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FLUID MECHANICS

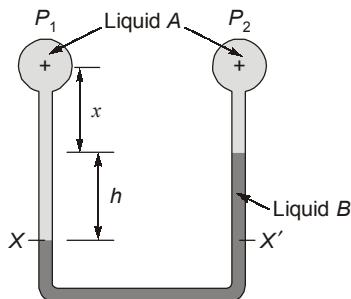
MECHANICAL ENGINEERING

Date of Test : 08/06/2022**ANSWER KEY ➤**

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (c) | 7. (c) | 13. (c) | 19. (b) | 25. (b) |
| 2. (b) | 8. (b) | 14. (d) | 20. (d) | 26. (c) |
| 3. (a) | 9. (b) | 15. (c) | 21. (a) | 27. (b) |
| 4. (c) | 10. (d) | 16. (b) | 22. (a) | 28. (c) |
| 5. (c) | 11. (b) | 17. (d) | 23. (a) | 29. (a) |
| 6. (a) | 12. (d) | 18. (a) | 24. (a) | 30. (c) |

DETAILED EXPLANATIONS

1. (c)



Taking point X and X' and equating the pressure on both sides

$$\begin{aligned}
 P_1 + (h+x) \times 0.88 \times 9.81 \times 10^3 &= P_2 + x \times 0.88 \times 9.81 \times 10^3 + h \times 2.95 \times 9.81 \times 10^3 \\
 \Rightarrow P_1 + h \times 0.88 \times 9.81 \times 10^3 + x \times 0.88 \times 9.81 &= x \times 0.88 \times 9.81 + P_2 + h \times 2.95 \times 9.81 \times 10^3 \\
 \Rightarrow P_1 - P_2 &= h(2.95 \times 9.81 - 0.88 \times 9.81) \times 10^3 \\
 860 &= h(2.3067 \times 10^3) \\
 42.35 \times 10^{-3} &= h \\
 h &= 42.35 \text{ mm}
 \end{aligned}$$

2. (b)

For geometrically similar model and prototype

$$\left(\frac{P}{N^3 D^5}\right)_{\text{model}} = \left(\frac{P}{N^3 D^5}\right)_{\text{prototype}}$$

Given,

$$\begin{aligned}
 N_m &= N_p \\
 \Rightarrow \frac{P_m}{N_m^3 D_m^5} &= \frac{P_p}{N_p^3 D_p^5} \\
 \frac{P_m}{P_p} &= \frac{N_m^3 D_m^5}{N_p^3 D_p^5} \\
 \frac{P_m}{P_p} &= \frac{2^3 N_p^3}{N_p^3} \times \frac{D_m^5}{16^5 D_m^5} \\
 P_m &= \frac{10 \times 10^6 \times 2^3}{16^5} W = 76.29 \text{ W}
 \end{aligned}$$

3. (a)

$$\begin{aligned}
 P_A - H \times 9.81 \times 1 - 0.18 \times 9.81 \times 0.827 &= P_B - 13.6 \times 9.81 \times (H + 0.53) \\
 - H \times 9.81 - 1.4603 &= 97 - 13.6 \times 9.81 \times H - 13.6 \times 9.81 \times 0.53 \\
 \Rightarrow H &= 0.2245 \text{ m} \\
 \therefore H &= 22.45 \text{ cm}
 \end{aligned}$$

4. (c)

$$\therefore \bar{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} d^2} \Rightarrow \frac{4Q}{\pi d^2} = \frac{4 \times 880 \times 10^{-9}}{\pi \times 0.50^2 \times 10^{-6}} = 4.48 \text{ m/s}$$

We know,

$$Q = \frac{\pi \Delta p D^4}{128 \mu L}$$

$$\Rightarrow \mu = \frac{\pi \Delta p D^4}{128 Q L} = \frac{\pi \times 10^6 \times (0.5)^4 \times 10^{-12}}{128 \times 880 \times 10^{-9} \times 1}$$

$$\mu = 1.74 \times 10^{-3}$$

5. (c)

$$Re_{\text{critical}} = 2000 = \frac{VD}{\nu} = \frac{\rho VD}{\mu}$$

$$2000 = \frac{950 \times V \times 0.15}{8 \times 10^{-2}}$$

$$V = 1.123 \text{ m/s}$$

$$\text{Head loss} = \frac{32 \mu VL}{\rho g D^2} = \frac{32 \times 8 \times 10^{-2} \times 1.123 \times 300}{950 \times 9.81 \times 0.15^2}$$

$$= 4.113. \Rightarrow \text{Maximum difference in oil elevations.}$$

6. (a)

Shear stress at boundary

$$\tau_0 = \left(\frac{-\partial P}{\partial x} \right) \left(\frac{B}{2} \right)$$

$$\text{Here, } B = 2 \times 1.5 \times 10^{-3} = 3 \times 10^{-3} \text{ m}$$

$$\tau_0 = 1.25 \times 10^3 \times 1.5 \times 10^{-3}$$

$$\tau_0 = 1.875 \text{ Pa}$$

$$\bar{U} = \left(\frac{-\partial P}{\partial x} \right) \times \frac{B^2}{12 \mu} = 1.25 \times 10^3 \times \frac{(2 \times 1.5 \times 10^{-3})^2}{12 \times 0.5}$$

$$\bar{U} = 1.875 \times 10^{-3}$$

$$\frac{Q}{W} = \bar{U} \times B = 1.875 \times 10^{-3} \times 3 \times 10^{-3} = 5.625 \times 10^{-6} \text{ m}^2/\text{s}$$

7. (c)

8. (b)

$$\text{Sensitivity} = \frac{1}{\sin \theta} = \frac{1}{\sin 30^\circ} = 2$$

9. (b)

Surface tension is due to cohesion between liquid particles at the surface.

10. (d)

$$P = \rho_{Hg} \times g \times H$$

$$6.8 \times 10^4 = 13.6 \times 10^3 \times 9.81 \times H$$

$$H_{Hg} = 0.5096 \text{ m}$$

$$H_{water} = \frac{13.6}{1} \times 0.5096$$

$$H_{water} = 6.931 \text{ m}$$

11. (b)

For just equilibrium condition,

$$\dot{m}[V \cos \theta] = \mu Mg$$

$$1000 \times \pi \times 0.25 \times 0.05^2 \times V^2 \times 0.5 = 0.55 \times M \times 9.81$$

$$1000 \times \pi \times 0.25 \times 0.05^2 \times 2 \times 9.81 \times 2 \times 0.5 = 0.55 \times M \times 9.81$$

$$\Rightarrow = 7.1399 \text{ kg}$$

12. (d)

$$\text{Volume of cube} = a^3$$

$$a^3 = 125 \times 10^{-3} \times 10^{-3} \text{ m}^3$$

$$\Rightarrow a = 5 \times 10^{-2}$$

$$\Rightarrow a = 0.05 \text{ m}$$

$$F = p \times A$$

$$\begin{aligned} P_{bottom} &= p_{atm} + h_1 g \rho_{oil} + h_2 \rho_{water} g \\ &= 101325 + 0.5 \times 0.8 \times 1000 \times 9.81 + 0.3 \times 1000 \times 9.81 \end{aligned}$$

$$P_{bottom} = 108192$$

$$F = P_{bottom} \times A = 108192 \times 0.05^2 = 270.48 \text{ N}$$

$$T = \text{Upthrust} - W$$

$$\begin{aligned} &= 125 \times 10^{-6} \times 1000 \times 9.81 - 125 \times 10^{-6} \times 0.77 \times 1000 \times 9.81 \\ &= 0.282 \text{ N} \end{aligned}$$

13. (c)

$$u = \frac{\partial \psi}{\partial y}$$

$$u = 2x^2 + (x + t) 2y$$

$$\therefore \text{for face } OB, \quad x \Rightarrow 0$$

$$u_{OB} = 2ty$$

Discharge through AB

$$\therefore Q_{AB} = \int_0^2 u_{OB} \cdot 5 dy = \int_0^2 2ty \cdot 5 dy$$

At $t = 1$

$$Q_{OB} = 20 \text{ units}$$

$$\therefore V = -\frac{\partial \psi}{\partial x} = -[4xy + y^2]$$

At $y = 0$

$$V = 0$$

$$\therefore Q_{AO} = 0$$

$$\begin{aligned} \therefore Q_{AB} &= Q_{OB} + Q_{OA} \\ &= 20 + 0 \\ &= 20 \text{ units} \end{aligned}$$

14. (d)

$$V_2 = \frac{Q}{A_2} = \frac{1.13 \times 10^{-6}}{\frac{\pi}{4} \times (0.0012)^2} \simeq 1 \text{ m/s}$$

$$\frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f$$

$$h_f = z_1 - z_2 - \frac{\alpha_2 V_2^2}{2g}$$

$$\Rightarrow h_f = 0.6 - 0 - \frac{(2)(1)^2}{2 \times 9.81} = 0.5 \text{ m}$$

$$h_f = \frac{32 \mu VL}{\rho g D^2}$$

$$\Rightarrow 0.5 = \frac{32 \times \mu \times 0.3 \times 1}{9000 \times 0.0012}$$

$$\Rightarrow \mu = 6.75 \times 10^{-4} \text{ Pa-s}$$

15. (c)

$$\mu = 0.97 \text{ poise} = 0.097 \text{ Ns/m}^2$$

$$\rho = 0.9 \times 998 = 898.2 \text{ kg/m}^3$$

$$\dot{m} = \text{mass rate of flow} = \frac{100}{30} = 3.333 \text{ kg/s}$$

Q = Volume rate of flow

$$= \frac{3.333}{988.2}$$

$$\Rightarrow 3.711 \times 10^{-3} \text{ m}^3/\text{s} = 3.711 \text{ L/s}$$

$$\text{Area of flow} = A = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2.$$

$$\bar{U} = \frac{3.711 \times 10^{-3}}{0.007854} = 0.4725 \text{ m/s}$$

$$-\Delta P = \frac{32\mu \bar{U}L}{D^2} = \frac{32 \times 0.097 \times 0.4725 \times 10.0}{(0.1)^2}$$

$$-\Delta P = 1467 \text{ Pa}$$

16. (b)

$$Re_L = \frac{UL}{v} = \frac{1.75 \times 5}{1.475 \times 10^{-5}} = 5.932 \times 10^5$$

$$C_f = \frac{0.074}{Re_L^{1/5}} = \frac{0.074}{(5.932 \times 10^5)^{1/5}} = 5.183 \times 10^{-3}$$

Drag force on one side of the plate,

$$F_d = C_f \times \text{area} \times \frac{1}{2} \rho U^2$$

$$= 5.183 \times 10^{-3} \times (1.8 \times 5) \times 1.22 \times \frac{(1.75)^2}{2}$$

$$F_d = 0.0871 \text{ N}$$

17. (d)

∴ Continuity equation holds,

$$\therefore \frac{\pi}{4} \times (5)^2 \times 2 = \frac{\pi}{4} \times 3^2 \times x$$

$$x = 5.55 \text{ m/s}$$

Mars flow rate

$$\Rightarrow \dot{m} = \int_{\dot{m}} A_1 V_1 = 100 \times \frac{\pi}{4} \times 0.05^2 \times 2 = 3.9269 \text{ kg/s}$$

Let f_x and f_y be the force in Right and vertically upward diversion respectively to hold the box in position.
 \therefore Now, $\Sigma f_x = 0$ [Box is stationary after applying force]

$$-\dot{m} \times V_1 \cos 65^\circ + f_x = -\dot{m} \times V_2 \cos 0^\circ$$

$$-3.9269 \times 2 \times \cos 65^\circ + f_x = -3.9269 \times 5.55 \times 1$$

$$f_x = -18.475 \text{ N.}$$

f_x must be in left as f_x comes out to be negative.

Similarly for vertical direction $\Sigma f_y = 0$

$$f_y - 3.9269 \times 2 \times \sin 65^\circ = 0$$

$$f_y = 7.11 \text{ N}$$

∴ It is towards vertically upward direction.

18. (a)

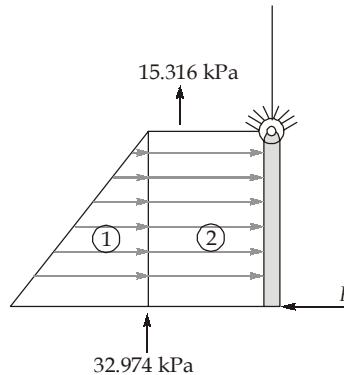
Gauge pressure at the upper end of the gate

$$= 80 + 0.9 \times 9.81 \times 4 - 100$$

$$= 15.316 \text{ kPa}$$

Gauge pressure at the lower end of the gate

$$= 80 + 0.9 \times 9.81 \times 6 - 100 = 32.974 \text{ kPa}$$



Using pressure diagram

For equilibrium,

Net moment about the hinge = 0

$$M_1 + M_2 = F \times 2$$

$$\frac{1}{2} \times (32.974 - 15.316) \times 2 \times 4 \times \left(\frac{2}{3} \times 2 \right) + 15.316 \times 2 \times 4 \times \frac{2}{2} = F \times 2$$

On solving

$$F = 108.352 \text{ kN}$$

19. (b)

$$H_m = H + \frac{fLV_d^2}{2gd} + 1.3 \times \frac{V_d^2}{2g} + \frac{V_d^2}{2g}$$

H_m = Net head required

$$H_m = 30 + \frac{4f'LV_d^2}{2gd} + 1.3 \times \frac{V_d^2}{2g} + \frac{V_d^2}{2g}$$

$$f' = 0.02 \quad A_d V_d = 50 \times 10^{-3} \text{ m}^3/\text{s}$$

$$d = 200 \text{ mm}$$

$$\frac{\pi}{4} \times 0.2^2 \times V_d = 50 \times 10^{-3}$$

$$V_d = 1.59 \text{ m/s}$$

$$\Rightarrow H_m = 30 + \frac{4 \times 0.02 \times 100 \times 1.59^2}{2 \times 9.81 \times 0.2} + \frac{1.3 \times 1.59^2}{2 \times 9.81} + \frac{1.59^2}{2 \times 9.81}$$

$$= 35.45 \text{ m}$$

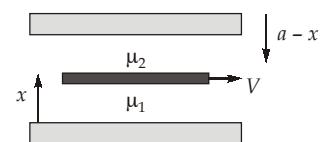
$$\Rightarrow \text{Power} = \rho Q g H_m = 17388.225 \text{ W} = 17.388 \text{ kW}$$

20. (d)

τ_1 = shear stress at bottom

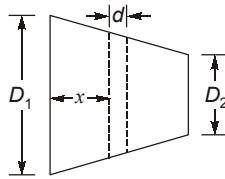
$$= \mu_1 \times \frac{V}{x}$$

$$\tau_2 = \text{shear stress at top} = \mu_2 \frac{V}{a-x}$$



$$\begin{aligned}
 \text{drag force} &= (\tau_1 + \tau_2) \times A = F_D \\
 &= F_D = A \times \left[\frac{\mu_1 V}{x} + \frac{\mu_2 V}{a-x} \right] \\
 \frac{dF_D}{dx} &= 0 = \frac{-\mu_1 V}{x^2} + \frac{\mu_2 V}{(a-x)^2} \Rightarrow \frac{\mu_1}{x^2} = \frac{\mu_2}{(a-x)^2} \\
 a-x &= \sqrt{\frac{\mu_2}{\mu_1}} x \\
 \Rightarrow \frac{a\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}} &= x
 \end{aligned}$$

21. (a)



$$\therefore D = D_1 - \frac{(D_1 - D_2)x}{L}$$

By Hagen-Poiseuille equation,

$$Q = \frac{\pi D^4}{128\mu} \frac{\partial P}{\partial x}$$

For a differential length dx , the differential pressure drop is,

$$\begin{aligned}
 d(\Delta_p) &= \frac{128\mu Q}{\pi D^4} dx \\
 \therefore dD &= -(D_1 - D_2) dx / L \\
 \therefore d(\Delta_p) &= \frac{128\mu Q - L}{\pi(D_1 - D_2)} \times \frac{1}{D^4} dD
 \end{aligned}$$

Integrating both sides to determine the pressure drop across the conical contraction,

$$\begin{aligned}
 \Delta_p &= \frac{-128\mu Q}{3x} \times \frac{-L}{(D_1 - D_2)} \left[\frac{1}{D^4} \right]_{D_1}^{D_2} \\
 \Rightarrow \Delta_p &= \frac{-128\mu Q}{3x} \times \frac{(D_1^2 + D_1 D_2 + D_2^2)}{(D_1^3 - D_2^3)}
 \end{aligned}$$

22. (a)

$$\begin{aligned}
 (1) \quad u &= x^2 \cos y \\
 v &= -2x \sin y
 \end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial u}{\partial x}(x^2 \cos y) + \frac{\partial}{\partial y}(-2x \sin y)\end{aligned}$$

$$(2x)\cos y - (2x)\cos y = 0$$

hence satisfy the continuity equation.

$$\begin{aligned}(2) \quad u &= x + 2 \\ v &= 1 - y\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}(x+2) + \frac{\partial}{\partial y}(1-y) = 1 - 1 = 0\end{aligned}$$

hence satisfy the continuity equation.

$$\begin{aligned}(3) \quad u &= xyt \\ v &= x^3 - y^2 \frac{t}{2}\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}(xyt) + \frac{\partial}{\partial y}\left(x^3 - \frac{y^2 t}{2}\right) = yt - yt = 0 \\ (4) \quad u &= \ln(x+y) \\ v &= xy - \frac{y}{x}\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}\ln(x+y) + \frac{\partial}{\partial y}\left(xy - \frac{y}{x}\right) = \frac{1}{x+y} + x - \frac{1}{x} \neq 0\end{aligned}$$

Hence, does not satisfy continuity equation.

23. (a)

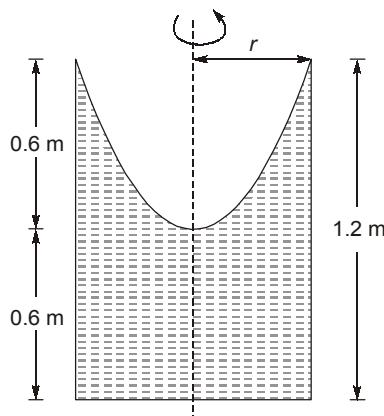
$$\text{Pressure head} = \frac{p}{\rho g} = \frac{19.62 \times 10^3}{1000 \times 9.81 \times 0.8} = 2.5 \text{ m of oil}$$

$$\text{Velocity head} = \frac{V^2}{2g} = \frac{Q^2}{2A^2g} = \frac{(0.12)^2}{2 \times \left(\frac{\pi}{4} \times 0.25^2\right)^2 \times 9.81} = 0.3 \text{ m of oil}$$

$$\text{Datum head} = 2.7 \text{ m}$$

$$\begin{aligned}\text{Total head} &= \text{Pressure head} + \text{Velocity head} + \text{Datum head} \\ &= 2.5 + 0.3 + 2.7 = 5.5 \text{ m}\end{aligned}$$

24. (a)



Original volume of

$$\text{Cylinder} = \pi r^2 h$$

$$V_1 = \pi r^2 \times 1.2$$

Volume of liquid spilled out

$$= \frac{1}{2} \pi r^2 \times h$$

$$V_2 = \frac{1}{2} \pi r^2 \times 0.6$$

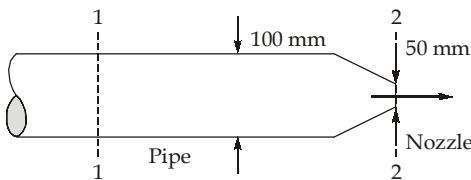
$$\therefore \frac{V_2}{V_1} = \frac{\frac{1}{2} \times 0.6 \pi r^2}{\pi r^2 \times 1.2} = \frac{1}{4}$$

25. (b)

$$V_1 A_1 = V_2 A_2$$

$$5 \times \frac{\pi}{4} (0.1)^2 = V_2 \times \frac{\pi}{4} (0.05)^2$$

$$\Rightarrow V_2 = 20 \text{ m/s}$$



From force balance in x-diagram

$$P_1 A_1 + R_x - P_2 A_2 = \rho Q (V_2 - V_1)$$

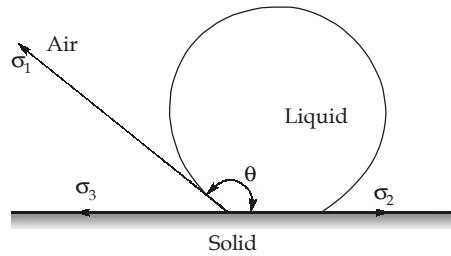
$$\Rightarrow R_x = \rho Q (V_2 - V_1) + P_2 A_2 - P_1 A_1$$

$$= 1000 \times \frac{\pi}{4} (0.1)^2 \times 5 (20 - 5) + 100 \times 10^3 \times \frac{\pi}{4} (0.05)^2 - 500 \times 10^3 \times \frac{\pi}{4} (0.1)^2$$

Force required to hold the nozzle, $R_x = -3141.6 \text{ N}$

26. (c)

From force balance at point of contact,



$$\sigma_1 \cos(180 - \theta) + \sigma_3 = \sigma_2$$

$$\text{or } \cos(180 - \theta) = \frac{\sigma_2 - \sigma_3}{\sigma_1} = -\cos\theta$$

∴

$$\sigma_1 = 0.0720 \text{ N/m} \quad (\text{liquid and air})$$

$$\sigma_2 = 0.0418 \text{ N/m} \quad (\text{liquid and solid})$$

$$\sigma_3 = 0.0008 \text{ N/m} \quad (\text{air and solid})$$

$$\cos\theta = \frac{0.0008 - 0.0418}{0.072} = -0.56944$$

$$\theta = 124.7^\circ$$

27. (b)

$$\therefore Re = \frac{16}{f} = \frac{16}{0.04} = 400$$

∴ The flow is viscous.

The shear stress in case of viscous flow through a pipe is given by

$$\tau = \frac{-\partial p}{\partial x} \left(\frac{r}{2} \right)$$

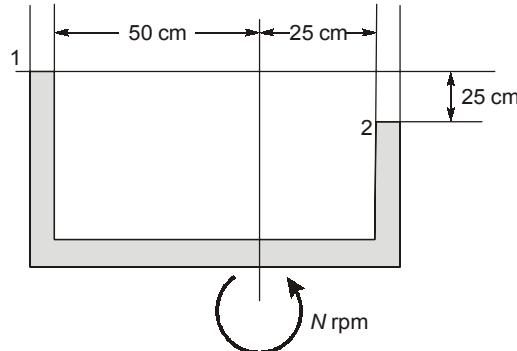
∴ $\frac{\partial p}{\partial x}$ is constant across a section.

$$\therefore \tau \propto r$$

$$\therefore \frac{\tau}{r} = \frac{\tau_0}{R} = \frac{0.00981}{40} = \frac{\tau_0}{100}$$

$$\therefore \tau_0 = \frac{100 \times 0.00981}{40} = 0.0245 \text{ N/cm}^2$$

28. (c)



$$\begin{aligned} \frac{\overset{0}{P_1}}{\rho g} - \frac{V_1^2}{2g} + Z_1 &= \frac{\overset{0}{P_2}}{\rho g} - \frac{V_2^2}{2g} + Z_2 \\ Z_1 - Z_2 &= \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \\ 0.25 &= \frac{1}{2g} \left\{ r_1^2 \omega^2 - r_2^2 \omega^2 \right\} \\ 0.25 \times 2 \times 9.81 &= \omega^2 \left\{ \left(\frac{50}{100} \right)^2 - \left(\frac{25}{100} \right)^2 \right\} \\ \omega &= 5.115 \text{ rad/s} \\ N &= \frac{60\omega}{2\pi} = \frac{60 \times 5.115}{2 \times \pi} \\ &= 48.8 \text{ rpm} \end{aligned}$$

29. (a)

$$\begin{aligned} \text{Re}_L &\leq 2000 \\ \Rightarrow \frac{\rho V D}{\mu} &\leq 2000 \\ \Rightarrow \frac{(0.92 \times 1000) \times \frac{Q}{A} \times (10 \times 10^{-2})}{0.9 \times 10^{-1}} &\leq 2000 \\ \Rightarrow Q &\leq \frac{2000 \times 0.9 \times 10^{-1} \times A}{(0.92 \times 1000) \times (10 \times 10^{-2})} \\ \Rightarrow Q &\leq 1.95652 \times \frac{\pi}{4} \times \left\{ \frac{10}{100} \right\}^2 \\ \Rightarrow Q &\leq 0.015366 \text{ m}^3/\text{s} \\ \Rightarrow Q &\leq 15.36 \simeq 15.4 \text{ L/s} \end{aligned}$$

30. (c)

$$\begin{aligned} f &= \frac{64}{\text{Re}} \\ \text{Re} &= \frac{UD}{v} = \frac{0.1 \times 0.1}{10^{-5}} = 1000 \\ f &= \frac{64}{1000} = 0.064 \end{aligned}$$

