CLASS TEST

S.No. : 04 SK_CE_H_290622

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ENGINEERING MECHANICS

CIVIL ENGINEERING

Date of Test : 29/06/2022

ANSWER KEY

1.	(d)	7.	(c)	13.	(c)	19.	(a)	25.	(a)
2.	(c)	8.	(a)	14.	(c)	20.	(a)	26.	(a)
3.	(b)	9.	(d)	15.	(c)	21.	(a)	27.	(d)
4.	(b)	10.	(c)	16.	(b)	22.	(c)	28.	(c)
5.	(c)	11.	(d)	17.	(c)	23.	(b)	29.	(a)
6.	(c)	12.	(d)	18.	(d)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (d)

When we resolve all the forces in the direction normal to F_2, the force F_2 vanishes and only the components of F_1 and R remain. So unknown force F_1 can be found by one equation.

2. (c)

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_0 = 0$$

$$\theta = \frac{1}{2}\times 2\times 10^2 = 100\,\text{rad}$$

$\therefore$ Number of revolutions $= \dfrac{100}{2\pi} = 15.92$

3. (b)

The position vector $\vec{P} = r\ \hat{r} + \theta\ \hat{\theta}$

The velocity vector $\vec{V} = \dfrac{d\vec{P}}{dt}\dot{r}\ \hat{r} + r\ \dot{\theta}\ \hat{\theta}$

The acceleration $\vec{a} = \dfrac{d\vec{V}}{dt}$

$$= (\ddot{r} - r\dot{\theta}^2)\ \hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

Thus radial acceleration

$$(a_m) = \ddot{r} + r\dot{\theta}^2 = \frac{dv_r}{dt} = \frac{v_t^2}{r}$$

Tangential acceleration $(a_t) = r\ \ddot{\theta} + 2\dot{r}\dot{\theta}$

4. (b)

The relative velocity of shot mass = 10 m/s

Let velocity of recoil = v

Absolute velocity of shot mass = $(10 - v)$

Using momentum equation

$0.002\ (10 - v) = 1 \times v$

$$v = \frac{0.02}{1.002} = \frac{20}{1002} = \frac{10}{501}\text{m/s}$$

5. (c)

$$I = mk^2 = 50(0.180)^2 = 1.62\ \text{kg.m}^2$$

$$M = I\alpha$$

$$3.5 = (1.62)\,\alpha$$

$\Rightarrow$ $\alpha = 2.1605\ \text{rad/s}^2$ (deceleration)

$$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi(3600)}{60} = 120\pi\ \text{rad/s}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \text{or} \quad 0 = (120\pi)^2 - 2 \times 2.1605 \times \theta$$

$\therefore$ $$\theta = 32.891 \times 10^3 \text{ rad}$$

$$\text{Number of revolutions} = \frac{\theta}{2\pi} = \frac{32.891 \times 10^3}{2 \times 3.14} = 5234.77$$

6. (c)

The velocity of block embedded with bullet

$$v = \frac{401 \times 0.01}{4 + 0.01} = 1 \text{ m/s}$$

Kinetic energy loss = $kE_i - kE_f$

$$= \frac{1}{2} \times 0.01 \times 401^2 - \frac{1}{2} \times 4.01 \times 1^2$$

$$= 802 \text{ N} - \text{m}$$

7. (c)

For perfectly elastic collision $e = 1.0$

8. (a)

$$P = \frac{W}{R} \cdot \mu$$

where, P = Rolling resistance, R = Radius of wheel, W = Weight of freight car

Coefficient of rolling resistance,

$$\mu = \frac{PR}{W} = \frac{30 \times 750}{1000000} = 0.0225 \text{ mm} = 22.5 \times 10^{-3} \text{ mm}$$

9. (d)

$$T = \frac{2m_1 m_2}{m_1 + m_2} g = \frac{2 \times 21 \times 28}{21 + 28} g = 24 \text{ gm wt}$$

10. (c)

Let, S be the distance by which a pile will move under a single blow of hammer.

Work done by hammer = Work done by the ground resistance

$$\frac{1}{2}(12 + 4)V^2 = 200 \times S$$

$$\Rightarrow \quad 8 \times 4^2 = 200 \times S$$

$$\Rightarrow \quad 128 = 200 \times S$$

$$\Rightarrow \quad S = 0.64 \text{ m}$$

11. (d)

For perfectly elastic spheres $e = 1$

Using momentum equation.

$$m\vec{v}_2 + m\vec{v}_1 = 2m\vec{u} + m\vec{u} = 3m\vec{u}$$

$$\vec{v}_2 - \vec{v}_1 = 3\vec{u}$$

Using Newton's Law of collision of elastic bodies

$\vec{v}_2 - \vec{v}_1 = e(2\vec{u} - \vec{u})\vec{u}$ $(\because e = 1)$

Solving

$$\vec{v}_2 = 2\vec{u}$$

$$\vec{v}_1 = \vec{u}$$

13. (c)

$$\omega = 12 + 9t - 3t^2$$

$$\frac{d\omega}{dt} = 9 - 6t = 0$$

$$\Rightarrow \quad t = 1.5\text{s}$$

$$\frac{d^2\omega}{dt^2} = -6 < 0$$

Hence, at $t = 1.5$ sec maximum value of angular velocity will occur

$$\therefore \quad \omega_{max} = 12 + 9 \times 1.5 - 3 \times 1.5^2$$

$$= 12 + 13.5 - 6.75$$

$$= 18.75 \text{ rad/s}$$

14. (c)

Shape	Area	Centroid from base
Square	$A_1 = d^2$	$y_1 = d/2$
Half circle	$A_2 = \pi d^2/8$	$y_2 = 2d/3\pi$

The centroid of hatched position from base.

$$\bar{y} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$$= \frac{d^2 \cdot \frac{d}{2} - \frac{\pi d^2}{8} \cdot \frac{2d}{3\pi}}{d^2 - \frac{\pi d^2}{8}} = \frac{10d}{3(8-\pi)}$$

15. (c)

Let the shortest distance between ships will occur at time thereafter the ship A passes point O.

The distance of ship A from O = $20\,t$

The distance of ship B from $O = 20\,(2 - t)$

The distance between ships

$$D = \sqrt{(20t)^2 + \{20(2-t)\}^2}$$

For shortest distance

$$\frac{dD}{dt} = 0 \quad \text{or} \quad \frac{d(D^2)}{dt} = 0$$

$$2 \times 20t - 20(2-t) \times 2 = 0$$

$$t = 1 \text{ hrs}$$

Shortest distance = $20\sqrt{2}$ km

16. (b)

Rectangle $ABCD$

Area $(A_1) = 8 \times 2 = 16$ cm^2

Centre of gravity (x_1) = 1 cm from AD

Rectangle $EFHK$

Area $(A_2) = 8 \times 1.5 = 12$ cm^2

Centre of gravity (x_2) = 2 + 4 = 6 cm from AD

The centre of gravity of lamina from AD

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = \frac{16 \times 1 + 12 \times 6}{16 + 12}$$

$$= \frac{22}{7} \text{ cm}$$

17. (c)

$$5g(2.1) = \frac{1}{2} \times 5 \times V^2 + \frac{1}{2}k\delta^2 \qquad [\because \; k = 10000 \text{ N/m}]$$

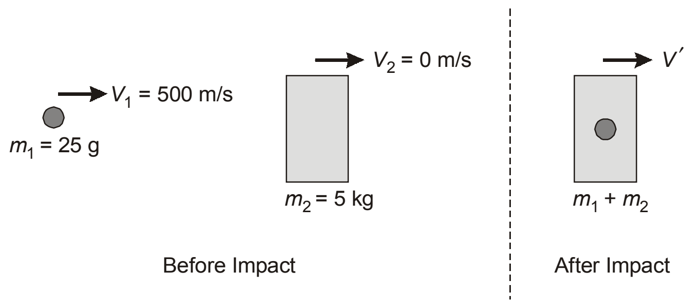
$$\Rightarrow \quad 10.5g = 2.5V^2 + \frac{1}{2} \times 10000 \times (0.1)^2$$

$$\Rightarrow \quad 10.5 \times 9.81 = 2.5\, V^2 + 50$$

$$\Rightarrow \quad V^2 = 21.202$$

$$\therefore \quad V = 4.6 \text{ m/s}$$

18. (d)



$$V' = \frac{0.025 \times 500}{5 + 0.025} = \frac{12.5}{5.025} = 2.488 \text{ m/s}$$

Change in kinetic energy,

$$\Delta KE = \frac{1}{2} \times 0.025 \times 500^2 - \frac{1}{2} \times 5.025 \times 2.488^2$$

$$= 3125 - 15.55 = 3109.45 \text{ J}$$

$$\text{Percentage of energy lost} = \frac{3109.45}{3125} \times 100 = 99.5\%$$

19. (a)

Maximum velocity of a particle $v_{max} = \omega\delta$

Velocity at any position y above the mean position

$$v = \omega\sqrt{\delta^2 - y^2}$$

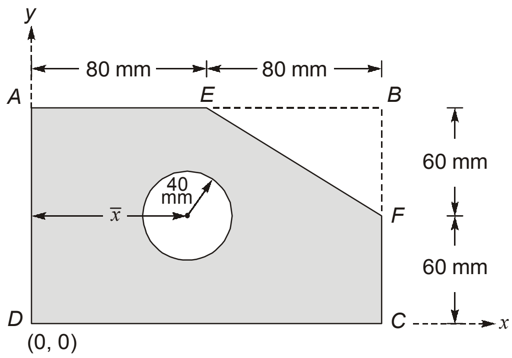
Given $\frac{1}{2} m v_{max}^2 = 2 \times \frac{1}{2} m v^2$

$\therefore \quad \delta_2 = 2(\delta^2 - y^2)$

$$2y^2 = \delta^2$$

$$y = \frac{\delta}{\sqrt{2}}$$

20. (a)



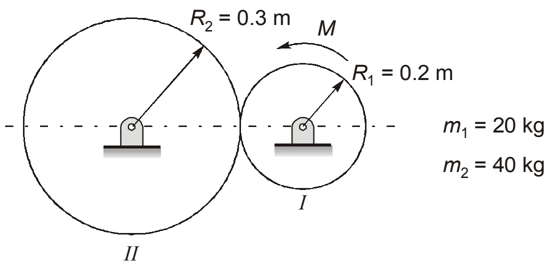
S. No.	Shape	Area (mm^2)	$\bar{x}$ (mm)	$a\bar{x}$ (mm^3)
1	*ABCD*	19200	80	1536000
2	Circle	–5026.55	$\bar{x}$	$-5026.55\bar{x}$
3	ΔEBF	–2400	133.33	–320000
		$\sum a = 11773.45$		$\sum a\bar{x} = 1216000 - 5026.55\bar{x}$

Now, $$\bar{x} = \frac{\sum a\bar{x}}{\sum a}$$

$\Rightarrow$ $$\bar{x} = \frac{1216000 - 5026.55\bar{x}}{11773.45}$$

$\Rightarrow$ $$\bar{x} = 72.38 \text{ mm}$$

21. (a)



Moment of inertia, $$I_2 = \frac{m_1 R_1^2}{2} = \frac{20 \times 0.2^2}{2} = 0.4 \text{ kgm}^2$$

$$I_2 = \frac{m_2 R_2^2}{2} = \frac{40 \times 0.3^2}{2} = 1.8 \text{ kgm}^2$$

A force of friction F acts between disc I and II which drives disc II.

$$F \times R_2 = I_2\alpha_2 \qquad ...(1)$$

$$R_1\alpha_1 = R_2\alpha_2$$

$$\Rightarrow \quad 0.2 \times 8.33 = 0.3 \times \alpha_2$$

$$\alpha_2 = 5.55 \text{ m/s}^2$$

Put α_2 value in (1)

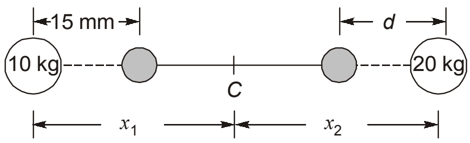
We get $F = 33.32$ N

$$M - FR_1 = I_1\alpha_1$$

$$\Rightarrow \quad M - 33.32 \times 0.2 = 0.4 \times 8.33$$

$$M = 9.996 \simeq 10 \text{ Nm}$$

22. (c)



To keep centre of mass at C

$$m_1x_1 = m_2x_2 \quad \rightarrow \quad \text{(Let 10 kg} = m_1\text{, 20 kg} = m_2)$$

and $m_1(x_1 - 15) = m_2(x_2 - d)$

$$15\,m_1 = m_2 d$$

$$d = \frac{15 \times 10}{20} = 7.5 \text{ mm}$$

23. (b)

From geometry it is clear that the:

Angle between R and T is $(\alpha + \beta)$

Angle between R and W is $(180 - \alpha)$

Angle between T and W is $(180 - \beta)$

Using Lami's theorem

$$\frac{W}{\sin(\alpha+\beta)} = \frac{T}{\sin(180-\alpha)} = \frac{R}{\sin(180-\beta)}$$

$$\therefore \frac{W}{\sin(\alpha+\beta)} = \frac{T}{\sin\alpha} = \frac{R}{\sin\beta}$$

24. (a)

Since no external torque has acted, angular momentum will be conserved.

Applying conservation of angular momentum,

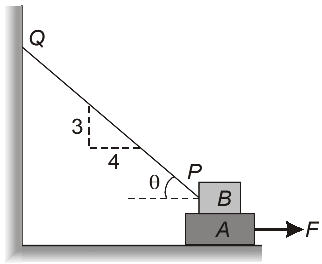
$$\therefore \quad I\omega = I'\omega'$$

$$MR^2 \times \omega = (MR^2 + 2mR^2)\omega'$$

$$5 \times (0.2)^2 \times 10 = [5 \times (0.2)^2 + 2 \times 0.5 \times (0.2)^2]\omega'$$

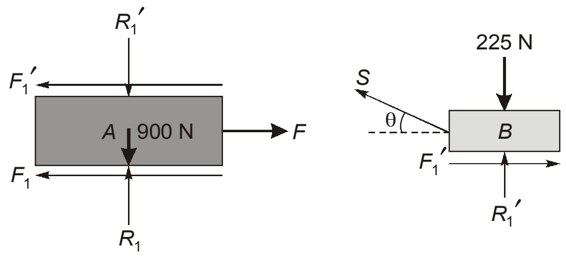
$$\Rightarrow \quad \omega' = 8.333 \text{ rad s}^{-1}$$

25. (a)



$$\tan\theta = \frac{3}{4}$$

The free body diagrams of the blocks are shown below.



$$F_1 = \mu R_1 \text{ and } F_1' = \mu R_1' \quad \text{...(i)}$$

From equilibrium of block A,

$$F - F_1 - F_1' = 0 \quad \text{...(ii)}$$

and

$$R_1 - W_1 - R_1' = 0 \quad \text{...(iii)}$$

$$\Rightarrow \quad R_1 = \frac{F_1}{\mu} = W_1 + \frac{F_1'}{\mu} \quad \text{...(iv)}$$

From the equilibrium of block B,

$$F_1' - S\cos\theta = 0 \quad \text{...(v)}$$

and

$$R_1' + S\sin\theta - W_2 = 0 \quad \text{...(vi)}$$

$$\Rightarrow \quad F_1' = \frac{W_2}{1/\mu + \tan\theta} \quad \text{...(vii)}$$

From equations (ii), (iv) and (vii), we get

$$F = \mu W_1 + \frac{2W_2}{\frac{1}{\mu} + \tan\theta} = 0.3 \times 900 + \frac{2 \times 225}{\frac{1}{0.3} + \frac{3}{4}} = 380.2\text{N}$$

26. (a)

$$x = 10 \sin 2t + 15 \cos 2t + 100$$

$$v = \frac{dx}{dt} = 20 \cos 2t - 30 \sin 2t$$

$$a = \frac{dv}{dt} = -40 \sin 2t - 60 \cos 2t \quad \text{...(i)}$$

For a_{max}, $\frac{da}{dt} = 0$

$\Rightarrow$ $-80 \cos 2t + 120 \sin 2t = 0$

$$\tan 2t = \frac{2}{3}$$

$\Rightarrow$ $2t = 33.69$

Now using equation (i), we get

$$a_{max} = -40 \sin (33.69) - 60 \times \cos (33.69) = -72.11 \text{ mm/s}^2$$

27. (d)

$$I = -2\hat{i} - \hat{j} + \hat{k}$$

$$r = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\text{Angular momentum} = H = r \times I$$

$$= \left(2\hat{i} - 3\hat{j} + \hat{k}\right) \times \left(-2\hat{i} - \hat{j} + \hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

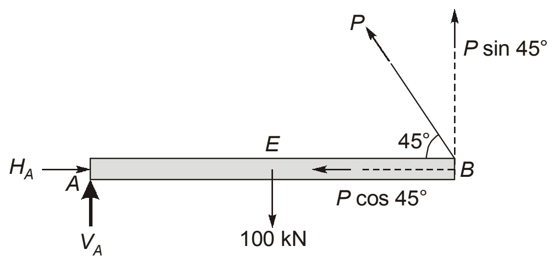
$$= \hat{i}(-3+2) - \hat{j}(2+4) + \hat{k}(-2-6)$$

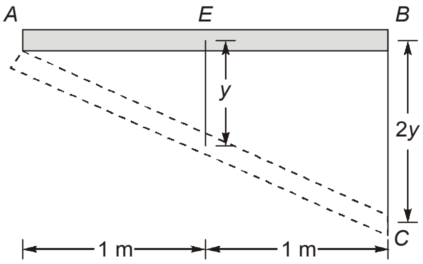
$$= \hat{i}(-1) - 6\hat{j} - 8\hat{k} = -\hat{i} - 6\hat{j} - 8\hat{k}$$

$$|H| = \sqrt{1^2 + 6^2 + 8^2} = 10.01 \text{ kg m}^2\text{/s} \simeq 10 \text{ kg m}^2\text{/s}$$

28. (c)

Free body diagram of beam *AB*,



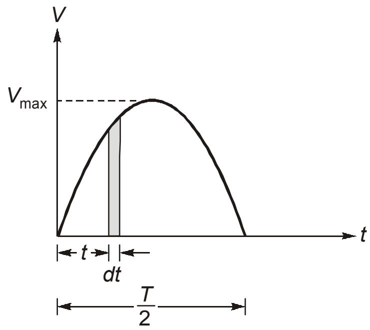


Now using the principle of virtual work done, if C.G. of beam AB shifts by an amount 'y' then end B must shift by '$2y$' (using similar triangles).

$$\therefore \quad 100 \times y - P\sin 45° \times 2y = 0$$

$$\Rightarrow \quad P = 70.71 \text{ kN}$$

29. (a)



Velocity at any instant, $V = V_{max}\sin\left(\frac{2\pi t}{T}\right)$

Consider the distance travelled through a small interval dt

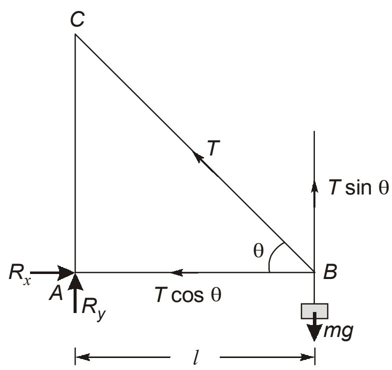
$$dS = vdt = V_{max}\sin\left(\frac{2\pi t}{T}\right)dt$$

$$\Rightarrow \quad S = \int_0^{T/2} V_{max}\sin\left(\frac{2\pi t}{T}\right)dt$$

$$= V_{max}\frac{T}{2\pi}\left|-\cos\left(\frac{2\pi t}{T}\right)\right|_0^{T/2}$$

$$= V_{max}\frac{T}{\pi}$$

30. (c)



$$T\sin\theta + R_y = mg$$

$$T\cos\theta = R_x$$

Now, $\tan\theta = \frac{125}{275}$

$$\theta = 24.44°$$

Taking moments about A,

$$l \times T\sin\theta = l \times mg$$

$$\Rightarrow \quad T = \frac{35 \times 9.81}{\sin 24.44°} = 829.87 \text{ N}$$

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