CLASS TEST

Sl. 01 SK_EE_EFSGH_020722 [G]

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ELECTROMAGNETIC FIELDS

ELECTRICAL ENGINEERING

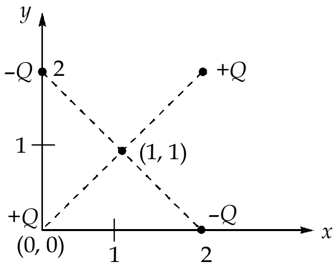
Date of Test : 02/07/2022

ANSWER KEY

1.	(a)	7.	(d)	13.	(d)	19.	(a)	25.	(b)
2.	(c)	8.	(a)	14.	(b)	20.	(c)	26.	(b)
3.	(d)	9.	(d)	15.	(b)	21.	(a)	27.	(d)
4.	(c)	10.	(a)	16.	(c)	22.	(c)	28.	(c)
5.	(c)	11.	(a)	17.	(a)	23.	(b)	29.	(b)
6.	(a)	12.	(d)	18.	(c)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (a)



The two positive charges Q are diagonally opposite in position and at the same distance from the point (1, 1, 0) fields produced by them are equal and opposite and so their resultant field is zero. Similarly for negative charges.

2. (c)

If the divergence of a given vector is zero, then it is said to be solenoidal.

$$\nabla.\vec{A} = 0$$

By Divergence theorem,

$$\int_V (\nabla.\vec{A})dv = \oint_s \vec{A}.\vec{ds}$$

So, for a solenoidal field,

$$\nabla.\vec{A} = 0 \text{ and } \oint_s \vec{A}.\vec{ds} = 0$$

3. (d)

From Biot savart law,

$$\vec{H} = \int_0^{2\pi} \frac{IRd\phi\hat{a}_\phi \times (-\hat{a}_\rho)}{4\pi R^2}$$

$$= \left(\frac{I}{4\pi}\int_0^{2\pi}\frac{Rd\phi}{R^2}\right)\hat{a}_z$$

$$\vec{H} = \frac{I}{2R}\hat{a}_z$$

4. (c)

$$\text{Electric field intensity, } \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\hat{a}_y$$

$$= \frac{4\times10^{-9}}{2\pi\times8.854\times10^{-12}\times3}\hat{a}_y$$

$$\vec{E} = 24\,\hat{a}_y \text{ V/m}$$

5. (c)

Force, $\vec{F} = I(\vec{L} \times \vec{B})$

$= 10(2\hat{a}_z \times 0.02(\hat{a}_y - \hat{a}_x))$

$= 10(0.04(-\hat{a}_x) - 0.04\hat{a}_y)$

$= -0.4\hat{a}_x - 0.4\hat{a}_y$

Force acting per unit length,

$$\frac{\vec{F}}{L} = \frac{-0.4\hat{a}_x - 0.4\hat{a}_y}{2}$$

$$= -0.2\hat{a}_x - 0.2\hat{a}_y$$

$$\frac{\vec{F}}{L} = -0.2(\hat{a}_x + \hat{a}_y)$$

6. (a)

$$\vec{P} = x^3 y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\vec{a}_z$$

For solenoidal, $\nabla \cdot \vec{P} = 0$

$\Rightarrow$ $$\nabla \cdot \vec{P} = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z}$$

$$= 3x^2y - 2x^2y - x^2y$$

$$= 0$$

$\Rightarrow$ $\vec{P}$ is solenoidal

For irrotational, $\nabla \times \vec{P} = 0$

$\Rightarrow$ $$\nabla \times \vec{P} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$$

$$= \vec{a}_x(-x^2z) + \vec{a}_y(2xyz) + \vec{a}_z(-2xy^2 - x^3)$$

$$\neq 0$$

$\Rightarrow$ $\vec{P}$ is not irrotational.

7. (d)

$$\vec{R}_{21} = (\vec{R}_1 - \vec{R}_2)$$

$$= (1, -2, 3) - (2, -1, 0)$$

$$= (-1, -1, 3)$$

$$= -\hat{i} - \hat{j} + 3\hat{k}$$

$$F_{21} = \frac{Q_1Q_2}{4\pi \in_0 |\vec{R}_{21}|^2}\hat{R}_{21}$$

$$= \frac{25\times 20\times 10^{-12}\times 9\times 10^{9}}{\left(\sqrt{1+1+9}\right)^2}\left(\frac{-\hat{i}-\hat{j}+3\hat{k}}{\sqrt{1+1+9}}\right)$$

$$= 0.123(-\hat{i}-\hat{j}+3\hat{k})\text{N}$$

8. **(a)**

Vector from the line to the point P

$$\vec{r} = -2u_x + 3u_y$$

$$\vec{E} = \frac{\lambda}{2\pi \in_0 r}\hat{r}$$

$$\vec{E} = \frac{0.1\times 10^{-6}}{2\pi \in_0 \sqrt{4+9}}\left(\frac{-2\hat{u}_x+3\hat{u}_y}{\sqrt{13}}\right)$$

$$= -276.92\hat{u}_x + 415.38\hat{u}_y$$

9. **(d)**

$$\text{Net flux} = \iiint \rho_v \cdot dv = \int_0^{2\pi}\int_0^{\pi}\int_1^{2}\frac{10}{r^2}r^2\sin\theta\, dr\, d\theta\, d\phi$$

$$= 10 \times 1 \times 2 \times 2\pi = 40\pi \text{ mC}$$

10. **(a)**

$$E = -N\frac{d\phi}{dt} = -100(3t^2-2)\times 10^{-3}$$

$$= -100(12 - 2) \times 10^{-3} = -1 \text{ V}$$

11. **(a)**

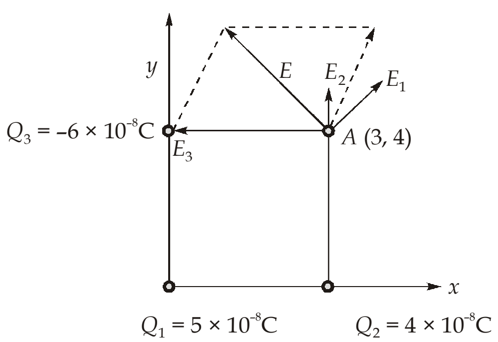
Given : $Q_1 = +5\times 10^{-8}$C, $Q_2 = +4\times 10^{-8}$ C and $Q_3 = -6\times 10^{-8}$ C

The potential at point A

$$V_A = \frac{Q_1}{4\pi\varepsilon_0 r_1}+\frac{Q_2}{4\pi\varepsilon_0 r_2}+\frac{Q_3}{4\pi\varepsilon_0 r_3}$$

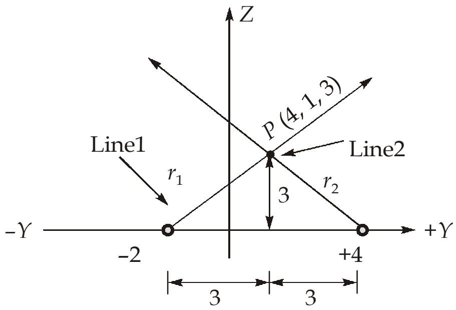
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times 10^{-8}}{5}+\frac{4\times 10^{-8}}{4}-\frac{6\times 10^{-8}}{3}\right]$$

$$= 0$$



12. (d)

Let r_1 and r_2 be the directed line segments from the lines 1 and 2 respectively to the point P(in Y-Z plane).



Then, $$r_1 = 3\hat{a}_y + 3\hat{a}_z$$

and $$r_2 = -3\hat{a}_y + 3\hat{a}_z$$

$$E_1 = \frac{\lambda}{2\pi\varepsilon_0 r_1}\left(\frac{3\hat{a}_y + 3\hat{a}_z}{r_1}\right) \quad \text{. . . (i)}$$

where, $$r_1^2 = 3^2 + 3^2 = r_2^2 \text{ (in magnitude)} = 18 = r^2$$

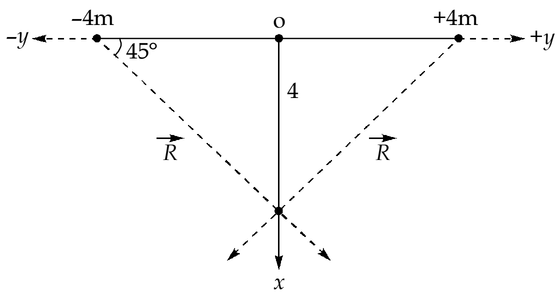
$$E_2 = \frac{\lambda}{2\pi\varepsilon_0 r_2}\left(\frac{-3\hat{a}_y + 3\hat{a}_z}{r_2}\right) \quad \text{. . . (ii)}$$

Adding (i) and (ii), we obtain the resultant field

$$E = E_1 + E_2 = \frac{\lambda}{2\pi\varepsilon_0 r^2}(2\times 3\hat{a}_z) \qquad \text{(replacing } r_1 \text{ and } r_2 \text{ by } r\text{)}$$

$$= \frac{5\times 10^{-9}}{2\pi\varepsilon_0(18)}(6\hat{a}_z) = 30\hat{a}_z \text{ V/m}$$

13. (d)



The y components of the fields produced by two lines of charge cancel out and only x components will exist in effect.

The resultant field is,

$$\vec{E} = \pm 2\frac{\lambda}{2\pi\varepsilon_0|\vec{R}|}\cdot\frac{\vec{R}}{|\vec{R}|}$$

where, $\vec{R} = 4\hat{a}_x$ and $|\vec{R}| = \sqrt{4^2+4^2} = 4\sqrt{2}$ m

$$\vec{E} = \pm 2 \times \frac{4\times 10^{-9}}{2\pi \times 8.854\times 10^{-12}} \times \frac{4\hat{a}_x}{(4\sqrt{2})^2}$$

$$\vec{E} = \pm 18\hat{a}_x \text{ V/m}$$

14. (b)

According to Gausss's law, the electric flux leaving the surface with R = 5 m is equal to the total flux enclosed by the surface.

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3$$

Ψ_1 = Electric flux leaving the spherical surface with R = 1m

$$= 20 \times 10^{-9} \times (4\,\pi R^2)$$

$$= 20 \times 10^{-9} \times 4\,\pi = 80\,\pi\text{nC}$$

$$\Psi_2 = -9 \times 10^{-9} \times (4\,\pi\,(2)^2)$$

$$= -144\,\pi\text{nC}$$

$$\Psi_3 = 2 \times 10^{-9} \times (4\pi \times 9) = 72\,\pi\text{ nC}$$

$$\Psi = 8\,\pi\text{ nC}$$

15. (b)

$$\nabla \times \vec{H} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & (z^2 - x^2) & 3y \end{vmatrix}$$

$$= \hat{i}(3-2z) + \hat{k}(-2x-2)$$

$$= (3-2z)\hat{i} - (2x+2)\hat{k}$$

At the origin , x = 0, z = 0

$$\nabla \times \vec{H} = 3\hat{i} - 2\hat{k}$$

$$|\nabla - \vec{H}| = \sqrt{3^2+2^2} = \sqrt{13}$$

16. (c)

The flux Φ_1, at i_1 = 5 A is

$$\Phi = B \times A$$

$$= 1 \times (30 \times 10^{-4})$$

$$\Phi_1 = 30 \times 10^{-4}\text{ Wb}$$

Φ_2 at i_2 = 10 A is,

$$\Phi_2 = 1.5 \times 30 \times 10^{-4}$$

Increase of flux when current is increased from 5 to 10 A

$$= 0.5 \times 30 \times 10^{-4}\text{ Wb}$$

$$\frac{d\phi}{dt}\text{(average)} = \frac{0.5 \times 30 \times 10^{-4}}{5}$$

$$\frac{d\phi}{dt}(\text{average}) = 3 \times 10^{-4} \text{ Wb/A}$$

$\therefore$ Mean value of inductance,

$$L = N\frac{d\phi}{di}$$

$$L = 2000 \times 3 \times 10^{-4}$$

$$L = 0.6 \text{ Henry.}$$

17. (a)

Let, $E = E_1,$

$$\text{Energy } E_1 = \frac{Q_1^2}{2C_1}$$

Electrically isolated

$\Rightarrow$ $Q_2 = Q_1$

$d_2 = 2d_1$

$\Rightarrow$ $C_2 = \frac{C_1}{2}$

$$E_2 = \frac{Q_2^2}{2C_2} = \frac{Q_1^2}{\frac{2C_1}{2}} = 2\left(\frac{Q_1^2}{2C_1}\right)$$

$$= 2E_1 = 2E$$

18. (c)

Given,

Voltage distribution across capacitance is in the ratio 2 : 3 : 4 and applied voltage is 135 V.

Then, $V_{C1} = 30$ V

$V_{C2} = 45$ V

$V_{C3} = 60$ V

Hence, $$C_1 = \frac{4500}{30} = 150\ \mu\text{F}$$

$$C_2 = \frac{4500}{45} = 100\ \mu\text{F}$$

$$C_3 = \frac{4500}{60} = 75\ \mu\text{F}$$

$\therefore$ $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_{eq} = 33.33\ \mu\text{F}$$

19. (a)

$$\mu_1 = 2\,\mu_0;$$
$$\mu_2 = 5\,\mu_0$$
$$B_2 = 10\hat{a}_\rho + 15\hat{a}_\phi - 20\hat{a}_z \text{ mWb/m}^2$$
$$B_{1n} = B_{2n} = 15\hat{a}_\phi$$
$$H_{1t} = H_{2t}$$
$$B_{1t} = \frac{\mu_1}{\mu_2}B_{2t} = \frac{2}{5}\left(10\hat{a}_\rho - 20\hat{a}_z\right)$$
$$B_{1t} = (4\hat{a}_\rho - 8\hat{a}_z)\text{mWb/m}^2$$
$$W_{m1} = \frac{1}{2}B_1 \cdot H_1 = \frac{B_1^2}{2\mu_1}$$
$$= \frac{(4^2 + 15^2 + 8^2)\times 10^{-6}}{2\times 2\times 4\pi \times 10^{-7}}$$
$$= \frac{305}{16\pi}\times 10 = 60.68 \text{ J/m}^3$$

20. (c)

$$E = \int (v\times B)\cdot dl$$
$$v \times B = -0.15 \sin 10^3 t\; u_x \text{ V/m}$$
$$E = \int_0^{0.25} -0.15 \sin 10^3 t\, dx$$
$$E = -0.15 \sin 10^3 t [x]_0^{0.25}$$
$$E = -0.0375 \sin 10^3\, t \text{ V}$$

21. (a)

$$E = \frac{\rho_s}{2\,\epsilon_0}\hat{a}_n$$
$$E = E_1 + E_2 + E_3$$
$$= \left(\frac{10}{2\,\epsilon_0}\hat{a}_x + \frac{(-20)}{2\,\epsilon_0}\hat{a}_y + \frac{30}{2\,\epsilon_0}(-\hat{a}_z)\right)\times 10^{-6}$$
$$= \frac{10}{2\,\epsilon_0}\left[\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z\right]\times 10^{-6}$$
$$|E| = \frac{10}{2\,\epsilon_0}\sqrt{1+2^2+3^2}\times 10^{-6}$$
$$= \frac{10}{2\,\epsilon_0}\sqrt{14} = \frac{18.71}{\epsilon_0}\times 10^{-6} = 2.11 \times 10^6 \text{ V/m}$$

22. (c)

$$H = H_x + H_y$$

$$H = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$H_x = \frac{5}{4\pi \times 2}(\cos 0° - \cos 90°)(-\hat{a}_x \times \hat{a}_z) = \frac{5}{8\pi}\hat{a}_y \text{A/m}$$

$$H_y = \frac{5}{4\pi \times 2}(\cos 0° - \cos 90°)(\hat{a}_y \times \hat{a}_z) = \frac{5}{8\pi}\hat{a}_x$$

$$\vec{H} = \frac{5}{8\pi}(\hat{a}_x + \hat{a}_y) \ \text{A/m}$$

23. (b)

$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$

$$\rho = \sqrt{5^2 + 5^2} = \sqrt{50}\,\text{m}$$

$$= \frac{2}{4\pi\sqrt{5^2 + 5^2}}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$\cos\alpha_2 = \frac{10}{\sqrt{50 + 100}} = \frac{10}{\sqrt{150}}$$

$$\cos\alpha_1 = \cos 90° = 0$$

$$\hat{a}_\phi = \hat{a}_l \times \hat{a}_\rho = \hat{a}_z \times \left(\frac{5\hat{a}_x + 5\hat{a}_y}{5\sqrt{2}}\right) = \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$$

$$\vec{H} = \frac{2}{4\pi \times 5\sqrt{2}} \times \left(\frac{10}{\sqrt{150}} - 0\right) \times \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right)$$

$$= \frac{1}{20\pi}\left(-\hat{a}_x + \hat{a}_y\right) \times \frac{10}{5\sqrt{6}}$$

$$= \frac{1}{10\pi\sqrt{6}}\left(-\hat{a}_x + \hat{a}_y\right)\text{A/m}$$

24. (a)

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\frac{\rho dV}{d\rho} + \frac{1}{\rho^2}\frac{d^2V}{d\phi^2} + \frac{d^2V}{dz^2} = \frac{-\rho_v}{\in} = \frac{-\rho_v}{\in_0 \in_r}$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{\rho dV}{dp}\right) = \frac{-\rho_v}{\in} = \frac{-10}{\rho} \times \frac{10^{-12}}{3.6 \times 1} \times 36\pi \times 10^9$$

$$\nabla^2 V = \frac{-\rho_v}{\in}$$

$$\frac{d}{d\rho}\frac{\rho dV}{d\rho} = -0.1\,\pi$$

$$\frac{\rho dV}{d\rho} = -0.1\,\pi\rho + A$$

$$V = -0.1\,\pi\rho + A\ln\rho + B$$

$$-0.1\,\pi \times 2 + A\ln 2 + B = 0$$

$$A\ln 2 + B = 0.2\,\pi$$

$$-0.1\,\pi \times 5 + A\ln 5 + B = 60$$

$$A\ln 5 + B = 60 + 0.5\,\pi$$

$$A\ln 2.5 = (60 + 0.3\,\pi)$$

$$A = \left(\frac{60+0.3\pi}{\ln 2.5}\right) = 66.51$$

$$E = -\nabla V$$

$$= -\left(-0.1\pi + \frac{A}{\rho}\right)\hat{a}_\rho$$

At $\rho = 1$,
$$E = -\left(-0.1\pi + \frac{66.51}{1}\right)\hat{a}_\rho = -66.19\hat{a}_\rho \text{ V/m}$$

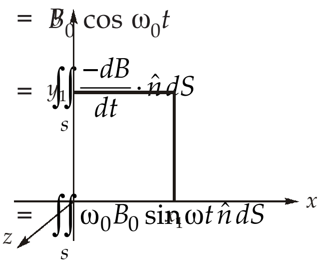
25. (b)

$$x_1 = y_1 = 1 \text{ m}$$

$$B_0 = \sin\pi x \sin\pi y \text{ T}$$

$$B = B_0 \cos\omega_0 t$$

$$E(t) = \iint_s \frac{-dB}{dt}\cdot\hat{n}\,dS$$



$$= \iint_s \omega_0 B_0 \sin\omega t\,\hat{n}\,dS$$

$$E_{max} = \omega_0 \int_{y=0}^{1}\int_{x=0}^{1} \sin\pi x \sin\pi y\,dx\,dy$$

$$E_{max} = 1000 \times 2\pi \times \frac{4}{\pi^2} = \frac{8000}{\pi}\text{ V/turns}$$

$$E_{rms} = \frac{1}{\sqrt{2}} \times \frac{8000}{\pi} \times 10 = 18 \text{ kV}$$

26. (b)

$$C = 4\pi\epsilon_0\left(\frac{ab}{a-b}\right)$$

$$A_a = 4\pi a^2$$

$$A_b = 4\pi b^2$$

$$a = \sqrt{\frac{A_a}{4\pi}};\; b - \sqrt{\frac{A_b}{4\pi}}$$

$$ab = \frac{\sqrt{A_a A_b}}{4\pi};$$

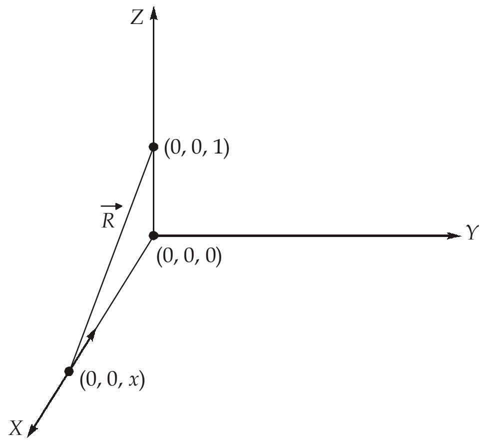
$$a - b = \frac{\sqrt{A_a} - \sqrt{A_b}}{\sqrt{4\pi}}$$

$$C = 4\pi \in_0 \left[\frac{\sqrt{A_a A_b}}{4\pi} \times \frac{\sqrt{4\pi}}{\left(\sqrt{A_a} - \sqrt{A_b}\right)} \right]$$

$$= \sqrt{4\pi} \in_0 \frac{\sqrt{A_a A_b}}{\sqrt{A_a} - \sqrt{A_b}}$$

27. (d)

According to the Biot-Savart's law,



$$\vec{H} = \int \frac{I\, d\vec{L} \times \vec{R}}{4\pi R^3}$$

$$\vec{R} = -x\hat{a}_x + \hat{a}_z$$

$$R = \sqrt{1 + x^2}$$

$$\vec{H} = \int_{\infty}^{0} \frac{10\, dx(-\hat{a}_x) \times (-x\hat{a}_x + \hat{a}_z)}{4\pi (x^2 + 1)^{3/2}}$$

$$\vec{H} = \frac{10}{4\pi} \int_{\infty}^{0} \frac{dx}{(x^2 + 1)^{3/2}} \hat{a}_y$$

$$\vec{H} = \frac{10}{4\pi} \left. \frac{x}{\sqrt{1 + x^2}} \right|_{\infty}^{0} \hat{a}_y = \frac{10}{4\pi} \hat{a}_y$$

28. (c)

The flux in the circuit is,

$$\Psi = \frac{\text{ö}}{\text{U}} = \frac{N_i i_1}{l / \mu S} = \frac{N_1 i_1 \mu S}{2\pi\rho_0}$$

ö = magneto motive force

U = reluctance

l = mean length

S = cross-sectional area of magnetic core

According to Faraday's Law, the emf induced in the second coil is,

$$V_2 = -N_2 \frac{d\Psi}{dt} = -\frac{N_1 N_2 \mu S}{2\pi\rho_0} \frac{di_1}{dt}$$

$$V_2 = -\frac{100 \times 200 \times 500 \times (4\pi \times 10^{-7}) \times 10^{-3} \times 300\pi \cos 100\pi t}{2\pi(10 \times 10^{-2})}$$

$$= -6\pi \cos 100\pi t \text{ V}$$

29. **(b)**

According to Ampere's law,

$$I_{enc} = \oint_{r=r_0} \vec{H} \cdot dl$$

$$= \int_0^{2\pi} \frac{10^4}{r_0}\left(\frac{4r_0^2}{\pi^2}\sin\frac{\pi}{2} - \frac{2r_0^2}{\pi}\cos\frac{\pi}{2}\right) \cdot r_0\, d\phi$$

$$= 10^4 \int_0^{2\pi} \frac{4r_0^2}{\pi^2} d\phi$$

$$I_{enc} = 10^4 \cdot \frac{4r_0^2}{\pi} \times 2 = \frac{8}{\pi} \text{ Ampere}$$

30. **(c)**

The potential is given by:

$$V_{AB} = -\int_B^A \bar{E} \cdot d\bar{l}$$

Now, we know that $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$ for infinite line charge

$$\vec{E} = \frac{10^{-9}}{2\pi\left(\frac{10^{-9}}{36\pi}\right)\rho}\hat{a}_\rho = \frac{18}{\rho}\hat{a}_\rho \text{ V/m}$$

$$d\bar{l} = d\rho \cdot \hat{a}_\rho + \rho d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

$$\bar{E} \cdot d\bar{l} = \frac{18}{\rho} d\rho$$

$$V_{AB} = -\int_4^2 \frac{18}{\rho} d\rho = \left[-18\ln\rho\right]_4^2 = -18\left[\ln 2 - \ln 4\right]$$

$$= 18 \ln 2 = 12.48 \text{ volts}$$

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