



MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Lucknow | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | E-mail: info@madeeasy.in | Ph: 011-45124612

ENGINEERING MATHEMATICS

Date of Test : 16/05/2022

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (d) | 7. (b) | 13. (a) | 19. (a) | 25. (c) |
| 2. (b) | 8. (b) | 14. (b) | 20. (c) | 26. (a) |
| 3. (a) | 9. (a) | 15. (a) | 21. (a) | 27. (b) |
| 4. (a) | 10. (b) | 16. (d) | 22. (a) | 28. (b) |
| 5. (a) | 11. (b) | 17. (b) | 23. (c) | 29. (a) |
| 6. (a) | 12. (d) | 18. (c) | 24. (a) | 30. (c) |

DETAILED EXPLANATIONS

1. (d)

For function to be differentiable i.e. continuous $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$

$$\begin{aligned} f(0^-) &= \lim_{x \rightarrow 0^-} \frac{\sin(3p-1)x}{3x} \times \frac{(3p-1)}{(3p-1)} \\ &= \lim_{x \rightarrow 0^-} \frac{\sin(3p-1)x}{(3p-1)x} \times \frac{(3p-1)}{3} = \frac{(3p-1)}{3} \\ f(0^+) &= \lim_{x \rightarrow 0^+} \frac{\tan(3p+1)x}{2x} \times \frac{(3p+1)}{(3p+1)} \\ &= \lim_{x \rightarrow 0^+} \frac{\tan(3p+1)x}{(3p+1)x} \times \frac{3p+1}{2} = \frac{3p+1}{2} \end{aligned}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\frac{5}{3}$

2. (b)

We have

$$\begin{aligned} y &= e^x (A \cos x + B \sin x) \\ y' &= e^x (A \cos x + B \sin x) + e^x (-A \sin x + B \cos x) \\ &= y + e^x [-A \sin x + B \cos x] \\ y'' &= y' + e^x (-A \sin x + B \cos x) + e^x (-A \cos x - B \sin x) \\ &= y' + y' - y - y \\ &= 2y' - 2y \end{aligned}$$

\Rightarrow Order = 2

Degree = 1

3. (a)

$$\begin{aligned} \frac{dy}{dx} &= e^{ax} \times e^{by} \\ \frac{dy}{e^{by}} &= e^{ax} \times dx \\ \frac{e^{-by}}{-b} &= \frac{e^{ax}}{a} + c \end{aligned}$$

$y(0) = 0$

$$\Rightarrow c = -\left[\frac{1}{b} + \frac{1}{a}\right] = -\left[\frac{a+b}{ab}\right]$$

4. (a)

$$\nabla \cdot \vec{F} = 0 \quad \text{[For solenoidal vector]}$$

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $a = 2$

5. (a)

Greatest rate of increase of ϕ is magnitude of directional derivative at that point.

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + x^2z\hat{j} + (x^2y + 8xz)\hat{k}$$

$$\nabla\phi|_{(1,-2,1)} = \hat{j} + 6\hat{k}$$

Greatest rate of increase = $\sqrt{1^2 + 6^2} = \sqrt{37} = 6.08$

6. (a)

$$\begin{bmatrix} 3 & 7.5 \\ -6 & 4.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -90 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ -6 & 4.5 & -90 \end{bmatrix}$$

$$R_2 \leftarrow R_2 + 2R_1$$

$$\begin{bmatrix} 3 & 7.5 & 6 \\ 0 & 19.5 & -78 \end{bmatrix}$$

$$19.5y = -78$$

or

$$y = -4$$

$$3x + 7.5y = 6$$

$$3x + 7.5(-4) = 6$$

$$3x = 36$$

\Rightarrow

$$x = 12$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$

7. (b)

$$\lim_{x \rightarrow 0} \frac{\ln(1+5x)}{e^{7x} - 1} \quad \left(\frac{0}{0} \text{ indeterminate form} \right)$$

Applying L' Hospital's rule

$$\lim_{x \rightarrow 0} \frac{\ln(1+5x)}{e^{7x} - 1} = \lim_{x \rightarrow 0} \frac{5}{(1+5x)7e^{7x}} = \frac{5}{7}$$

8. (b)

Probability of first item being defective,

$$P_1 = \frac{15}{50}$$

Probability of second item being defective,

$$P_2 = \frac{14}{49}$$

Probability of third item being defective,

$$P_3 = \frac{13}{48}$$

Probability that all three are defective,

$$P = P_1 P_2 P_3 = \frac{15}{50} \times \frac{14}{49} \times \frac{13}{48} = \frac{13}{560}$$

9. (a)

Given, $x = b(2 - \cos\theta)$, $y = b(\sin\theta + 1)$

$$\therefore \frac{dx}{d\theta} = b\sin\theta,$$

$$\frac{dy}{d\theta} = b(\cos\theta + 1)$$

$$\frac{dx}{dy} = \frac{dx/d\theta}{dy/d\theta} = \frac{b\sin\theta}{b(\cos\theta + 1)}$$

$$= \frac{2b\sin\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{2}\right)}{b \times 2\cos^2\left(\frac{\theta}{2}\right)} = \tan\left(\frac{\theta}{2}\right)$$

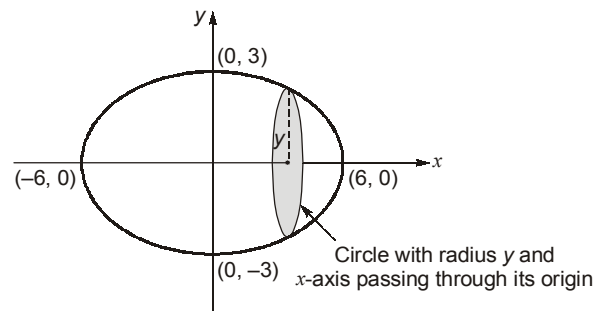
10. (b)

$$P(T) = 0.5$$

Probability of getting tails exactly 6 times is

$${}^8C_6(0.5)^6(0.5)^2 = \frac{7}{64}$$

11. (b)



$$\begin{aligned} \text{Volume generated} &= \int_{-6}^6 \pi y^2 dx = \int_{-6}^6 \pi \left(\frac{36 - x^2}{4} \right) dx \\ &= \frac{\pi \times 2}{4} \int_0^6 (36 - x^2) dx = \frac{\pi}{2} \left[36x - \frac{x^3}{3} \right]_0^6 \\ &= 72\pi \text{ unit}^3 \end{aligned}$$

12. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF \cdot f(x) \cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)} \cdot f(x) \cdot f'(x)dx$$

Let

$$f(x) = t$$

$$f'(x) dx = dt$$

$$y \times e^t = \int e^t \cdot t dt$$

$$y \cdot e^t = t \cdot e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log [y + 1 - f(x)] + f(x) = c'$$

13. (a)

For particular integral,

$$PI = \frac{96x^2}{D^2(D^2 + 4)} = 96 \frac{1}{4D^2 \left(1 + \frac{D^2}{4}\right)} x^2 = \frac{96}{4} \left[\frac{\left(1 - \frac{D^2}{4}\right) x^2}{D^2} \right]$$

$$= 24 \frac{\left(x^2 - \frac{1}{2}\right)}{D^2}$$

$$PI = 24 \left[\frac{x^4}{4 \times 3} - \frac{x^2}{4} \right] = 2x^2(x^2 - 3)$$

$$PI|_{x=2} = 2 \times 2^2(4 - 3) = 8$$

14. (b)

$$u(x, y) = 2x(1 - y)$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy = -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$

$$dv = (2x)dx + 2(1 - y)dy$$

$$v = x^2 + 2y - y^2 + c$$

15. (a)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_0^2 kx dx + \int_2^4 2k dx + \int_4^6 (-kx + 6k) dx = 1$$

$$\left. \frac{kx^2}{2} \right|_0^2 + 2kx \Big|_2^4 + \left. \left(\frac{-kx^2}{2} + 6kx \right) \right|_4^6 = 1$$

$$\frac{k}{2}(2^2 - 0) + 2k(4 - 2) - \frac{k}{2}(6^2 - 4^2) + 6k(6 - 4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow \quad k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2 dx + \int_2^4 \frac{1}{4}x dx + \int_4^6 \left(-\frac{1}{8}x^2 + \frac{3}{4}x \right) dx$$

$$= \left. \frac{1}{8} \frac{x^3}{3} \right|_0^2 + \left. \frac{1}{4} \frac{x^2}{2} \right|_2^4 - \left. \frac{1}{8} \frac{x^3}{3} \right|_4^6 + \left. \frac{3}{4} \frac{x^2}{2} \right|_4^6$$

$$= \frac{1}{3} + \frac{3}{2} - \frac{19}{3} + \frac{15}{2} = 3$$

16. (d)

$$\begin{aligned} I &= \int_0^{\pi/2} \sqrt{1 + \sec x} dx = \int_0^{\pi/2} \sqrt{1 + \frac{1}{\cos x}} dx \\ &= \int_0^{\pi/2} \frac{\sqrt{1 + \cos x}}{\sqrt{\cos x}} dx = \int_0^{\pi/2} \frac{\sqrt{2} \cos(x/2)}{\sqrt{1 - 2\sin^2(x/2)}} dx \end{aligned}$$

$$\begin{aligned} \text{Let} \quad \sin \frac{x}{2} &= t, & \begin{cases} x = 0, & t = 0 \\ x = \frac{\pi}{2}, & t = \frac{1}{\sqrt{2}} \end{cases} \\ \frac{1}{2} \cos \frac{x}{2} dx &= dt \end{aligned}$$

$$\begin{aligned} I &= \int_0^{1/\sqrt{2}} \frac{2\sqrt{2} dt}{\sqrt{1 - 2t^2}} \\ &= 2 \sin^{-1}(\sqrt{2}t) \Big|_0^{1/\sqrt{2}} = 2 \sin^{-1}\left(\sqrt{2} \times \frac{1}{\sqrt{2}}\right) - 2 \sin^{-1}(0) \\ &= 2 \times \frac{\pi}{2} = \pi = 3.14 \end{aligned}$$

17. (b)

$$(2y - 3x)dx + xdy = 0$$

$$\frac{dy}{dx} + \frac{2}{x}y = 3$$

$$IF = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$$

$$y \cdot x^2 = 3 \int x^2 dx = x^3 + c$$

For $x = 0, y = 0$

$$\Rightarrow 0 = 0 + c$$

$$\Rightarrow c = 0$$

For $x = 2,$

$$y \times 2^2 = 2^3$$

$$y = 2$$

18. (c)

$$\frac{4C_1 \cdot 4C_1 \cdot 4C_1 \cdot 4C_1}{52C_4} = \frac{4 \times 4 \times 4 \times 4}{(52 \times 51 \times 50 \times 49) / (4 \times 3 \times 2 \times 1)}$$

$$= \frac{4 \times 4 \times 4 \times 4 \times 3 \times 2}{52 \times 51 \times 50 \times 49} = \frac{256}{270725}$$

19. (a)

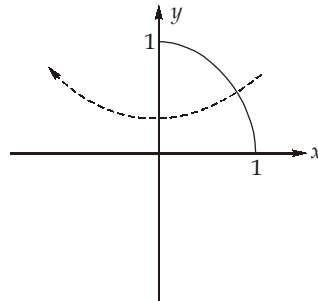
$$x = \sin\left(\frac{\pi k}{2}\right), y = \cos\left(\frac{\pi k}{2}\right)$$

Just by seeing, we can know that it represents a circle in $x - y$ plane, given by

$$x^2 + y^2 = 1$$

Given $0 \leq k \leq 1$, which gives $0 \leq x \leq 1$; $0 \leq y \leq 1$

or
$$0 \leq \frac{\pi k}{2} \leq \frac{\pi}{2}$$



So we get a quarter circle, when this is rotated with respect to y -axis by 360 degree, it creates a hemisphere of radius 1.

Surface area of hemisphere,

$$A_s = 2\pi r^2$$

$$= 2\pi(1)^2 = 2\pi$$

20. (c)

$$f(y) = y^2 e^{-y}$$

$$f'(y) = y^2(-e^{-y}) + e^{-y} \times 2y$$

$$= e^{-y}(2y - y^2)$$

Putting $f'(y) = 0$

$$e^{-y}(2y - y^2) = 0$$

$$e^{-y}y(2 - y) = 0$$

$y = 0$ or $y = 2$ are the stationary points

Now,

$$f''(y) = e^{-y}(2 - 2y) + (2y - y^2)(-e^{-y})$$

$$= e^{-y}(2 - 2y - 2y + y^2)$$

$$= e^{-y}(y^2 - 4y + 2)$$

At $y = 0$,

$$f''(0) = e^{-0}(0 - 0 + 2) = 2$$

Since $f''(0) = 2$ is > 0 at $y = 0$ we have a minima

$$\begin{aligned} \text{Now, at } y = 2f''(2) &= e^{-2}(2^2 - 4 \times 2 + 2) \\ &= e^{-2}(4 - 8 + 2) \\ &= -2e^{-2} < 0 \end{aligned}$$

\therefore At $y = 2$ we have a maxima.

21. (a)

$$\begin{aligned} P(x) &= \frac{\mu^x e^{-\mu}}{x!} \\ P(x < 3) &= P(0) + P(1) + P(2) \\ &= \frac{\mu^0 e^{-\mu}}{0!} + \frac{\mu^1 e^{-\mu}}{1!} + \frac{\mu^2 e^{-\mu}}{2!} \\ &= \frac{1}{e^\mu} + \frac{\mu}{e^\mu} + \frac{\mu^2}{2e^\mu} \end{aligned}$$

As $\mu(\text{mean}) = 6.8$

$$\therefore P(x < 3) = \frac{1 + 6.8 + \left(\frac{6.8^2}{2}\right)}{e^{6.8}} = \frac{30.92}{897.85} \approx 0.034$$

22. (a)

$$\sin x \cos y dx + \cos x \sin y dy = 0$$

Divide by $\cos x \cos y$, we get,

$$\tan x dx + \tan y dy = 0$$

Integrating the equation,

$$\log \sec x + \log \sec y = C_1$$

$$\begin{aligned} \log \frac{1}{\cos x \cos y} &= C_1 \\ \cos x \cos y &= C \end{aligned}$$

Since it passes through $\left(0, \frac{\pi}{3}\right)$

$$\cos(0) \cos\left(\frac{\pi}{3}\right) = C$$

$$\frac{1}{2} = C$$

\Rightarrow The equation of curve is,

$$\cos x \cos y = \frac{1}{2}$$

23. (c)

$$P(x) = x^5 + x + 2$$

It has a real root at $x = -1$

$$\Rightarrow P(x) = (x^4 - x^3 + x^2 - x + 2)(x + 1)$$

Now, $x^4 - x^3 + x^2 + x + 2$ will give other 4 roots

To find roots,

$$\Rightarrow x^4 - x^3 + x^2 - x + 2 = 0$$

$$\Rightarrow x^3(x-1) + x(x-1) + 2 = 0$$

$$\Rightarrow x(x^2+1)(x-1) + 2 = 0$$

In the above expression, $x^2 + 1$ is always positive. So, either 'x' or 'x - 1' should be negative in order to satisfy the equation.

For $x > 1$, both (x) and (x - 1) are positive and,

For $x < 0$, both (x) and (x - 1) are negative

∴ x should lie within 0 and 1 in order to have real roots.

As $x \in (0, 1)$

$$\Rightarrow |x| < 1$$

$$\Rightarrow |x^2 + 1| < 2, |x| < 1 \text{ and } |x - 1| < 1$$

∴ The product of these three will be less than 2 and hence, no real value of 'x' can satisfy the equation

$$x^4 - x^3 + x^2 - x + 2 = 0$$

∴ The equation will have four imaginary roots apart from one real roots.

24. (a)

$$I = \int \sec^3 \theta d\theta = \int \sec \theta \cdot \sec^2 \theta d\theta$$

$$= \sec \theta \int \sec^2 \theta d\theta - \int \tan \theta (\sec \theta \tan \theta) d\theta$$

$$= \sec \theta \tan \theta - \int \tan^2 \theta \sec \theta d\theta$$

$$\Rightarrow I = \sec \theta \tan \theta - \int (\sec^2 \theta - 1) \sec \theta d\theta$$

$$= \sec \theta \tan \theta - \int \sec^3 \theta d\theta + \int \sec \theta d\theta$$

$$\Rightarrow I = \sec \theta \tan \theta - I + \ln |\sec \theta + \tan \theta| + c$$

$$\Rightarrow I = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + c$$

$$\therefore a + b = \frac{1}{2} + \frac{1}{2} = 1$$

25. (c)

$$\int_C \vec{F} \cdot d\vec{r} = \int_C x^2 y^2 dx + y \cdot dy$$

For curve C,
and

$$y^2 = 4x$$

$$2y dy = 4 dx$$

$$\Rightarrow \int_C \vec{F} \cdot d\vec{r} = \int_0^4 x^2 (4x) dx + 2dx$$

$$= \int_0^4 (4x^3 + 2) dx = 264$$

26. (a)

To obtain maximum value of $f(x)$, first $f'(x)$ should be equated to zero.

$$\Rightarrow f'(x) = 6x^2 - 6x - 36 = 0$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$$\therefore f'(x) = 0 \text{ at } x = 3 \text{ and } -2$$

Now, $f''(x) = 12x - 6$

$$f''(3) = 30 > 0$$

at $x = 3$, there is local minima

and $f''(2) = -30 < 0$

∴ at $x = -2$, a local maxima is observed.

27. (b)

$$\text{Length of curve} = \int_0^1 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

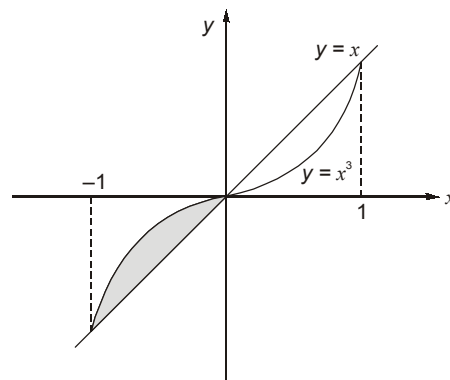
Curve:

$$3x^2 = y^3$$

$$\Rightarrow \frac{dx}{dy} = \frac{\sqrt{3y}}{2}$$

$$\begin{aligned} \therefore \text{Length} &= \int_0^1 \sqrt{1 + \frac{3y}{4}} dy \\ &= \frac{1}{2} \int_0^1 \sqrt{4 + 3y} dy \\ &= \frac{1}{2} \left[\frac{(4 + 3y)^{3/2}}{\frac{3}{2} \times 3} \right]_0^1 \\ &= \frac{1}{9} (7\sqrt{7} - 8) \end{aligned}$$

28. (b)

Point of inter-section of the two curves are $x = 0, 1, -1$ 

$$\text{Area} = \int_{-1}^0 (x^3 - x) dx = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 = \frac{0 - (-1)^4}{4} - \frac{0 - (-1)^2}{2} = \frac{1}{4}$$

29. (a)

$$(x+1) \frac{dy}{dx} + 1 = 2e^{-y}$$

$$\Rightarrow (x+1) \frac{dy}{dx} = (2e^{-y} - 1)$$

$$\Rightarrow \frac{dy}{(2e^{-y} - 1)} = \frac{dx}{x+1}$$

$$\Rightarrow \frac{e^y dy}{2 - e^y} = \frac{dx}{x+1}$$

$$\Rightarrow -\log(2 - e^y) = \log(x+1) + c$$

$$\Rightarrow (x+1)(2 - e^y) = k$$

30. (c)

$$\frac{d^2y}{dx^2} = y$$

$$\Rightarrow D^2y = y \quad (\because d/dx = D)$$

$$(D^2 - 1)y = 0$$

$$D^2 - 1 = 0$$

$$D = \pm 1$$

$$y = C_1 e^x + C_2 e^{-x}$$

Given point passes through origin

$$\Rightarrow 0 = C_1 + C_2$$

$$C_1 = -C_2 \quad \dots(i)$$

Also, point passes through (ln 2, 3/4)

$$\Rightarrow \frac{3}{4} = C_1 e^{\ln 2} + C_2 e^{-\ln 2}$$

$$\frac{3}{4} = 2C_1 + \frac{C_2}{2}$$

$$\Rightarrow C_2 + 4C_1 = 1.5 \quad \dots(ii)$$

From (i) $C_1 = -C_2$, putting in (ii), we get

$$\Rightarrow -3C_2 = 1.5$$

$$C_2 = -0.5$$

$$\therefore C_1 = 0.5$$

$$\Rightarrow y = 0.5(e^x - e^{-x})$$

$$y = \frac{e^x - e^{-x}}{2}$$

