Controllable}$$

Check for observability:

$$Q_o = [C^T \ : \ A^T C^T]$$

$$= \begin{bmatrix} 1 & : & \begin{pmatrix} 1 & 4 \\ -2 & -5 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ 1 & : & \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 1 & -7 \end{bmatrix}$$

$$|Q_C| \neq 0 \quad \Rightarrow \quad \text{Observable}$$

26. (d)

As per root locus transfer function

$$G(s)\,H(s) = \frac{k}{(s+4)(s^2+2s+2)}$$

CE: $\quad 1 + G(s)\,H(s) = 0$

$$s(s^2 + 2s + 2) + 4\,(s^2 + 2s + 2) + k = 0$$

$$s^3 + 6s^2 + 10s + (8 + k) = 0$$

s^3	1	10
s^2	6	$8 + k$
s^1	$-\dfrac{(8+k)-60}{6}$	0
s^0	$8 + k$	0

Row, $\quad s^1 = 0$

$\Rightarrow \quad 8 + k = 60$

$$k = 52$$

For calculation of intersection points,

$$6s^2 + (8 + k) = 0$$

$$6s^2 + (60) = 0$$

$$s^2 = -10$$

$$s = \pm j\sqrt{10}$$

Thus points of intersection are,

$$s = \pm j\omega = \pm j\sqrt{10}$$

27. (b)

The gain margin of the system can be given as,

$$\text{GM} = 20\log_{10}\frac{1}{|G(j\omega_{pc})|}$$

ω_{pc} is independent of the value of K.

So, $\text{GM} = C - 20\log_{10}(K)$

Where, C is a term independent of "K".

For $K = 2$, $\text{GM} = 32$ dB

So, $C = 32 + 20\log_{10}(2)$

When GM = 25 dB, $25 = C - 20\log_{10}(K)$

$$25 = 32 + 20\log_{10}(2) - 20\log_{10}(K)$$

$$20\log_{10}(K) = 7 + 20\log_{10}(2) = 13.02$$

$$K = 10^{(13.02/20)} = 4.48$$

28. (b)

The maximum phase lead is given by,

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

For high pass filter/lead compensator,

$$\tau = R_1C$$

and $\alpha = \dfrac{R_2}{R_1+R_2}$; $\alpha < 1$

By putting the value of α in the above relation, we get,

$$\phi_m = \sin^{-1}\left(\frac{1-\dfrac{R_2}{R_1+R_2}}{1+\dfrac{R_2}{R_1+R_2}}\right)$$

$$= \sin^{-1}\left(\frac{R_1+R_2-R_2}{R_1+R_2+R_2}\right) = \sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$$

29. (b)

The characteristic equation is given by,

$$|sI - A| = 0$$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 4 & 1 \\ -5 & -2 & -2 \end{bmatrix} = \begin{bmatrix} s & -3 & -1 \\ -2 & s-4 & -1 \\ 5 & 2 & s+2 \end{bmatrix}$$

$$\therefore \quad \begin{vmatrix} s & -3 & -1 \\ -2 & s-4 & -1 \\ 5 & 2 & s+2 \end{vmatrix} = 0$$

$$s[(s-4)(s+2)+2] + 3[-2(s+2)+5] - 1[-4-5(s-4)] = 0$$

$\Rightarrow$ $s[s^2 - 2s - 8 + 2] + 3[-2s - 4 + 5] - [-4 - 5s + 20] = 0$
$\Rightarrow$ $s[s^2 - 2s - 6] + 3[-2s + 1] - [-5s + 16] = 0$
$\Rightarrow$ $s^3 - 2s^2 - 6s - 6s + 3 + 5s - 16 = 0$
$\Rightarrow$ $s^3 - 2s^2 - 7s - 13 = 0$

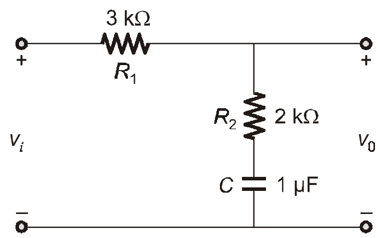
Using Routh's tabular form,

s^3	1	−7
s^2	−2	−13
s^1	−13.5	0
s^0	−13	0

Here, the total number of sign changes in the first column of Routh array is 1, therefore only one pole lie in the RHS of s-plane.

30. **(a)**

For the circuit shown,



$$G(s) = \frac{V_0(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}}$$

$$= \left(\frac{R_2}{R_1 + R_2}\right) \cdot \frac{\left(s + \frac{1}{R_2 C}\right)}{\left[s + \left(\frac{R_2}{R_1 + R_2}\right)\frac{1}{R_2 C}\right]}$$

$\therefore$ $$\alpha = \frac{R_1 + R_2}{R_2} = \frac{3\text{ k}\Omega + 2\text{ k}\Omega}{2\text{ k}\Omega} = \frac{5}{2} = 2.50$$

■■■■