CLASS TEST

S.No. : 01SK_EC_ABCD_160522

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Lucknow | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

COMMUNICATIONS

ELECTRONICS ENGINEERING

Date of Test : 16/05/2022

ANSWER KEY

1.	(b)	7.	(d)	13.	(c)	19.	(a)	25.	(b)
2.	(a)	8.	(d)	14.	(a)	20.	(a)	26.	(c)
3.	(d)	9.	(c)	15.	(c)	21.	(a)	27.	(c)
4.	(c)	10.	(b)	16.	(c)	22.	(d)	28.	(c)
5.	(b)	11.	(b)	17.	(b)	23.	(c)	29.	(d)
6.	(c)	12.	(b)	18.	(b)	24.	(a)	30.	(a)

Detailed Explanations

1. **(b)**

$$\mu_1 = \frac{2}{20} = \frac{1}{10} \text{ and } \mu_2 = \frac{10}{20} = \frac{1}{2}$$

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\mu^2 = \left(\frac{1}{10}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{100} + \frac{1}{4} = \frac{26}{100} = 0.26$$

$$P_{SB} = P_c \frac{\mu^2}{2}$$

$$\frac{P_{SB}}{P_c} = \frac{\mu^2}{2} = \frac{0.26}{2} = 0.13$$

2. **(a)**

For properly designed PLL based demodulator,

$$y(t) = \frac{k_f}{k_v} m(t) = \frac{5}{10} m(t) = \frac{1}{2} m(t)$$

$$P_y = \left(\frac{1}{2}\right)^2 P_m = \frac{1}{4}(20)\text{ W} = 5\text{ W}$$

3. **(d)**

$$H(X) = -\sum_{i=1}^{N} P(x_i)\log_2 P(x_i)$$

$$Y = 4X$$

$$P(y_i) = P(x_i)$$

$$H(Y) = H(X)$$

4. **(c)**

Entropy of the source,

$$H(X) = \frac{2}{8}\log_2(8) + \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 \text{ bits/symbol}$$

$$= \frac{6}{8} + \frac{2}{4} + \frac{1}{2} = \frac{14}{8} = 1.75 \text{ bits/symbol}$$

The entropy of the 2nd order extension of the source will be,

$$H(X^2) = 2H(X) = 2(1.75) = 3.50 \text{ bits/block}$$

5. **(b)**

The capacity of BSC with $p = 0.25$ will be,

$$C = 1 + p\log_2 p + (1-p)\log_2(1-p) \text{ bits/symbol}$$

$$= 0.1887 \text{ bits/symbol}$$

6. (c)

For zero mean Gaussian random variable, the differential entropy can be given by,

$$H(X) = \frac{1}{2}\log_2(2\pi e\sigma^2)$$

Given that, $\sigma^2 = 2$

So, $$H(X) = \frac{1}{2}\log_2(4\pi e) = \frac{1}{2}\log_2(4) + \frac{1}{2}\log_2(\pi e)$$

$$= 1 + \frac{1}{2}\log_2(\pi e)$$

7. (d)

For matched filter,

$$(\text{SNR})_{\max} = \frac{2E_s}{N_0}$$

$$E_s = \text{Energy of the signal } s(t)$$

$$= \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_0^2 (4)^2 dt = 32$$

So, $$(\text{SNR})_{\max} = \frac{2(32)}{N_0} = \frac{64}{N_0}$$

8. (d)

Bit rate, $R_b = 100$ kbps

Chip rate, $R_c = 7.2$ Mcps

$$\text{Processing gain} = \frac{R_c}{R_b} = \frac{7.2\times1000}{100} = 72$$

9. (c)

The condition required to eliminate the slope-overload distortion is,

$$\frac{\Delta}{T_s} \ge \left|\frac{dm(t)}{dt}\right|_{\max} = |2\pi f_m \sin(2\pi f_m t)|_{\max}$$

$$2f_s \ge 2\pi f_m$$

$$f_s \ge \pi f_m \approx 3.14 f_m$$

$$f_{s(\min)} = 3.14 f_m$$

10. (b)

From the given angle modulated signal,

Modulation index, $\beta = 4$

Message signal frequency, $f_m = \dfrac{2000\pi}{2\pi} = 1000\text{ Hz} = 1\text{ kHz}$

According to Carson's rule,

$$\text{BW} = (1 + \beta)2f_m = (1 + 4)\,(2 \times 1)\text{ kHz}$$

$$= 10\text{ kHz}$$

11. (b)

The transmission efficiency of an AM signal can be given by,

$$\eta = \frac{k_a^2 P_m}{1 + k_a^2 P_m}$$

Here, k_a = amplitude sensitivity of the modulator

$= 0.25\ \text{V}^{-1}$

P_m = Power of the message signal

For the given message signal,

$$P_m = A^2 = (2)^2 = 4$$

So,
$$\eta = \frac{(0.25)^2 (4)}{1 + (0.25)^2 (4)} = \frac{0.25}{1 + 0.25} = \frac{1}{5} = 0.20 \text{ (or) } 20\%$$

12. (b)

The pre-envelope of a real valued signal $x(t)$ can be given by,

$$x_{pe}(t) = x(t) + j\hat{x}(t)$$

where, $\hat{x}(t)$ is the Hilbert transform of $x(t)$.

$$x(t) \xleftrightarrow{\text{Fourier Transform}} X(f)$$

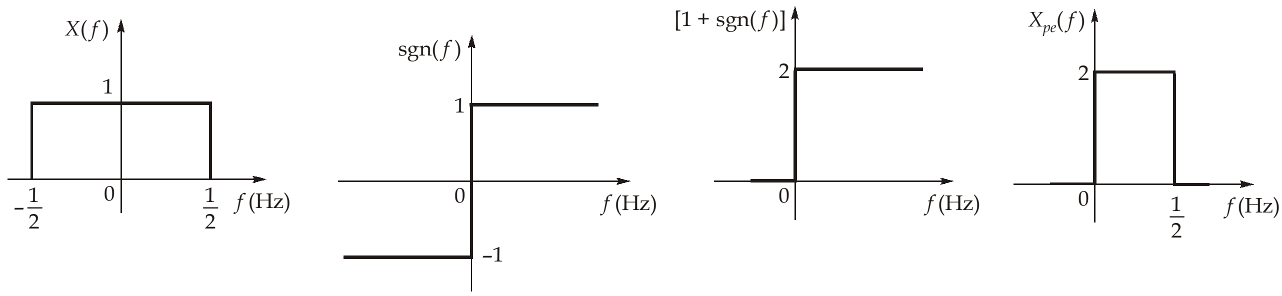
$$\hat{x}(t) \xleftrightarrow{\text{Fourier Transform}} (-j\,\text{sgn}(f))\, X(f)$$

$$x_{pe}(t) \xleftrightarrow{\text{Fourier Transform}} X_{pe}(f)$$

So,
$$X_{pe}(f) = X(f) + j(-j\,\text{sgn}(f))\, X(f)$$
$$= X(f)\,[1 + \text{sgn}(f)]$$

From the basic Fourier transform pairs, we can give,

$$x(t) = \frac{\sin \pi t}{\pi t} = \text{sinc}(t) \xleftrightarrow{\text{Fourier Transform}} X(f) = \text{rect}(f)$$



The Fourier transform of $x_{pe}(t)$ can be expressed in terms of $\text{rect}(f)$ as,

$$X_{pe}(f) = 2\,\text{rect}\left(2f - \frac{1}{2}\right)$$

$$\text{sinc}(t) \xleftrightarrow{\text{Fourier Transform}} \text{rect}(f)$$

$$\text{sinc}(t)\, e^{j2\pi\left(\frac{1}{2}\right)t} \xleftrightarrow{\text{Fourier Transform}} \text{rect}\left(f - \frac{1}{2}\right)$$

$$x_{pe}(t) = \text{sinc}\left(\frac{t}{2}\right)e^{j\pi\left(\frac{t}{2}\right)} \xleftrightarrow{\text{Fourier Transform}} X_{pe}(f) = 2\,\text{rect}\left(2f - \frac{1}{2}\right)$$

$$x_{pe}(t) = \text{sinc}\left(\frac{t}{2}\right)e^{j\frac{\pi t}{2}}$$

13. (c)

For Hilbert transform, the impulse response is,

$$h(t) = \frac{1}{\pi t}$$

$$H(\omega) = -j\,\text{sgn}(\omega)$$

$$|H(\omega)|^2 = 1$$

So, the input and output power spectral densities are related as,

$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega) = (1)\, S_X(\omega) = S_X(\omega)$$

Since $S_Y(\omega) = S_X(\omega)$, $R_Y(\tau) = R_X(\tau) \Rightarrow$ Hence, R_1 is correct

$$R_{XY}(\tau) = R_X(\tau) * h(\tau)$$

$$R_{XY}(-\tau) = R_X(-\tau) * h(-\tau)$$

$$h(\tau) = \frac{1}{\pi\tau} \text{ and } h(-\tau) = -h(\tau)$$

$$R_X(-\tau) = R_X(\tau) \qquad \because \text{ ACF of a WSS process is an even function}$$

So, $$R_{XY}(-\tau) = R_X(\tau) * [-h(\tau)] = -[R_X(\tau) * h(\tau)]$$

$$R_{XY}(-\tau) = -R_{XY}(\tau) \Rightarrow \text{Hence, } R_2 \text{ is correct}$$

So, both the given relations are correct.

14. (a)

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

$$\frac{P_c\left(1 + \frac{\mu^2}{2}\right)}{P_c\left(1 + \frac{\mu_1^2}{2}\right)} = 2$$

$$(2 + \mu^2) = 2(2 + \mu_1^2)$$

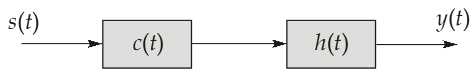
$$\mu^2 - 2\mu_1^2 = 2$$

So, $$\mu_2^2 - \mu_1^2 = 2 \qquad \because \mu^2 = \mu_1^2 + \mu_2^2$$

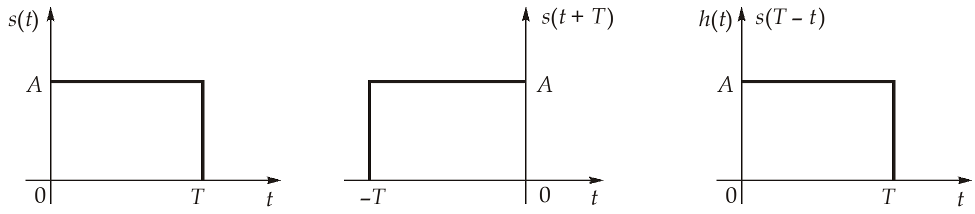
$$\mu_2^2 = 2 + \mu_1^2 = 2 + (0.80)^2 = 2.64$$

$$\mu_2 = \sqrt{2.64} = 1.625$$

15. (c)



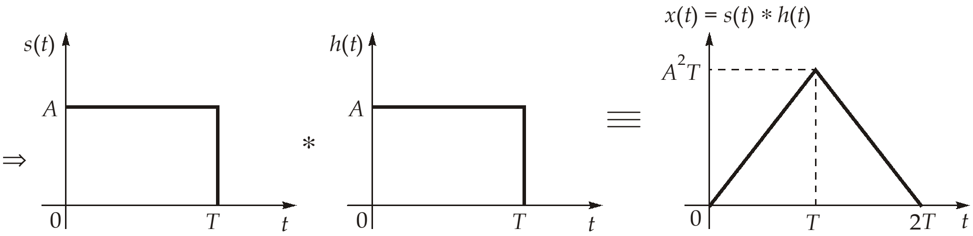
$$h(t) = s(T - t)$$



Given that, $$c(t) = \delta(t)+\frac{1}{2}\delta\left(t-\frac{T}{2}\right)$$

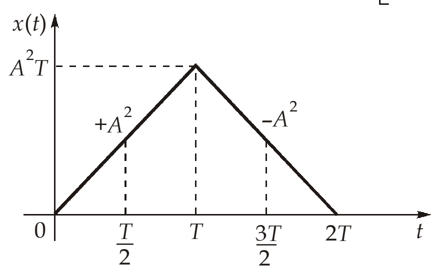
So, $$y(t) = s(t) * c(t) * h(t) = [s(t) * h(t)] * c(t)$$

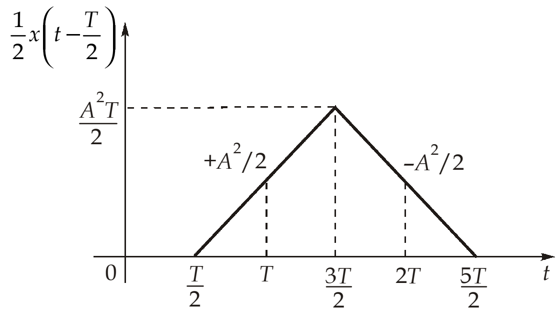
$x(t) = [s(t) * h(t)] \Rightarrow$

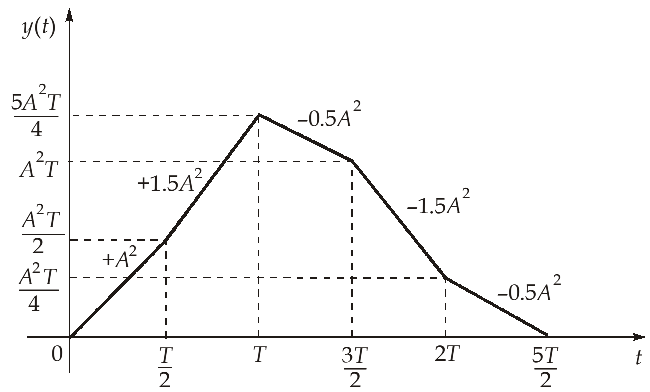


$$y(t) = x(t) * c(t)$$

$$= x(t)*\left[\delta(t)+\frac{1}{2}\delta\left(t-\frac{T}{2}\right)\right] = x(t)+\frac{1}{2}x\left(t-\frac{T}{2}\right)$$







So, the peak value of the filter output is,

$$y_{max} = \frac{5}{4}A^2T$$

16. (c)

Given that,
$$f_X(x) = \begin{cases} \frac{1}{2}; & -1 < x < 1 \\ 0; & \text{otherwise} \end{cases}$$

$$\bar{X} = E[X] = \int_{-\infty}^{\infty} x f_X(x) dx = \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{1} = 0$$

Covariance of X and Y,
$$C_{XY} = E\left[(X-\bar{X})(Y-\bar{Y})\right] = E[XY] - \bar{X}\bar{Y}$$
$$= E[XY] - (0)\bar{Y} = E[XY]$$

So,
$$C_{XY} = E[XY] = E(XX^n) = E[X^{n+1}]$$
$$= \int_{-\infty}^{\infty} x^{n+1} f_X(x)\, dx = \frac{1}{2}\int_{-1}^{1} x^{n+1}\, dx$$
$$= \frac{1}{2}\left[\frac{x^{n+2}}{n+2}\right]_{-1}^{1} = \frac{1}{2}\left[\frac{1}{n+2} - \frac{(-1)^{n+2}}{n+2}\right]$$
$$C_{XY} = \begin{cases} \frac{1}{n+2}; & n = \text{odd} \\ 0 \;; & n = \text{even} \end{cases}$$

17. (b)

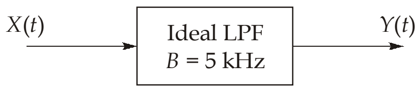
$$f_Z(z) = f_X(z) * f_Y(z)$$
$$f_X(z) = ae^{-az}\, u(z)$$
$$f_Y(z) = be^{-bz}\, u(z)$$

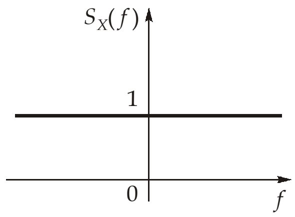
$$L\{f_X(z)\} = \frac{a}{s+a} \quad \text{and} \quad L\{f_Y(z)\} = \frac{b}{s+b}$$

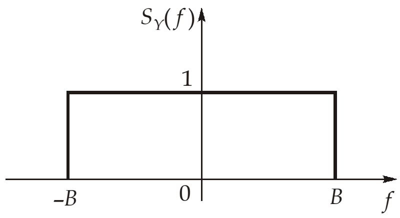
$$f_Z(z) = L^{-1}\left\{\frac{ab}{(s+a)(s+b)}\right\} = L^{-1}\left\{\frac{ab}{(b-a)}\left[\frac{1}{(s+a)} - \frac{1}{(s+b)}\right]\right\}$$

$$= \frac{ab}{(b-a)}\left[e^{-az} - e^{-bz}\right]u(z)$$

18. (b)







$$S_Y(f) = \text{rect}\left(\frac{f}{2B}\right)$$

$$R_Y(\tau) \xleftrightarrow{\text{CTFT}} S_Y(f)$$

So, $$R_Y(\tau) = 2B\,\text{sinc}\,(2B\tau)$$

For sampling interval T_s, if the adjacent samples are to be correlated, then

$$R_Y(T_s) \neq 0$$

$$2B\text{sinc}(2BT_s) \neq 0$$

So, $$2BT_s \neq n\ ;\ n = 1, 2, 3\ \ldots\ldots$$

$$T_s \neq \frac{n}{2B};\ n = 1, 2, 3\ \ldots\ldots$$

$$\frac{1}{2B} = \frac{1}{2(5)}\text{msec} = 100\,\mu\text{sec}$$

So, T_s should not be the integer multiple of 100 μsec.

Only option (b) satisfies this.

19. (a)

Any CDF will vary from 0 to 1 only. So, the random variable Y will also vary from 0 to 1.

$$0 \leq y \leq 1$$

The probability density function of Y can be given as,

$$f_Y(y) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|} = \frac{f_X(x)}{\left|\frac{d}{dx}(F_X(x))\right|} = \frac{f_X(x)}{f_X(x)} = 1\ ; \qquad 0 \leq y \leq 1$$

So, $$f_Y(y) = \begin{cases} 1\ ; & 0 \leq y \leq 1 \\ 0\ ; & \text{otherwise} \end{cases}$$

$$E[Y^2] = \int_{-\infty}^{\infty} y^2 f_Y(y)\,dy = \int_0^1 (1)y^2\,dy = \left[\frac{y^3}{3}\right]_0^1 = \frac{1}{3}$$

$$= 0.33$$

20. (a)

Given that, $$f_X(x) = \begin{cases} \frac{1}{2}\ ; & 0 < x < 2 \\ 0\ ; & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{4} \ ; & 0 < y < 4 \\ 0 \ ; & \text{otherwise} \end{cases}$$

$$f_{XY}(x, y) = f_X(x)\, f_Y(y) \qquad \because X \text{ and } Y \text{ are independent}$$

The CDF of Z can be given by,

$$F_Z(z) = P(Z \le z) = P[\min(X, Y) \le z]$$

$$= P(X \le z, Y > X) + P(Y \le z, X > Y)$$

$$= \int_{x=0}^{z} \frac{1}{2}\left(\int_{y=x}^{4} \frac{1}{4} dy\right) dx + \int_{y=0}^{z} \frac{1}{4}\left(\int_{x=y}^{2} \frac{1}{2} dx\right) dy \ ; \quad 0 < z < 2$$

$$= \int_{x=0}^{z} \frac{1}{2}\left(1 - \frac{x}{4}\right) dx + \int_{y=0}^{z} \frac{1}{4}\left(1 - \frac{y}{2}\right) dy \ ; \quad 0 < z < 2$$

$$f_Z(z) = \frac{d}{dz}(F_Z(z)) = \frac{1}{2}\left(1 - \frac{z}{4}\right) + \frac{1}{4}\left(1 - \frac{z}{2}\right) \ ; \quad 0 < z < 2$$

$$= \frac{3}{4} - \frac{z}{4} = \frac{1}{4}(3 - z) \ ; \quad 0 < z < 2$$

$$E[Z] = \int_{-\infty}^{\infty} z\, f_Z(z) dz = \frac{1}{4}\int_0^2 (3z - z^2)\, dz = \frac{1}{4}\left[\frac{3}{2}z^2 - \frac{1}{3}z^3\right]_0^2$$

$$= \frac{1}{4}\left(6 - \frac{8}{3}\right) = \frac{10}{12} = 0.83$$

21. (a)

The probability density function of the input variable r can be given by,

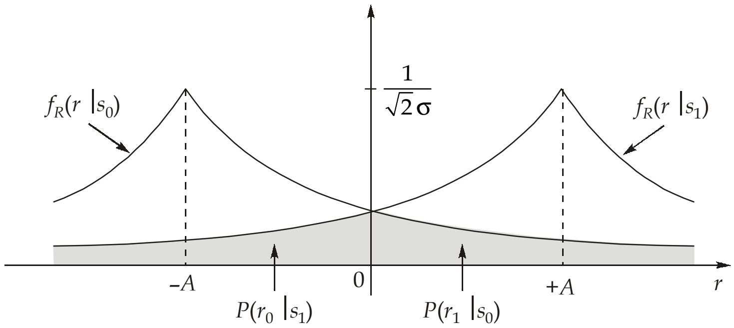
$$f_R(r \mid s_0) = \frac{1}{\sqrt{2}\,\sigma} e^{\frac{-\sqrt{2}|r + A|}{\sigma}} \ ; \quad \text{when "0" is transmitted}$$

$$f_R(r \mid s_1) = \frac{1}{\sqrt{2}\,\sigma} e^{\frac{-\sqrt{2}|r - A|}{\sigma}} \ ; \quad \text{when "1" is transmitted}$$

Given that, $r_{th} = 0$. So, the detector will make the decision in favour of logic-0 when $r < 0$ and in favour of logic-1 when $r > 0$.

The probability of error can be given by,

$$P_e = \begin{pmatrix} \text{Probability of decision} \\ \text{made in favour of 0} \\ \text{when 1 is transmitted} \end{pmatrix} P(s_1) + \begin{pmatrix} \text{Probability of decision} \\ \text{made in favour of 1} \\ \text{when 0 is transmitted} \end{pmatrix} P(s_0)$$



It is given that, $P(s_0) = P(s_1) = \frac{1}{2}$ and from the above graph it is clear that $P(r_0|s_1) = P(r_1|s_0)$.

So,
$$P_e = \frac{1}{2}P(r_0|s_1) + \frac{1}{2}P(r_1|s_0) = P(r_1|s_0) = P(r_0|s_1)$$
$$= \int_0^\infty f_R(r|s_0)\,dr = \frac{1}{\sqrt{2}\sigma}\int_0^\infty e^{-\frac{\sqrt{2}|r+A|}{\sigma}}\,dr$$
$$= \frac{e^{-\frac{\sqrt{2}A}{\sigma}}}{\sqrt{2}\sigma}\int_0^\infty e^{-\sqrt{2}r/\sigma}\,dr = \frac{e^{-\frac{\sqrt{2}A}{\sigma}}}{\sqrt{2}\sigma}\left[-\frac{e^{-\sqrt{2}r/\sigma}}{\sqrt{2}/\sigma}\right]_0^\infty$$
$$P_e = \frac{1}{2}e^{-\frac{\sqrt{2}A}{\sigma}}$$

The SNR at the input of the detector can be given by,
$$\text{SNR} = \frac{(\pm A)^2}{\sigma^2} = \left(\frac{A}{\sigma}\right)^2$$

So, the relation between P_e and SNR can be given by,
$$P_e = \frac{1}{2}e^{-\sqrt{2(\text{SNR})}}$$

To achieve a maximum error probability of 10^{-5},
$$\frac{1}{2}e^{-\sqrt{2(\text{SNR})}} \le 10^{-5}$$
$$-\sqrt{2(\text{SNR})} \le \ln(2 \times 10^{-5})$$
$$\text{SNR} \ge 58.534 \text{ (or) } 17.6741 \text{ dB}$$

So, $$(\text{SNR})_{\min} = 58.538 \text{ (or) } 17.6741 \text{ dB}$$

22. (d)

Maximum distance separable (MDS) codes satisfy the Singleton bound.

Singleton bound, $d_{\min} = n - k + 1$

For (6, 1, 6) $\Rightarrow$ $n - k + 1 = 6$ and $d_{\min} = 6$

For (6, 5, 2) $\Rightarrow$ $n - k + 1 = 2$ and $d_{\min} = 2$

For (7, 6, 2) $\Rightarrow$ $n - k + 1 = 2$ and $d_{\min} = 2$

For (7, 4, 3) $\Rightarrow$ $n - k + 1 = 4$ and $d_{\min} = 3$

So, the code given in option (d) is not an MDS code.

23. (c)

$$E_1 = \frac{1}{4}\left[(0)^2 + (2A)^2 + (\sqrt{2}A)^2 + (\sqrt{2}A)^2\right] = 2A^2$$

$$E_2 = \frac{1}{4}\left[(A)^2 + (A)^2 + (A)^2 + (A)^2\right] = A^2$$

So, $\quad E_1 > E_2$

The distance between adjacent symbols is same in both the constellations. So, both the modulation schemes provide same average symbol error under similar circumstances.

Hence, $\quad p_1 = p_2$

24. (a)

The quantization level for region R_3 will be,

$$x_{q3} = \frac{1+5}{2} = 3$$

So, the quantization noise due to rounding off of the samples lie in the region R_3 alone is,

$$N_{Q3} = E\left[(X - x_{q3})^2\right] = \int_{<R_3>} (x-3)^2 f_X(x)\,dx$$

$$f_X(x) = \begin{cases} \dfrac{1}{5}; & -3 \le x \le 2 \\ 0\,; & \text{otherwise} \end{cases}$$

So, $$N_{Q3} = \frac{1}{5}\int_1^2 (x^2 - 6x + 9)\,dx = \frac{1}{5}\left[\frac{x^3}{3} - 3x^2 + 9x\right]_1^2$$

$$= \frac{1}{5}\left[\left(\frac{8}{3} - 12 + 18\right) - \left(\frac{1}{3} - 3 + 9\right)\right] = \frac{7}{15}$$

25. (b)

$$s(t) \longrightarrow \boxed{\begin{array}{c}\text{Filter matched}\\ \text{to } s(t)\end{array}} \longrightarrow y(t)$$

For a matched filter, peak value of the output will be numerically equal to the energy of the input signal.

So, $$|y(t)|_{\max} = \int_{-\infty}^{\infty} |s(t)|^2\,dt$$

$$s(t) = \begin{cases} \left(3 - \dfrac{3}{2}|t|\right)\text{V}\,; & 0 \le |t| \le 2 \\ \quad 0 \quad ; & \text{otherwise} \end{cases}$$

So, $$|y(t)|_{\max} = 2\int_0^2 \left(3 - \frac{3}{2}t\right)^2 dt$$

$$= \frac{9}{2}\int_0^2 (t^2 + 4 - 4t)\,dt = \frac{9}{2}\left[\frac{t^3}{3} + 4t - 2t^2\right]_0^2 = 12\ \text{V}$$

26. (c)

Bandwidth of the baseband signal with Nyquist pulse shaping is,

$$(BW)_{baseband} = \frac{R_b}{2}$$

Bandwidth of M-ary PSK modulated signal is,

$$(BW)_{M\text{-PSK}} = \frac{2[(BW)_{baseband}]}{\log_2(M)} = \frac{R_b}{\log_2(M)}$$

For proper transmission of the signal,

$$(BW)_{M\text{-PSK}} \le (BW)_{channel}$$

$$\frac{R_b}{\log_2(M)} \le 200\text{ kHz}$$

$$\log_2(M) \ge \frac{10^6}{200\times 10^3} = 5$$

$$M \ge 2^5 = 32$$

$$M_{min} = 32$$

27. (c)

$$x(t) = m(t) + c(t) = m(t) + \cos(2\pi f_c t)$$

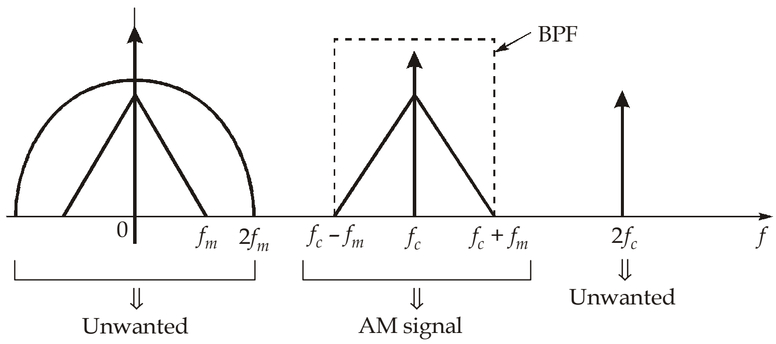
$$y(t) = x(t) + x^2(t)$$

$$= m(t) + \cos(2\pi f_c t) + m^2(t) + 2m(t)\cos(2\pi f_c t) + \cos^2(2\pi f_c t)$$

$$\cos^2(2\pi f_c t) = \frac{1}{2} + \frac{\cos(4\pi f_c t)}{2}$$

So, $$y(t) = \left[\frac{1}{2} + m(t) + m^2(t)\right] + [\cos(2\pi f_c t) + 2m(t)\cos(2\pi f_c t)] + \left[\frac{\cos(4\pi f_c t)}{2}\right]$$

The spectrum of the signal $y(t)$ can be plotted as follows:



To get the desired AM signal at the output of the filter, the following conditions to be satisfied:

$$f_c + f_m < 2f_c \Rightarrow f_c > f_m \quad \text{...(i)}$$

$$f_c - f_m > 2f_m \Rightarrow f_c > 3f_m \quad \text{...(ii)}$$

From (i) $\cap$ (ii), the necessary condition to be satisfied is "$f_c > 3f_m$".

28. (c)

The output of the narrowband FM modulator can be given by,

$$x(t) = A\cos[2\pi f_0 t + \phi(t)]\ ;\ |\phi(t)|_{max} = 0.10\text{ radians}$$

The signal at the output of upper frequency multiplier can be given by,

$$y(t) = A\cos[2\pi n_1 f_0 t + n_1 \phi(t)]$$

After mixing $y(t)$ with the output signal of the lower frequency multiplier, we get,

$$z(t) = A^2 \cos[2\pi n_1 f_0 t + n_1 \phi(t)] \cos[2\pi n_2 f_0 t]$$

$$= \frac{A^2}{2}\cos[2\pi(n_1 + n_2) f_0 t + n_1 \phi(t)] + \frac{A^2}{2}\cos[2\pi(n_1 - n_2) f_0 t + n_1\phi(t)]$$

It is given that the mixer is designed for up-conversion. So, the signal $s(t)$ can be given by,

$$s(t) = \frac{A^2}{2}\cos[2\pi(n_1 + n_2) f_0 t + n_1 \phi(t)] \quad ...(i)$$

It is given that, f_c = 104 MHz and Δf_{max} = 75 kHz for $s(t)$.

So, the modulation index of the wideband signal $s(t)$ will be,

$$\beta = \frac{\Delta f_{max}}{f_{m(max)}} = n_1 |\phi(t)|_{max}$$

$$n_1(0.10) = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5$$

$$n_1 = \frac{5}{0.10} = 50$$

$$f_c = (n_1 + n_2)f_0 = 104 \text{ MHz}$$

$$(n_1 + n_2) \times 100 = 104 \times 1000$$

$$n_2 = 1040 - n_1 = 1040 - 50 = 990$$

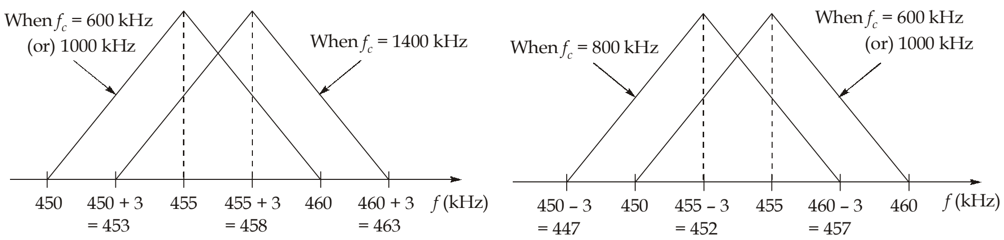
29. (d)

Given that the maximum frequency of the modulating signal is $f_{m\,(max)}$ = 5 kHz. So, the bandwidth of AM modulated signal is 10 kHz.

For ideal tracking (i.e. zero tracking error for any input carrier frequency), the minimum bandwidth required by the IF amplifier is 10 kHz. In this case, the IF amplifier has a flat band from 450 kHz to 460 kHz.

But it is given that the tracking is not ideal. From the given tracking error curve it is clear that only for the carrier frequency 600 kHz and 1000 kHz the tracking error is zero. So, only for these frequencies the spectrum will lie in the range (450 to 460) kHz at the input of the IF amplifier.

From the given tracking error curve, it is clear that, maximum deviation occur from ideality at f_c = 800 kHz, 1400 kHz. The spectrum produced at the input of IF amplifier for these carrier frequencies will be in the range as shown in the figure below.



So, the minimum bandwidth required by the IF amplifier can be given by,

$$\text{BW}_{\text{IF(min)}} = f_{\text{max}} - f_{\text{min}}$$

$$f_{\text{max}} = f_{\text{IF}} + |\text{Max. positive tracking error}| + f_{m(\text{max})}$$

$$= 455 + 3 + 5 = 463 \text{ kHz}$$

$$f_{\text{min}} = f_{\text{IF}} - |\text{Max negative tracking error}| - f_{m(\text{max})}$$

$$= 455 - 3 - 5 = 447 \text{ kHz}$$

So, $$\text{BW}_{\text{IF(min)}} = 463 - 447 = 16 \text{ kHz}$$

30. (a)

The signals $x_1(t)$ and $x_2(t)$ can be given by,

$$x_1(t) = m(t)\cos(2\pi f_1 t) = \cos(2\pi f_1 t)\cos(2\pi f_m t)$$

$$= \frac{1}{2}[\cos 2\pi(f_1 + f_m)t + \cos 2\pi(f_1 - f_m)t]$$

$$x_2(t) = m(t)\sin(2\pi f_1 t) = \sin(2\pi f_1 t)\cos(2\pi f_m t)$$

$$= \frac{1}{2}[\sin 2\pi(f_1 + f_m)t + \sin 2\pi(f_1 - f_m)t]$$

$|f_1 - f_m| < f_1$ and $f_1 = 2f_m$. So, after passing $x_1(t)$ and $x_2(t)$ through respective low-pass filters, we get,

$$y_1(t) = \frac{1}{2}\cos 2\pi(f_1 - f_m)t = \frac{1}{2}\cos(2\pi f_m t)$$

$$y_2(t) = \frac{1}{2}\sin 2\pi(f_1 - f_m)t = \frac{1}{2}\sin(2\pi f_m t)$$

The signals $z_1(t)$ and $z_2(t)$ can be given by,

$$z_1(t) = \cos(2\pi f_2 t)\, y_1(t) = \frac{1}{2}\cos(2\pi f_2 t)\cos(2\pi f_m t)$$

$$z_2(t) = \sin(2\pi f_2 t)\, y_2(t) = \frac{1}{2}\sin(2\pi f_2 t)\sin(2\pi f_m t)$$

Now, the modulated signal $s(t)$ can be given by,

$$s(t) = z_1(t) + z_2(t)$$

$$= \frac{1}{2}[\cos(2\pi f_2 t)\cos(2\pi f_m t) + \sin(2\pi f_2 t)\sin(2\pi f_m t)]$$

$$= \frac{1}{2}\cos 2\pi(f_2 - f_m)t \quad \Rightarrow \text{USSB signal}$$

$$= A\cos 2\pi(f_c + f_m)t \Rightarrow \text{USSB signal}$$

From the above equations, it is clear that,

$f_2 - f_m = f_c + f_m \Rightarrow$ $$f_2 = f_c + 2f_m = 1 \text{ MHz} + 2(5 \text{ kHz}) = 1010 \text{ kHz}$$

■■■■