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FLUID MECHANICS

CIVIL ENGINEERING

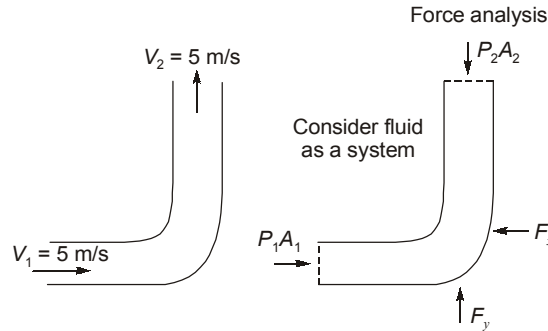
Date of Test : 08/06/2022

ANSWER KEY > Fluid Mechanics

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (d) | 7. (b) | 13. (a) | 19. (c) | 25. (b) |
| 2. (c) | 8. (b) | 14. (d) | 20. (c) | 26. (c) |
| 3. (b) | 9. (c) | 15. (b) | 21. (d) | 27. (d) |
| 4. (a) | 10. (b) | 16. (b) | 22. (c) | 28. (a) |
| 5. (d) | 11. (a) | 17. (c) | 23. (d) | 29. (b) |
| 6. (c) | 12. (b) | 18. (c) | 24. (a) | 30. (c) |

DETAILED EXPLANATIONS

1. (d)



$$P_1 = P_2 = 4000 \text{ Pa}$$

$$D_1 = D_2 = 30 \text{ cm}$$

F_x, F_y = Force exerted on the fluid

Momentum eq. in y-direction.

$$F_y - P_2 A_2 = \dot{m} V_2 - \dot{m}(0)$$

$$F_y = \dot{m} V_2 + P_2 A_2$$

$$= (\rho A_2 V_2) V_2 + P_2 A_2$$

$$= [(1000)(5)^2] + 4000 \left[\frac{\pi}{4} (0.3)^2 \right]$$

$$= 2.05 \text{ kN}$$

2. (c)

Whirlpool is an example of free vortex flow.

So, $v \propto \frac{1}{r}$ i.e. $vr = \text{constant}$

Now, $v_1 = 10 \text{ m/s}, r_1 = 20 \text{ cm}$

When $r_2 = 50 \text{ cm}$

$$v_2 = \frac{v_1 r_1}{r_2} = \frac{10 \times 20}{50} = 4 \text{ m/s}$$

According to fundamental equation of vortex flow,

$$dP = \frac{\rho v^2 dr}{r} - \rho g dz \left\{ v = \frac{c}{r} \right\}$$

$$0 = \frac{\rho C^2}{r^2} dr - \rho g dz$$

$$dz = \frac{\rho C^2}{r^3 g} dr$$

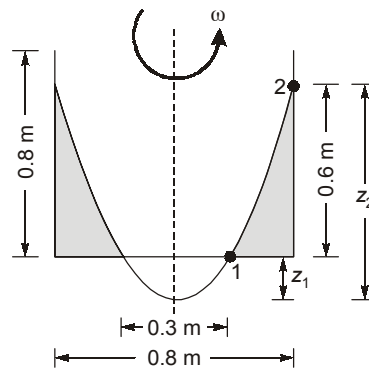
Int. it

$$z = \frac{c^2}{g} \times -\frac{1}{2} \times \left[\frac{1}{r^2} \right]_{0.5}^{\infty}$$

$$[c = vr = 0.2 \times 10]$$

$$z = 0.816 \text{ m}$$

3. (b)



$$z_2 - z_1 = \frac{\omega^2}{2g} [R_2^2 - R_1^2]$$

$$0.6 = \frac{\omega^2}{2g} [0.4^2 - 0.15^2]$$

$$\omega = 9.253 \text{ rad/sec}$$

$$\frac{2\pi N}{60} = 9.253$$

$$N = 88.36 \text{ rpm}$$

4. (a)

$$\begin{aligned} \text{Shear stress, } \tau &= \mu \frac{du}{dy} = 0.44 \times \frac{4}{0.018} \\ &= 97.8 \text{ Pa} \end{aligned}$$

5. (d)

In Navier's-Stoke equation viscous force term is considered and in all the above mentioned flow viscous force can't be neglected.

6. (c)

$$K = 2.1 \times 10^9 \text{ Pa; } E = 2.1 \times 10^{11} \text{ Pa}$$

$$\rho = 1000 \text{ kg/m}^3; \quad D = 400 \text{ mm}$$

$$t = 4 \text{ mm}$$

Velocity of propagation of water hammer pressure

$$= \sqrt{\frac{K/\rho}{1 + \frac{KD}{Et}}} = \sqrt{\frac{2.1 \times \frac{10^9}{1000}}{1 + 1}} = 1024.7 \text{ m/s}$$

7. (b)

In a turbulent boundary layer, near the boundary large velocity change occurs in a relatively small vertical distance, and hence at the boundary the velocity gradient $\left(\frac{du}{dy}\right)$ is steeper in turbulent boundary layer than in a laminar boundary layer. The velocity distribution in a turbulent boundary layer follows a logarithm law, which can also be represented by a power law of the type.

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^n$$

The value of the exponent n is approximately (1/7) for the moderate Reynolds number.

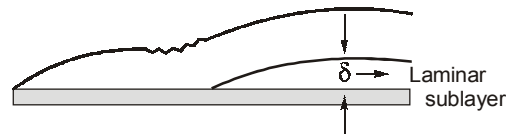
8. (b)

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(1 - \left(\frac{y}{\delta}\right)^{1/6}\right) dy$$

$$\delta^* = \left[y - \frac{6y^{7/6}}{7\delta^{1/6}} \right]_0^\delta = \delta - \frac{6\delta}{7} = \frac{\delta}{7}$$

$$\therefore \frac{\delta^*}{\delta} = \frac{1}{7}$$

9. (c)



11. (a)

For parallel connection in pipe,

Total discharge, $Q = 250 \text{ lt/s} = 0.25 \text{ m}^3/\text{s}$

$$Q = Q_1 + Q_2$$

$$Q_1 + Q_2 = 0.25 \text{ m}^3/\text{s} \quad \dots \text{ (i)}$$

For parallel connection head loss in both pipes will be same so,

$$h_{f1} = h_{f2}$$

$$\frac{fL_1 Q_1^2}{12.1 \times d_1^5} = \frac{fL_2 Q_2^2}{12.1 \times d_2^5} \quad [\because L_1 = L_2]$$

$$\frac{Q_1^2}{0.8^5} = \frac{Q_2^2}{0.6^5}$$

$$\frac{Q_1}{Q_2} = \left(\frac{0.8}{0.6}\right)^{5/2} = 2.05$$

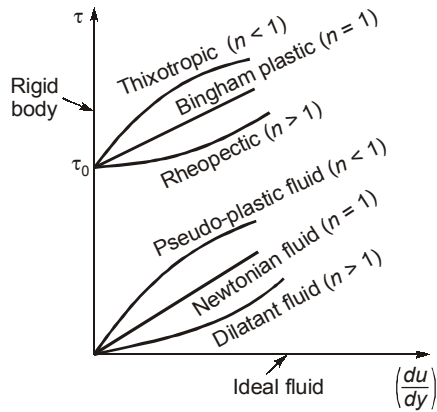
$$Q_1 = 2.05 Q_2 \quad \dots \text{ (ii)}$$

By equations (i) and (ii),

$$2.05Q_2 + Q_2 = 0.25, \quad Q_2 = 0.0814 \text{ m}^3/\text{s}$$

$$Q_1 = 0.25 - 0.082 = 0.168 \text{ m}^3/\text{s}$$

12. (b)



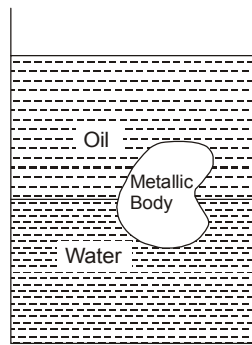
13. (a)

Let V = Volume of metallic body

45% of volume is in oil

i.e., $V_{oil} = 45\% \text{ of } V = 0.45V$

and $V_{water} = 0.55V$



For equilibrium condition,

$$\begin{aligned} \text{Net buoyant force} &= (F_B)_w + (F_B)_{oil} \\ &= \text{Weight of body} \end{aligned}$$

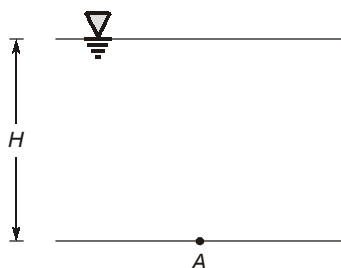
$$M_b g = \rho_w V_w g + \rho_{oil} V_{oil} g$$

$$M_b = \rho_w V_w + \rho_{oil} V_{oil}$$

$$\rho_b V = \rho_w \times 0.55V + \rho_{oil} \times 0.45V$$

$$\begin{aligned} \text{or } \rho_b &= \rho_w \times 0.55 + \rho_{oil} \times 0.45 \\ &= 1000 \times 0.55 + 700 \times 0.45 \\ &= 550 + 315 = 865 \text{ kg/m}^3 \end{aligned}$$

15. (b)



$$\begin{aligned}
 P_A &= 4.2 \text{ MPa} \quad [\text{Absolute pressure}] \\
 P_{\text{atm}} &= 101 \text{ kPa} \\
 \rho &= 1050 \text{ kg/m}^3 \\
 g &= 9.8 \text{ m/s}^2 \\
 P_A &= P_{\text{atm}} + \rho g H \\
 4.2 \times 10^6 &= (101 \times 10^3) + [1050 \times 9.81 \times H] \\
 H &= 397.94 \text{ or } 398 \text{ m}
 \end{aligned}$$

16. (b)

For 2D-flow velocity field is:

$$\vec{V} = \frac{x}{x^2 + y^2} \hat{i} + \frac{y}{x^2 + y^2} \hat{j}$$

so $u = \frac{x}{x^2 + y^2}$ and $v = \frac{y}{x^2 + y^2}$

$$\begin{aligned}
 a_x &= u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\
 &= \frac{x}{x^2 + y^2} \frac{\partial}{\partial x} \left[\frac{x}{x^2 + y^2} \right] + \frac{y}{x^2 + y^2} \frac{\partial}{\partial y} \left[\frac{x}{x^2 + y^2} \right] \\
 &= \frac{x}{x^2 + y^2} \left[\frac{x(-2x)}{(x^2 + y^2)^2} + \frac{1}{x^2 + y^2} \right] + \frac{y}{x^2 + y^2} \left[\frac{-x(2y)}{(x^2 + y^2)^2} \right] \\
 &= \frac{-2x^3}{(x^2 + y^2)^3} + \frac{x}{(x^2 + y^2)^2} - \frac{2xy^2}{(x^2 + y^2)^3} \\
 &= \frac{-2x^3 + x(x^2 + y^2) - 2xy^2}{(x^2 + y^2)^3} \\
 &= \frac{-2x^3 + x^3 + xy^2 - 2xy^2}{(x^2 + y^2)^3} = \frac{-x^3 - xy^2}{(x^2 + y^2)^3} \\
 &= \frac{-x(x^2 + y^2)}{(x^2 + y^2)^3}
 \end{aligned}$$

$$a_x = \frac{x}{(x^2 + y^2)^2}$$

$$\begin{aligned}
 a_y &= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\
 &= \frac{x}{x^2 + y^2} \frac{\partial}{\partial x} \left[\frac{y}{x^2 + y^2} \right] + \frac{y}{x^2 + y^2} \frac{\partial}{\partial y} \left[\frac{y}{x^2 + y^2} \right] \\
 &= \frac{x}{x^2 + y^2} \frac{(-2xy)}{(x^2 + y^2)^2} + \frac{y}{x^2 + y^2} \times \left[\frac{-2y^2}{(x^2 + y^2)^2} + \frac{1}{x^2 + y^2} \right] \\
 &= \frac{-2x^2y}{(x^2 + y^2)^3} - \frac{2y^3}{(x^2 + y^2)^3} + \frac{y}{(x^2 + y^2)^2} \\
 &= \frac{-2x^2y - 2y^3 + y(x^2 + y^2)}{(x^2 + y^2)^3} = \frac{-x^2y - y^3}{(x^2 + y^2)^3} = \frac{-y(x^2 + y^2)}{(x^2 + y^2)^3}
 \end{aligned}$$

$$a_y = \frac{-y}{(x^2 + y^2)^2}$$

17. (c)

Given velocity field,

$$\vec{v} = (-x^2 + 3y)\hat{i} + (2xy)\hat{j}$$

where $u = -x^2 + 3y$ and $v = 2xy$

The acceleration components along x and y-axis.

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

and

$$\begin{aligned} a_y &= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \\ &= (-x^2 + 3y) \times (-2x) + 2xy \times 3 \\ &= 2x^3 - 6xy + 6xy = 2x^3 \end{aligned}$$

and

$$\begin{aligned} a_y &= (-x^2 + 3y) \times 2y + 2xy \times 2x \\ &= -2yx^2 + 6y^2 + 4x^2y = 2yx^2 + 6y^2 \end{aligned}$$

At point (1, -1),

$$a_x = 2$$

and

$$\begin{aligned} a_y &= 2 \times (-1) \times 1 + 6 \times (-1)^2 \\ &= -2 + 6 = 4 \end{aligned}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j} = 2\hat{i} + 4\hat{j}$$

Resultant acceleration,

$$\begin{aligned} a &= \sqrt{4 + 16} = \sqrt{20} \\ &= \sqrt{4 \times 5} = 2\sqrt{5} \end{aligned}$$

18. (c)

$$\frac{\partial(u^2)}{\partial x} + \frac{\partial(uv)}{\partial y}$$

By differentiating:

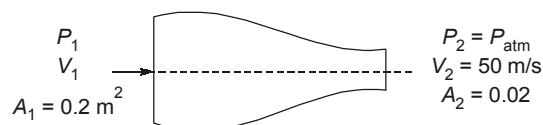
$$\Rightarrow 2u \left[\frac{\partial u}{\partial x} \right] + u \frac{\partial v}{\partial y} + v \frac{\partial u}{\partial y}$$

$$\Rightarrow u \frac{\partial u}{\partial x} + \left[u \frac{\partial u}{\partial x} + u \frac{\partial v}{\partial y} \right] + v \frac{\partial u}{\partial y}$$

$$\text{According to continuity eq. : } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{So, } u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

19. (c)

As per given data, $\rho_{\text{air}} = 1.23 \text{ kg/m}^3$ 

$$\begin{aligned} A_1 V_1 &= A_2 V_2 \\ 0.2 \times V_1 &= 0.02 \times 50 \end{aligned}$$

$$V_1 = \frac{1}{10} \times 50 = 5 \text{ m/s}$$

Applying BE

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2g} + z_2 \quad (\because z_1 = z_2)$$

$$\frac{P_1 - P_2}{\rho_{air} g} = \frac{V_2^2 - V_1^2}{2g}$$

Pressure difference,

$$P_1 - P_2 = \left(\frac{50^2 - 5^2}{2} \right) \times 1.23$$

$$= 1522.125 \text{ Pa}$$

$$= 1.52 \text{ kPa}$$

20. (c)

Reynold number,

$$Re_x = \frac{u_\infty x}{\nu} = \frac{2 \times 1}{1.5 \times 10^{-5}} = 1.33 \times 10^5$$

$$\delta = \frac{4.64x}{\sqrt{Re_x}} = \frac{4.64 \times 1}{\sqrt{1.33 \times 10^5}} = 0.0127$$

Now, $\frac{du}{dy} = u_\infty \left[\frac{3}{2} \cdot \frac{1}{\delta} - \frac{3}{2} \left(\frac{y^2}{\delta^3} \right) \right]$

$$\left. \frac{du}{dy} \right|_{y=0} = \frac{3u_\infty}{2\delta}$$

Now, shear stress,

$$\tau_0 = \mu \left(\frac{du}{dy} \right)_{y=0} = \mu \times \frac{3u_\infty}{2\delta}$$

$$= \frac{3u_\infty \times \nu \times \rho}{2\delta}$$

$$= \frac{3 \times 2 \times 1.5 \times 10^{-5} \times 1.23}{2 \times 0.0127}$$

$$= 4.36 \times 10^{-3} \text{ N/m}^2$$

$$\because \mu = \nu \rho$$

21. (d)

Radius: $r = 10 \text{ mm}$

\therefore Diameter: $d = 2r = 2 \times 10 = 20 \text{ mm} = 0.02 \text{ m}$

$$\dot{m} = 36 \text{ kg/hr} = \frac{36}{3600} = 0.01 \text{ kg/s}$$

$$\mu = 0.001 \text{ kg/ms}$$

Reynolds number: $Re = \frac{\rho V d}{\mu}$

From continuity equation, $\dot{m} = \rho A V$

or $V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho \pi d^2} = \frac{4 \dot{m}}{\rho \pi d^2}$

$$\begin{aligned} \therefore Re &= \frac{\rho d}{\mu} \times \frac{4\dot{m}}{\rho\pi d^2} = \frac{4\dot{m}}{\pi\mu d} \\ &= \frac{4 \times 0.01}{3.14 \times 0.001 \times 0.02} = 637 \end{aligned}$$

22. (c)

Instantaneous velocity : $u = \bar{u} + u'$

The time-average of the fluctuating velocity

$$\begin{aligned} \bar{u}' &= \frac{1}{T} \int_0^T u' dt = \frac{1}{T} \int_0^T (u - \bar{u}) dt \\ &= \frac{1}{T} \int_0^T u dt - \frac{1}{T} \bar{u} \int_0^T dt = \bar{u} - \frac{\bar{u}}{T} T \\ &= \bar{u} - \bar{u} = 0 \end{aligned}$$

24. (a)

Shear stress on wall,

$$\tau_w = -\frac{\partial p}{\partial x} \frac{R}{2}$$

where $-\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$

 Δp = Pressure drop

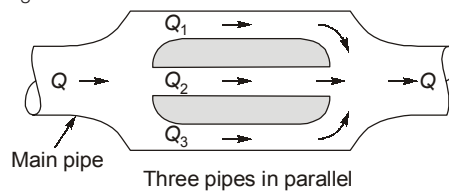
and $R = \frac{D}{2}$

$$\therefore \tau_w = \frac{\Delta p}{L} \times \frac{D}{2 \times 2} = \frac{\Delta p D}{4L}$$

25. (b)

Total discharge,

$$Q = Q_1 + Q_2 + Q_3$$

Head loss: $h_L = h_{L1} = h_{L2} = h_{L3}$ 

For the pipe connected in series,

Total discharge,

$$Q = Q_1 = Q_2 = Q_3$$

Head loss; $h_L = h_{L1} + h_{L2} + h_{L3}$

26. (c)

$$\text{Head loss: } h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$$

$$h_1 = \frac{32\mu\bar{u}L}{\rho g D^2}$$

and
$$h_2 = \frac{32\mu \times 2\bar{u}L}{\rho g(D/2)^2}$$

$$= \frac{32\mu 2\bar{u}L \times 4}{\rho g D^2} = \frac{8 \times 32\mu \bar{u}L}{\rho g D^2}$$

$$\frac{h_2}{h_1} = \frac{8 \times 32\mu \bar{u}L}{\rho g D^2} \times \frac{\rho g D^2}{32\mu \bar{u}L} = 8$$

27. (d)

$$F_x = \rho g \bar{h} A_v = (10^3)(10)\left(\frac{5}{2}\right)(5 \times 1)$$

$$= 125 \text{ kN per unit width}$$

$$F_y = \rho g \nabla$$

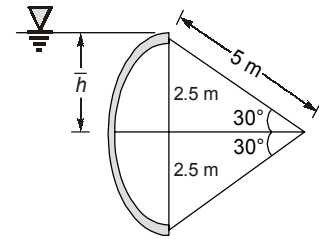
where $\nabla = \frac{\pi(5)^2}{6} - \left(\frac{1}{2} \times 5 \times 5 \cos 30^\circ\right) = 2.264 \text{ m}^3$

$$F_y = (10)^3 (10) (2.264) = 22.64 \text{ kN}$$

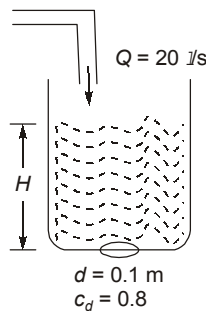
Resultant force per unit width,

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{125^2 + 22.64^2}$$

$$= 127.03 \text{ kN}$$



28. (a)



Assume H is the level of water in the tank in steady condition.

For steady water level in the tank

Discharge through orifice

= Water enters in the tank

$$c_d \cdot a \cdot \sqrt{2gH} = 20 \times 10^{-3}$$

$$0.8 \times \frac{\pi}{4} (0.1)^2 \sqrt{2gH} = 0.02$$

$$H = 0.52 \text{ m}$$

29. (b)

$$Q = 0.21 \text{ m}^3/\text{s}$$

$$\text{Allowable velocity} = 0.75 \text{ m/s}$$

$$f = 0.01$$

$$g = 9.81 \text{ m/s}^2$$

$$Q = AV$$

$$0.21 = \left(\frac{\pi}{4}d^2\right)(0.75)$$

$$\Rightarrow d = 0.597 \text{ m}$$

$$\Rightarrow h_f = \frac{fLV^2}{2gd} = \frac{0.01 \times 100 \times (0.75)^2}{2 \times 9.81 \times 0.597} \text{ m}$$

$$= 4.8 \text{ cm}$$

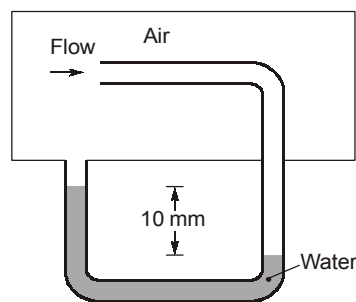
$$= 0.048 \text{ m}$$

$$\Rightarrow \text{Min. gradient} = \frac{h_f}{l} = \frac{4.8 \text{ cm}}{100 \text{ m}}$$

Hence, **answer is 4.8.**

30. (c)

Given data:



Density of air,

$$\rho_a = 1.2 \text{ kg/m}^3$$

Density of water,

$$\rho_w = 1000 \text{ kg/m}^3$$

Differential head in manometer,

$$h = 10 \text{ mm of water} = 0.01 \text{ m of water}$$

This reading is the dynamic pressure head. Hence dynamic pressure,

$$\begin{aligned} p_{\text{dyn}} &= (\rho gh)_{\text{water}} = \rho_w gh \\ &= 1000 \times 9.81 \times 0.01 = 98.1 \text{ N/m}^2 \end{aligned}$$

$$\text{also } p_{\text{dyn}} = \left(\frac{1}{2}\rho V^2\right)_{\text{air}} = \frac{1}{2}\rho_a V^2$$

$$\therefore 98.1 = \frac{1}{2} \times 1.2 \times V^2$$

$$\text{or } V^2 = 163.5$$

$$V = 12.78 \text{ m/s}$$

Alternatively

$$V = \sqrt{2gh}$$

$$h = x \left(\frac{\rho_m}{\rho} - 1 \right)$$

where

$$\begin{aligned} x &= \text{Reading of differential manometer} \\ &= 10 \times 10^{-3} \text{ m} \end{aligned}$$

$$\begin{aligned} \rho_m &= \text{Density of manometric fluid} \\ &= 1000 \text{ kg/m}^3 \end{aligned}$$

$$\rho = \text{Density of flowing fluid} = 1.2 \text{ kg/m}^3$$

$$\therefore V = \sqrt{2 \times 9.81 \times 10 \times 10^{-3} \left(\frac{1000}{1.2} - 1 \right)}$$

$$V = 12.779 \text{ m/s} \approx \mathbf{12.8 \text{ m/s}}$$

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