

CLASS TEST

S.No. : 04 GH1_ME_W_160619

Engineering Mechanics



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CLASS TEST 2019-2020

MECHANICAL ENGINEERING

Date of Test : 16/06/2019

ANSWER KEY > Engineering Mechanics

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (a) | 7. (c) | 13. (a) | 19. (b) | 25. (c) |
| 2. (b) | 8. (b) | 14. (d) | 20. (a) | 26. (b) |
| 3. (a) | 9. (c) | 15. (c) | 21. (a) | 27. (a) |
| 4. (b) | 10. (d) | 16. (d) | 22. (d) | 28. (a) |
| 5. (b) | 11. (c) | 17. (a) | 23. (a) | 29. (b) |
| 6. (b) | 12. (c) | 18. (d) | 24. (b) | 30. (a) |

DETAILED EXPLANATIONS

1. (a)

$$a = \frac{f}{m} = -\frac{bv}{m}$$

but,

$$a = v \frac{dv}{dx}$$

 \therefore

$$\frac{v dv}{dx} = -\frac{bv}{m}$$

(at time infinity means steady state)

$$\int_u^0 dv = -\frac{b}{m} \int_0^x dx$$

$$-u = -\frac{b}{m} \times x$$

 \Rightarrow

$$x = mu/b$$

2. (b)

Resolving the forces in horizontal and vertical components.

$$\text{Horizontal components, } \Sigma F_x = 60 \cos 30^\circ - 80 \cos 45^\circ = -4.607$$

$$\text{Vertical components, } \Sigma F_y = 80 \sin 45^\circ + 60 \sin 30^\circ = 86.568$$

$$\begin{aligned} \text{Resultant, } R &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-4.607)^2 + (86.568)^2} \\ &= 86.69 \text{ N} \end{aligned}$$

3. (a)

As the body is in equilibrium, using Lami's theorem

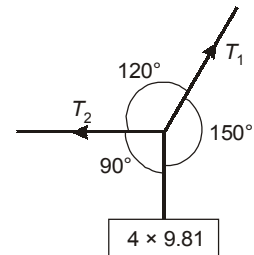
$$\therefore \frac{T_1}{\sin 90^\circ} = \frac{4 \times 9.81}{\sin(120^\circ)}$$

$$\therefore T_1 = 45.310 \text{ N}$$

$$\frac{T_2}{\sin 150^\circ} = \frac{4 \times 9.81}{\sin 120^\circ}$$

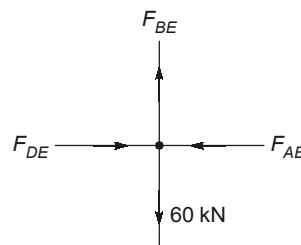
 \Rightarrow

$$T_2 = 22.65 \text{ N}$$



4. (b)

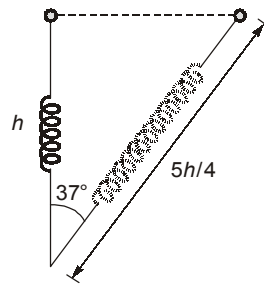
Consider joint (E)



$$F_{BE} = 60 \text{ kN (Tensile)}$$

6. (b)

 \therefore The kinetic energy of the ring will be given by the potential energy of spring. \therefore Let V be the speed of the ring when the spring becomes vertical



$$\frac{1}{2}mV^2 = \frac{1}{2}k[X]^2$$

$$X = \frac{5h}{4} - h = \frac{h}{4}$$

$$mV^2 = k\left[\frac{h}{4}\right]^2$$

$$V = \frac{h}{4}\sqrt{\frac{k}{m}}$$

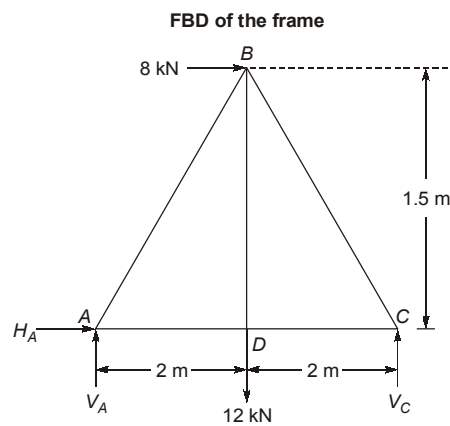
8. (b)

Using Lami's Theorem,

$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin(360^\circ - (90^\circ + 120^\circ))}$$

$$\frac{T_1}{T_2} = \frac{\sin 120^\circ}{\sin 150^\circ} = 1.732$$

9. (c)



∴ Taking moments about A,

$$V_C \times 4 = 8 \times 1.5 + 12 \times 2$$

$$V_C = \frac{12 + 24}{4} = \frac{36}{4} = 9 \text{ kN}$$

Reaction of support C, $V_C = 9 \text{ kN}$

10. (d)

Let u, v, w be the components of velocity in x, y and z direction respectively.

$$u = \frac{dx}{dt} = 2 \cos t$$

Similarly,

$$v = -3 \sin t$$

$$w = \sqrt{5} \cos t$$

$$V = \sqrt{u^2 + v^2 + w^2}$$

$$= \sqrt{(2 \cos t)^2 + (-3 \sin t)^2 + (\sqrt{5} \cos t)^2}$$

$$V = \sqrt{4 \cos^2 t + 9 \sin^2 t + 5 \cos^2 t}$$

$$V = \sqrt{9(\sin^2 t + \cos^2 t)} = 3 \text{ units}$$

11. (c)

$$a = \frac{dv}{dt}$$

Let resisting force,

$$F = Kv^2$$

if m is mass of the bullet then,

$$a = \frac{F}{m} = \frac{Kv^2}{m}$$

$$\Rightarrow \frac{dv}{dt} = \frac{Kv^2}{m}$$

$$\Rightarrow \frac{1}{v^2} dv = \frac{K}{m} \cdot dt$$

$$\Rightarrow \left[\frac{v^{-1}}{-1} \right]_u^v = \frac{K}{m} \int_0^t dt$$

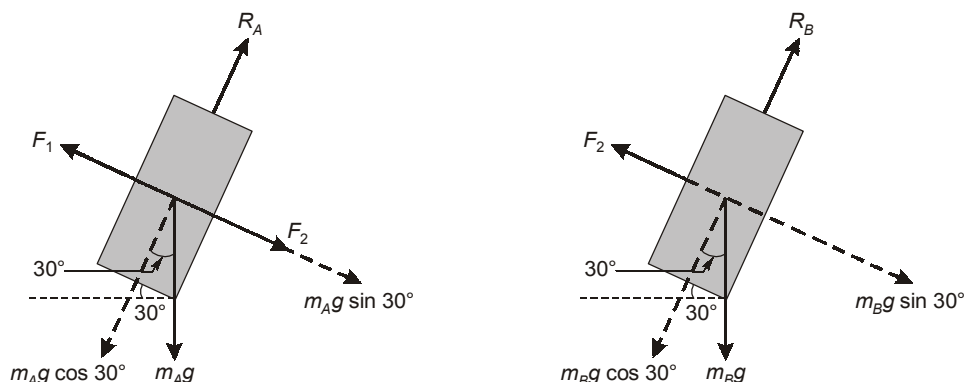
$$\Rightarrow \left[\frac{v-u}{uv} \right] = \frac{K}{m} t$$

$$\Rightarrow t = \frac{(u-v)}{uv} \times \frac{-m}{K}$$

$$\therefore t \propto (u-v)(uv)^{-1}$$

12. (c)

The FBD of the blocks A and B are shown below



Here F_1 and F_2 are the spring forces.

$$F = k\Delta z = k(x_0 - x_{\text{unstretched}})$$

$$F_1 = 1000 \times (0.3 - 0.25) = 50 \text{ N}$$

and

$$F_2 = 1000 \times (0.28 - 0.25) = 30 \text{ N}$$

At equilibrium,

Σ Forces along the plane for mass A = 0

$$\Rightarrow -F_1 + F_2 + m_A g \sin 30^\circ = 0$$

$$\Rightarrow m_A = \frac{F_1 - F_2}{g \sin 30^\circ} = \frac{50 - 30}{9.81 \times 0.5} = 4.08 \text{ kg}$$

and Σ Forces along the plane for mass B = 0

$$\Rightarrow -F_2 + m_B g \sin 30^\circ = 0$$

$$\Rightarrow m_B = \frac{F_2}{g \sin 30^\circ} = \frac{30}{9.81 \times 0.5} = 6.12 \text{ kg}$$

13. (a)

$$\text{K.E.} = \frac{1}{2} I \omega^2$$

$$I = \frac{mr^2}{2} = \frac{20 \times (0.2)^2}{2} = 0.4 \text{ kgm}^2$$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 500}{60} = 52.33 \text{ rad/s}$$

$$\text{K.E.} = \frac{1}{2} \times 0.4 \times 52.33^2 = 547.68 \text{ J}$$

14. (d)

Let speed of car moving in opposite direction is V m/s.

From relative velocity approach

$$\frac{12}{V+50} = \frac{5}{60}$$

$$12 \times 60 = 5v + 250$$

$$V = 94 \text{ km/hr}$$

15. (c)

\therefore Velocities are in opposite directions,

$\therefore I$ will lie between A and B,

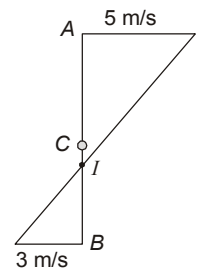
$$\frac{IA}{IB} = \frac{V_a}{V_b} = \frac{5}{3}$$

$$\Rightarrow \frac{0.5 - IB}{IB} = \frac{5}{3}$$

$$IB = 0.1875 \text{ m}$$

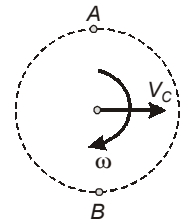
$$IA = 0.3125 \text{ m}$$

$$\omega = \frac{V_A}{IA} = \frac{5}{0.3125} = 16 \text{ rad/s}$$



Alternatively,

$$\begin{aligned} \therefore V_A &= V_C + R\omega \\ V_B &= R\omega - V_C \\ \therefore V_C + R\omega &= 5 \\ R\omega - V_C &= 3 \\ V_C + 0.25\omega &= 5 \quad \dots(a) \\ 0.25\omega - V_C &= 3 \quad \dots(b) \end{aligned}$$



On solving (a) and (b),

$$\begin{aligned} \omega &= 16 \text{ rad/s} \\ V_C &= 1 \text{ m/s} \end{aligned}$$

where V_C = velocity of centre C.

16. (d)

$$\begin{aligned} E &= \frac{1}{2} I \omega^2 \\ I &= MR^2 \\ E &= \frac{1}{2} MR^2 \omega^2 \\ \frac{E_1}{E_2} &= \frac{MR_1^2 \omega^2}{MR_2^2 \omega^2} = 4 \end{aligned}$$

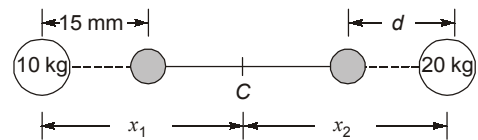
17. (a)

$$I_y = I_x = \frac{1}{2} I_{\text{circle}} = \frac{1}{2} \times \pi \times \frac{D^4}{64} = \frac{\pi r^4}{8}$$

18. (d)

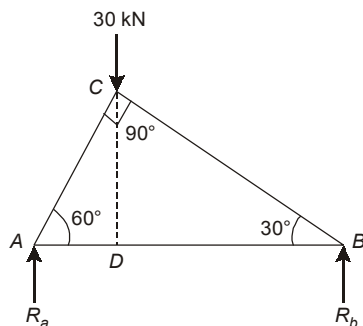
To keep centre of mass at C

$$\begin{aligned} \rightarrow m_1 x_1 &= m_2 x_2 \\ \text{(Let } 10 \text{ kg} &= m_1, 20 \text{ kg} = m_2) \\ \text{and } m_1(x_1 - 15) &= m_2(x_2 - d) \\ 15 m_1 &= m_2 d \end{aligned}$$



$$d = \frac{15 \times 10}{20} = 7.5 \text{ mm}$$

19. (b)



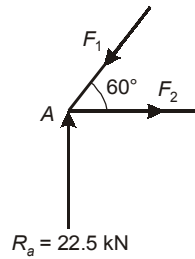
$$\begin{aligned} AC &= AB \cos 60^\circ = 2.5 \text{ m} \\ AD &= AC \cos 60^\circ = 2.5 \times 0.5 = 1.25 \end{aligned}$$

\therefore Taking moments about A,

$$R_b \times 5 = 30 \times 1.25$$

$$R_b = 7.5 \text{ kN}, \quad R_a = 30 - 7.5 = 22.5 \text{ kN}$$

Considering joint A,



$$\sum F_x = 0, \quad F_2 - F_1 \cos 60^\circ = 0$$

$$F_1 \sin 60^\circ - R_a = 0$$

$$F_1 = \frac{R_a}{\sin 60^\circ} = \frac{22.5}{\sin 60^\circ} = 25.97 \text{ kN} \quad (\text{compressive})$$

$$F_2 = F_1 \cos 60^\circ = 12.99 \text{ kN} \quad (\text{tensile})$$

∴ AB is in tension.

20. (a)

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$g = \frac{4\pi^2 L}{T^2}$$

In given problem $T = \frac{36}{20} = 1.8 \text{ s}$

∴

$$g = \frac{4 \times \pi^2 \times 0.8}{1.8^2} = 9.74 \text{ m/s}^2$$

21. (a)

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2$$

$$2.5 = \frac{1}{2} \alpha (1)^2$$

$$\alpha = 5 \text{ rad/s}^2$$

The angle rotated during 1st two second

$$= \frac{1}{2} \times 5 \times 2^2 = 10 \text{ radian}$$

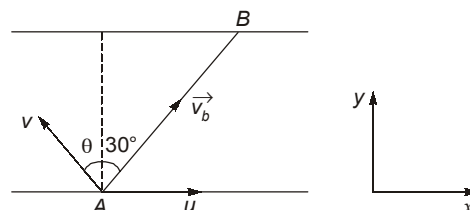
then

Angle rotated during the 2nd second is

$$10 - 2.5 = 7.5 \text{ radian}$$

22. (d)

Let v be the speed of boatman in still water



Resultant of u and v should be along AB . Components of \vec{v}_b (absolute velocity of boatman) along x and y -direction are:

$$v_x = u - v \sin \theta, v_y = v \cos \theta$$

$$\tan 30^\circ = \frac{v_y}{v_x}$$

$$\Rightarrow 0.577 = \frac{v \cos \theta}{u - v \sin \theta}$$

$$0.577u - 0.577v \sin \theta = v \cos \theta$$

$$\Rightarrow v = \frac{0.577u}{0.577 \sin \theta + \cos \theta}$$

$$v = \frac{(0.577 \times \cos 30^\circ)u}{\sin 30^\circ \sin \theta + \cos 30^\circ \cos \theta}$$

$$v = \frac{0.49964}{\sin(\theta + 30^\circ)}$$

v is minimum at $\theta = 60^\circ$,

$$\Rightarrow v_{\min} = 0.49964$$

$$v_{\min} \approx 0.54$$

23. (a)

Velocity of A is v along AB and velocity of particle B is along BC , its component

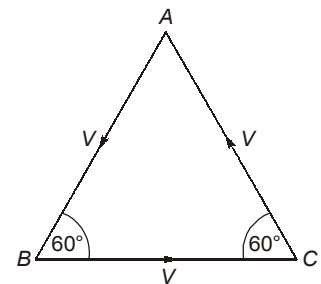
along BA is $v \cos 60^\circ = \frac{v}{2}$.

Thus separation AB decreases at the rate of

$$v + \frac{v}{2} = \frac{3v}{2}$$

Since this rate is constant, time taken in reducing separation from AB from d to zero is

$$t = \frac{d}{3v/2} = \frac{2d}{3v}$$



24. (b)

$$\Sigma M_A = 0$$

$$\Rightarrow P \times a \sin 60^\circ = 2a \cdot R_{cv}$$

$$\Rightarrow R_{cv} = 0.433 P \uparrow$$

$$R_{CH} = 0$$

$$\Rightarrow R_c = 0.433 P$$

$A \rightarrow (1)$

Reaction at A

$$\Sigma F_y = 0$$

$$\Rightarrow R_{AV} = 0.433 P$$

$$\Sigma F_x = 0; R_{AH} = P$$

$$R_A = \sqrt{(0.433P)^2 + P^2} = 1.09 P$$

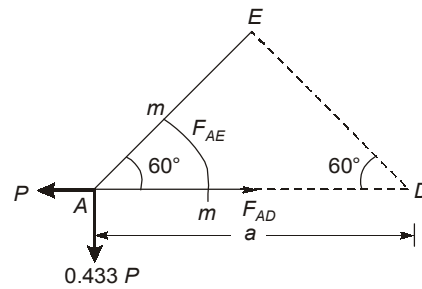
$B \rightarrow (4)$

At joint E , members AE and EB are collinear and member DE is joined at E .

$$\Rightarrow F_{DE} = 0$$

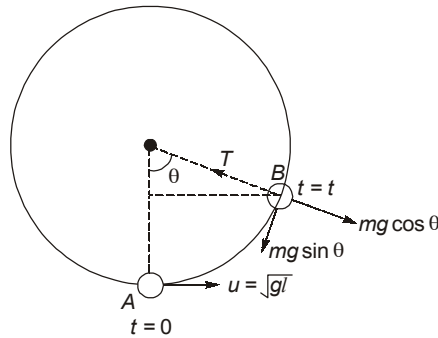
$D \rightarrow (3)$

Taking section mm as shown,



$$\begin{aligned} \Rightarrow \quad \Sigma M_E &= 0 \\ \Rightarrow \quad P \times a \times \sin 60^\circ &= 0.433 P \times a \sin 30^\circ + F_{AD} \times a \sin 60^\circ \\ \Rightarrow \quad 0.866 P &= 0.2165 P + 0.866 F_{AD} \\ \Rightarrow \quad F_{AD} &= P - 0.25 P = 0.75 P \\ C \rightarrow (2) \end{aligned}$$

25. (c)



Let $T = mg$ at angle θ shown in figure

$$h = l(1 - \cos \theta) \quad \dots(1)$$

Apply conservation of mechanical energy between points A and B,

$$\frac{1}{2} m(u^2 - v^2) = mgh$$

$$u^2 = gl \quad \dots(2)$$

v = Speed of particle in position on B

$$v^2 = u^2 - 2gh \quad \dots(3)$$

$$T - mg \cos \theta = \frac{mv^2}{l}$$

$$mg - mg \cos \theta = \frac{mv^2}{l}$$

$$\Rightarrow \quad v^2 = gl(1 - \cos \theta) \quad \dots(4)$$

Substituting the values of v^2 , u^2 and h from equations (4), (2) and (1) in equation (3).

$$gl(1 - \cos \theta) = gl - 2gl(1 - \cos \theta)$$

$$\cos \theta = \frac{2}{3}$$

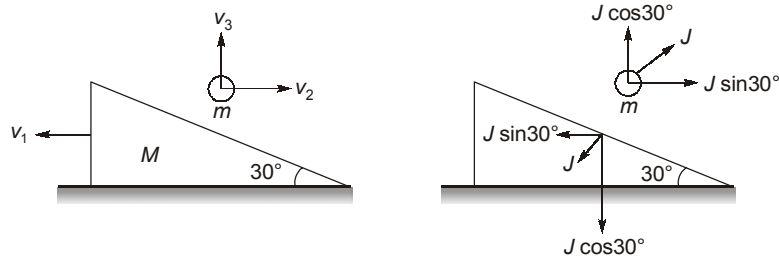
$$\theta = \cos^{-1} \left(\frac{2}{3} \right)$$

Substituting $\cos \theta = \frac{2}{3}$ in equation (4),

$$v = \sqrt{\frac{gl}{3}}$$

26. (b)
Given:

$$M = 2 \text{ kg and } m = 1 \text{ kg}$$



Let J be the impulse between ball and the wedge during collision and v_1, v_2 and v_3 be the components of the velocity of the wedge and the ball in horizontal and vertical directions respectively.

Impulse = Change in momentum

$$J \sin 30^\circ = Mv_1 - mv_2$$

$$\Rightarrow \frac{J}{2} = 2v_1 - v_2 \quad \dots(1)$$

$$J \cos 30^\circ = m(v_3 + v_o)$$

$$\Rightarrow \frac{\sqrt{3}}{2} J = v_3 + 2 \quad \dots(2)$$

$\frac{\text{Relative speed of separation}}{\text{Relative speed of approach}} = \text{Coefficient of restitution}$

$$\frac{(v_1 + v_2) \sin 30^\circ + v_3 \cos 30^\circ}{v_o \cos 30^\circ} = \frac{1}{2}$$

$$\Rightarrow v_1 + v_2 + \sqrt{3}v_3 = \sqrt{3} \quad \dots(3)$$

Solving equations (1), (2) and (3),

$$v_1 = \frac{-1}{\sqrt{3}} \text{ m/s}$$

$$v_2 = \frac{2}{\sqrt{3}} \text{ m/s and } v_3 = 0$$

$$\text{Thus velocity of wedge} = \frac{-1}{\sqrt{3}} \hat{i} \text{ m/s}$$

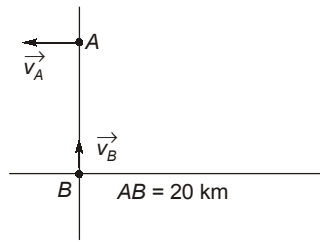
$$\text{Velocity of ball} = \frac{2}{\sqrt{3}} \hat{i} \text{ m/s}$$

27. (a)

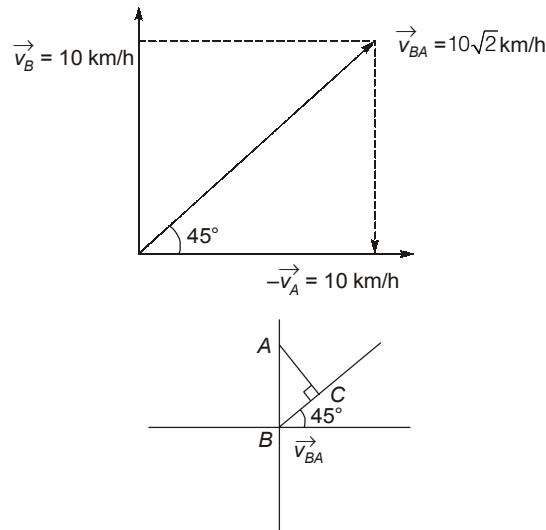
Boats A and B are moving with same speed 10 km/h in the directions shown in figure. It corresponds to a 2-dimensional, 2 body problem with zero acceleration.

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

$$|\vec{v}_{BA}| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ km/h}$$



It can be assumed that A is at rest and B is moving with \vec{v}_{BA} in the direction shown



$$\text{Minimum distance} = AC = AB \sin 45^\circ = \frac{20}{\sqrt{2}} \text{ km} = 10\sqrt{2} \text{ km}$$

$$\text{time is } t = \frac{BC}{|\vec{v}_{BA}|} = \frac{10\sqrt{2}}{10\sqrt{2}} = 1 \text{ hr}$$

28. (a)

Here,

$$\alpha = 45^\circ$$

We have:

$$a = \frac{dV}{dt} \Rightarrow a = \frac{dV}{dx} \times \frac{dx}{dt}$$

\therefore

$$a = \frac{dV}{dx} \times V$$

Also,

$$a = \frac{mg \sin \alpha - \mu mg \cos \alpha}{m}$$

$$a = g[\sin \alpha - \mu \cos \alpha]$$

\therefore

$$g[\sin \alpha - \mu \cos \alpha] = \frac{dV}{dx} \times V$$

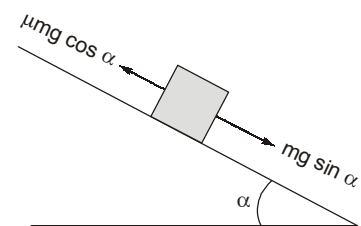
\therefore

$$g[\sin \alpha \cdot dx - \mu \cos \alpha \cdot dx] = V \cdot dV$$

On integrating,

$$g \left[\sin \alpha \cdot x - 5 \cos \alpha \times \frac{x^2}{2} \right] = \left[\frac{V^2}{2} \right]_0$$

$$g \left[\sin \alpha \cdot x - 5 \cos \alpha \times \frac{x^2}{2} \right] = 0$$



$$\Rightarrow \sin \alpha \cdot x = 5 \cos \alpha \times \frac{x^2}{2}$$

$$x = \frac{2 \tan \alpha}{5} \Rightarrow \frac{2 \tan 45^\circ}{5} = 0.4 \text{ m}$$

29. (b)

We have,

 \therefore

Torque = $I\alpha$

$3F \sin 30^\circ \times 0.5 = I\alpha$

$3 \times 0.5 \times \frac{1}{2} \times 0.5 = 1.5 \times \frac{0.5^2}{2} \times \alpha$

 \therefore

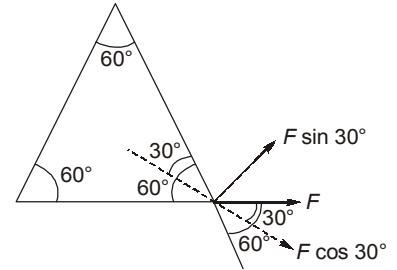
$\alpha = 2 \text{ rad/s}^{-1}$

 \therefore

$\omega = \omega_0 + \alpha t$

$\omega = 0 + 2 \times 1$

$\omega = 2 \text{ rad s}^{-1}$



30. (a)

$a = \frac{dV}{dt}$

 \Rightarrow

$\alpha \sqrt{V} = \frac{dV}{dt}$

 \Rightarrow

$\alpha \int_{t=0}^t dt = \int_{V_0}^0 \frac{dV}{\sqrt{V}}$

 \Rightarrow

$\alpha t = \frac{V_0^{-1/2+1}}{\frac{-1}{2}+1}$

 \Rightarrow

$t = \frac{2\sqrt{V_0}}{\alpha}$

■■■■