S.No. : 02 SK1_CE_B_240519

Strength of Material

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CLASS TEST 2019-2020

CIVIL ENGINEERING

Strength of Material

Date of Test : 24/05/2019

Answer Key

1.	(c)	7.	(b)	13.	(c)	19.	(b)	25.	(c)
2.	(b)	8.	(b)	14.	(a)	20.	(d)	26.	(a)
3.	(b)	9.	(d)	15.	(c)	21.	(a)	27.	(d)
4.	(b)	10.	(a)	16.	(a)	22.	(d)	28.	(a)
5.	(b)	11.	(a)	17.	(c)	23.	(b)	29.	(a)
6.	(c)	12.	(b)	18.	(c)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1. (c)

Theory of simple bending is only applicable to section of beam in which plane of loading is axis of symmetry  and  have symmetry about loading axis (vertical axis) so theory of simple bending is applicable only to these sections.

2. (b)

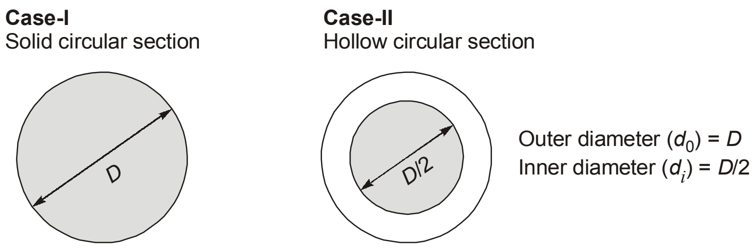
$$R_A = \frac{100}{2} - \frac{25 \times 5/3}{10} = \frac{275}{6} \text{ kN}$$

$\therefore$ Bending moment at D

$$M = R_A \times 5 - \frac{10 \times (5)^2}{2}$$

$$= \frac{275}{6} \times 5 - \frac{10 \times (5)^2}{2} = 104.167 \text{ kN-m}$$

3. (b)



According to question it is given that maximum shear stress would be same in both cases.

From the torsion formula

$$\frac{\tau}{r} = \frac{T}{I_p}$$

$$\frac{\tau_{max}}{R} = \frac{T}{I_p}$$

$$T = \frac{I_p}{R} \times \tau_{max}$$

$$\therefore \quad \left(\frac{T_h}{T_s}\right) = \frac{(I_p)_h}{(I_p)_s} \times \left(\frac{(\tau_{max}/R)_h}{(\tau_{max}/R)_s}\right) = \frac{\frac{\pi}{32}\left(d_0^4 - d_i^4\right)}{\frac{\pi}{32}.(D)^4}$$

$$= \frac{\frac{\pi}{32}\left(D^4 - (D/2)^4\right)}{\frac{\pi}{32}.(D)^4} = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\frac{T_h}{T_s} = \frac{15}{16}$$

5. (b)

Force on vertical bar

$$\Sigma M_C = 0$$

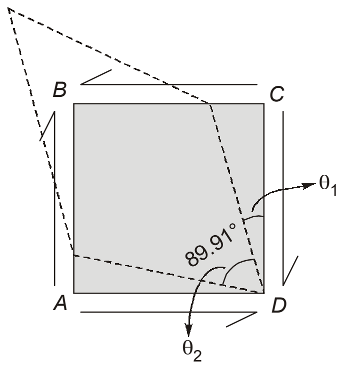
$$P \times L = R \times 2L \qquad \text{(where } R \text{ is reaction at } B\text{)}$$

$$R = \frac{P}{2}$$

$\therefore$ Normal stress in bar AB,

$$\sigma = \frac{R}{A} = \frac{P/2}{A} = \frac{P}{2A}$$

6. (c)



Shear strain,

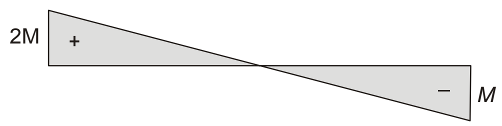
$$\gamma = (\theta_1 + \theta_2)$$

$$= 90° - (89.91°) = 0.09° = 0.09 \times \frac{\pi}{180}$$

$$\gamma = \frac{\pi}{2000}$$

($180° = \pi$ radian, $1° = \pi/180$ radian)

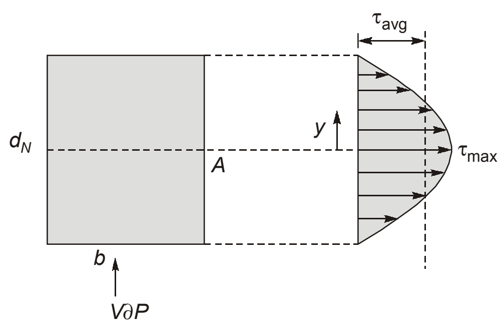
7. (b)



8. (b)

$$\tau = \frac{6V}{bd^3}\cdot\left(\frac{d^4}{4} - y^2\right)$$

$$\tau_{avg} = \frac{V}{bd}$$



According to question given that

$$\tau = \tau_{avg}$$

$$\frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right) = \frac{V}{bd}$$

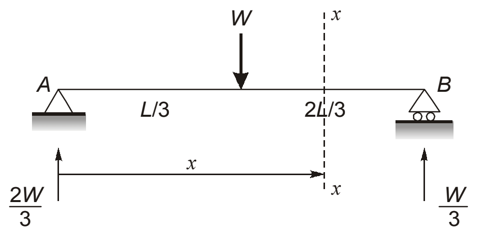
$$\frac{6}{b^2}\left(\frac{d^2}{4} - y^2\right) = 1$$

$$y^2 = \frac{d^2}{4} - \frac{d^2}{6} = \frac{3d^2 - 2d^2}{12}$$

$$y = \pm\frac{d}{\sqrt{12}}$$

$$y = \frac{d}{\sqrt{12}} = \frac{d}{2\sqrt{3}}$$

10. (a)



Using Macaulay's Method:

$$M_{XX} = EI.\frac{d^2y}{dx^2} = \frac{2W}{3}x - W\left(x-\frac{L}{3}\right)$$

Integrating above equation, we get

$$EI\frac{dy}{dx} = \left(\frac{2W}{3}\frac{x^2}{2}+C_1\right) - \frac{W\left(x-\frac{L}{3}\right)^2}{2}$$

Again integrating

$$EIy = \left(\frac{2W}{6}\left(\frac{x^3}{3}\right)+C_1x+C_2\right) - \frac{W\left(x-\frac{L}{3}\right)^3}{2\times 3}$$

At $x = 0$, $y = 0$

$\Rightarrow$ $$C_2 = 0$$

At, $x = L$, $y = 0$

$$EI\times 0 = \frac{2W}{18}L^3 + C_1L - \frac{W\left(L-\frac{L}{3}\right)^3}{2\times 3}$$

$$0 = \frac{WL^3}{9} + C_1L - \frac{8W.L^3}{27\times 6}$$

$$C_1L = -\frac{5}{81}WL^3$$

$$C_1 = -\frac{5}{81}WL^2$$

$\therefore$ $$EIy = \frac{2Wx^3}{18} - \frac{5}{81}WL^2x - \frac{W(x-L/3)^3}{6}$$

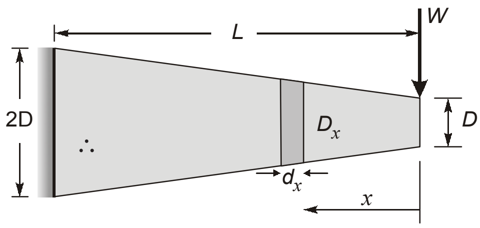
At $$x = \frac{L}{3}$$

$$EIy = \frac{2W(L/3)^3}{18} - \frac{5}{81}WL^2\times\frac{L}{3} - 0 = -\frac{4WL^3}{243}$$

$$y = \frac{4WL^3}{243EI}\downarrow \text{ (downward)}$$

11. (a)

Assuming a cross section at distance x from free end



$$D_x = D + \left(\frac{2D - D}{L}\right)x = D + \frac{Dx}{L}$$

∴

$$I_x = \frac{\pi D_x^4}{64} = \frac{\pi}{64}\cdot\left(D + \frac{Dx}{L}\right)^4$$

By unit load method,

$$\Delta_{\text{free end}} = \int_0^L \frac{Mmdx}{EI} = \int_0^L \frac{(Wx).x.dx}{E\frac{\pi}{64}D^4\left(1+\frac{x}{L}\right)^4}$$

Let,

$$\left(1+\frac{x}{L}\right) = t$$

differentiating w.r.t. t

$$dx = Ldt$$

$$x = (t-1)\,L$$

$$x^2 = (t-1)^2\,.\,L^2$$

∴

$$\Delta_{\text{freend}} = \int_1^2 \frac{W \times 64(t-1)^2 .L^3}{\pi E D^4 t^4} dt \qquad (x = 0,\ t = 1,\ x = L,\ t = 2)$$

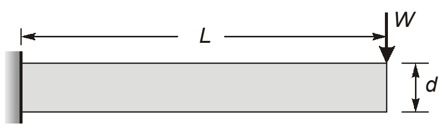
$$= \frac{64WL^3}{\pi ED^4}\left[\int_1^2 \left\{\frac{1}{t^2} + \frac{1}{t^4} - \frac{2}{t^3}\right\} dt\right]$$

$$= \frac{64WL^3}{\pi ED^4}\left[-\frac{1}{t} - \frac{1}{3t^3} + \frac{2}{2t^2}\right]_1^2$$

$$= \frac{64WL^3}{\pi ED^4}\left[-\frac{1}{2} - \frac{1}{24} + \frac{1}{4} + 1 + \frac{1}{3} - 1\right] = \frac{64}{24}\frac{WL^3}{\pi ED^4}$$

∴

$$\Delta_{\text{freend}} = \frac{64}{24}\frac{WL^3}{\pi ED^4} \qquad \text{...(i)}$$



$$\Delta_{\text{freend}} = \frac{WL^3}{3EI} = \frac{WL^3 \times 64}{3E \times \pi d^4} \qquad \text{...(ii)}$$

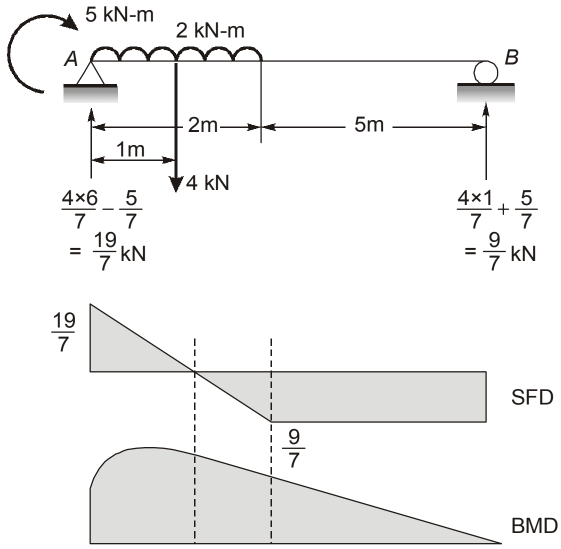
For (i) and (ii) to be equal

$$\frac{64}{24}\frac{WL^3}{\pi ED^4} = \frac{WL^3 \times 64}{3E \times \pi d^4}$$

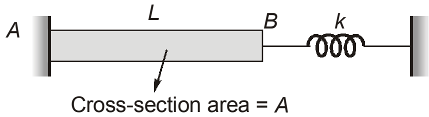
$$d^4 = \frac{24}{3}D^4$$

$$d = 1.6820\,D$$

12. (b)

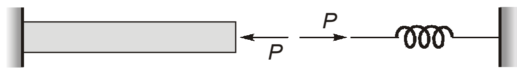


13. (c)



Free expansion of bar (without spring presence) = $L\,\alpha\,T$

$$\text{Compression of spring} = \frac{P}{k}$$



$$\therefore \quad L\alpha T - \frac{P}{k} = \frac{PL}{AE}$$

$$\Rightarrow \quad P\left(\frac{1}{k}+\frac{L}{AE}\right) = L\,\alpha\,T$$

$$\frac{P}{A} = \frac{\alpha TE}{1+\dfrac{AE}{kL}} = \sigma$$

14. (a)

As per Beltrami and Haigh,

$$\sigma_1^2+\sigma_2^2-2\mu\sigma_1\sigma_2 \le \sigma_y^2$$

As per J.J. Guest theory,

$$(\sigma_1-\sigma_2) = \sigma_y$$

As per Von mises and Hencky theory,

$$\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2 \le \sigma_y^2$$

As per St. Venant theory,

$$\sigma_1-\mu\sigma_2 \le \sigma_y$$

15. (c)

Given,
$$\text{Strain matrix} = \begin{bmatrix} 0.004 & 0.006 & 0.008 \\ 0.006 & 0.006 & 0.002 \\ 0.008 & 0.002 & 0.004 \end{bmatrix}$$

From strain matrix, $\quad \in = \begin{bmatrix} \epsilon_{xx} & \gamma_{xy}/2 & \gamma_{xz}/2 \\ \gamma_{yx}/2 & \epsilon_{yy} & \gamma_{zy}/2 \\ \gamma_{xz}/2 & \gamma_{zy}/2 & \epsilon_{zz} \end{bmatrix}$

Comparing these two

we get, $\quad \dfrac{\gamma_{zy}}{2} = 0.002$

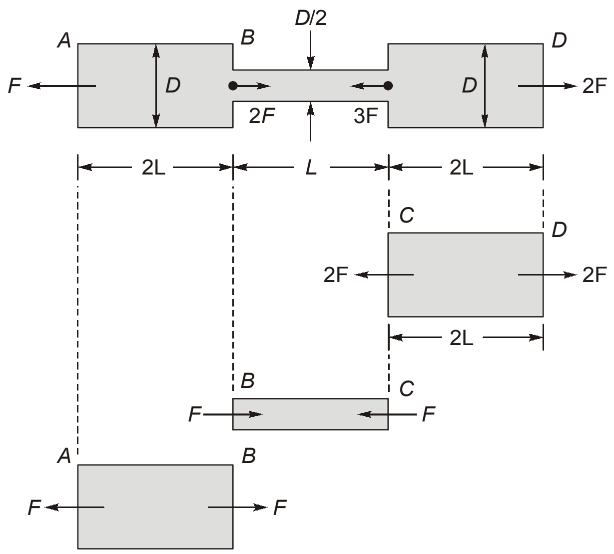
$\gamma_{zy} = 0.004$

Shear modulus, $\quad G = 100$ GPa

$\therefore$ Shear stress $= 100 \times 0.004 \times 10^3$ MPa

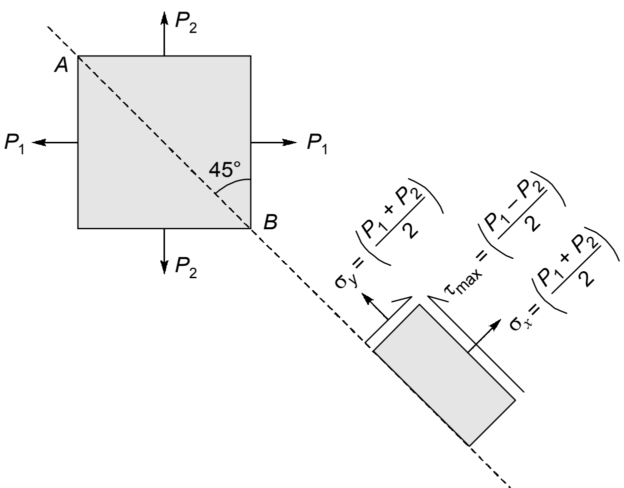
$= 400$ MPa

17. (c)



$$\sigma_{AB} : \sigma_{BC} : \sigma_{CD} = \frac{F}{\frac{\pi}{4}D^2} : \frac{F}{\frac{\pi}{4}\left(\frac{D}{2}\right)^2} : \frac{2F}{\frac{\pi}{4}D^2} = 1 : 4 : 2$$

18. (c)



If $\qquad P_1 = P_2$

then $\qquad \tau_{max} = 0$

19. (b)

Given, L = 2m, P = 120 N, L = 2000 mm

Central deflection = 4 mm

$$\therefore \quad \frac{PL^3}{48EI} = 4 \text{ mm}$$

$$\therefore \quad \frac{(120\text{ N})(2000\text{ mm}) \times L^2(\text{mm}^2)}{48E(\text{N/mm}^2).I(\text{mm}^4)} = 4$$

$$\frac{EI}{l^2} = \frac{120 \times 2000}{48 \times 4} = 1250 \text{ N}$$

Now, as per Euler,
$$P_e = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2} \qquad (l_e = L)$$

$$= \pi^2 \times 1250 \text{ N} = \frac{\pi^2 \times 1250}{1000} \text{ kN}$$

$$P_e = 12.34 \text{ kN}$$

20. (d)

Longitudinal stress,
$$\sigma_x = \frac{PD}{4t}$$

Hoop stress,
$$\sigma_\theta = \frac{PD}{2t}$$

Torsional shear stress,
$$\tau_{x\theta} = \frac{T}{z_p} = \frac{T}{\pi D^2 t} \qquad (z_p = \text{Polar modulus of section})$$

21. (a)

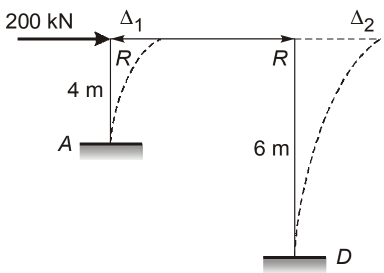
$$\Delta_1 = \Delta_2$$

$$\frac{(200-R)(4)^3}{3EI} = \frac{R(6)^3}{3EI}$$

$$\frac{200-R}{R} = \frac{27}{8}$$

$$R = 45.71 \text{ kN}$$

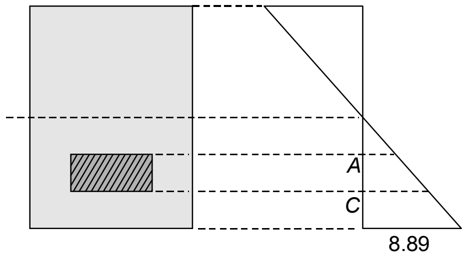
$$\therefore \quad H_A = 200 - 45.71 = 154.29 \text{ kN}$$



23. (b)

$$\sigma_{max} = \frac{M}{I} y_{max}$$

$$= \frac{20 \times 10^6 \times 150}{\left(\dfrac{150 \times (300)^3}{12}\right)} = 8.89 \text{ MPa}$$

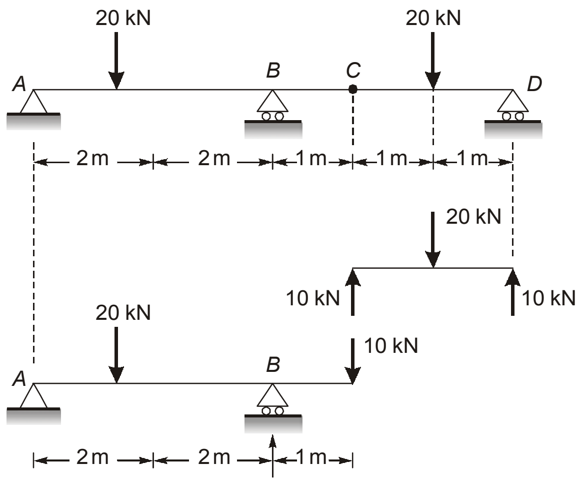


$$\sigma_A = \frac{8.89}{150} \times 50 = 2.96 \text{ MPa}$$

$$\sigma_C = \frac{8.89}{150} \times 100 = 5.93 \text{ MPa}$$

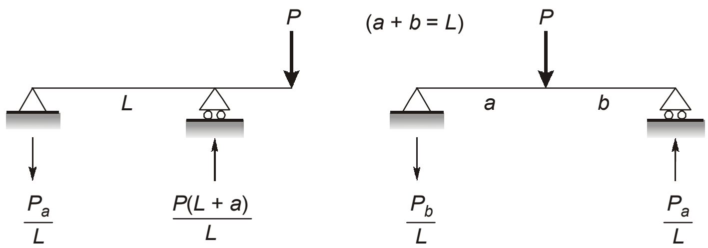
Tensile force in hatched area $= \frac{1}{2}(2.96 + 5.93) \times 50 \times 50 = 11.107$ kN

24. (d)



$$R_B = \frac{20}{2} + \frac{10(4+1)}{4} = 10 + 12.5 = 22.5 \text{ kN}$$

Note:



25. (c)

The vertical jump at 'c' indicates that a couple is acting at section 'c' from BMD,

Couple = 3000 – 1000 = 2000 N-m

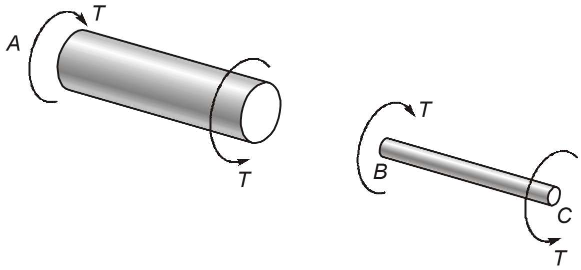
26. (a)

As we know that, (angle of twist)

$$\theta = \frac{TL}{GJ}$$

J for a circular bar of diameter 'd' is $\dfrac{\pi d^4}{32}$

The total angle of twist, θ_{total} is equal to the sum of the angles of twist for the two different sections. Torques is same for gboth sections.



$$\theta_{AC} = \theta_{AB} + \theta_{BC} = \frac{T.(2L)}{GJ_1} + \frac{T.L}{GJ_1}$$

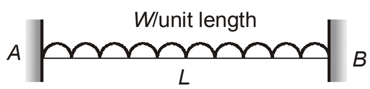
$$0.0225 = \frac{T.(2L)}{G\dfrac{\pi(2)^4}{32}} + \frac{T.L}{G\dfrac{\pi(1)^4}{32}}$$

$$0.0225 = \frac{T.(2L)}{G\pi}(32+4)$$

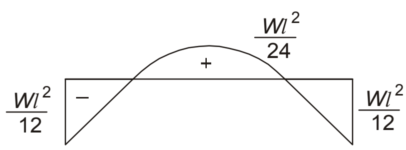
$$T = \frac{0.0225}{36} \times \frac{G\pi}{L} = 6.25\ 5\ 10^{-4}\ \frac{G\pi}{L}$$

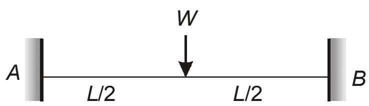
$$T = 0.000625\ \frac{G\pi}{L}$$

27. (d)

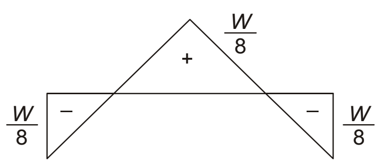


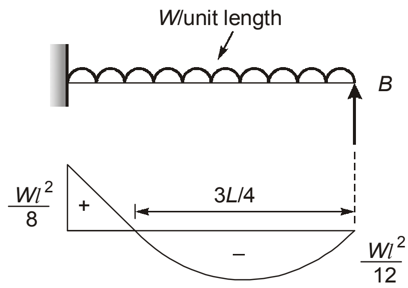
(a) Support moment $= \dfrac{\omega l^2}{12}$ (hogging)



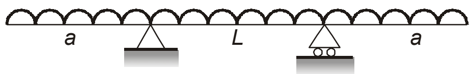


(b) Support moment $= \dfrac{WL}{8}$ (hogging)



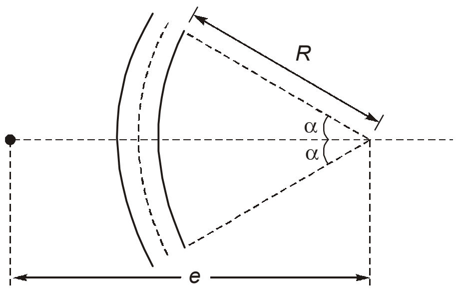


(c) Moment at fixed end $= \dfrac{WL^2}{8}$ (sagging)



(d) Support moment $= \dfrac{Wa^2}{2}$ (hogging)

28. (a)

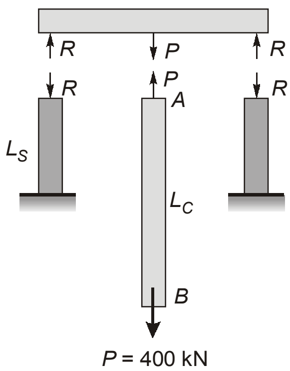


$$e = \frac{2R(\sin\alpha - \alpha \, . \cos\alpha)}{(\alpha - \sin\alpha \, . \cos\alpha)}$$

For circular arc $\alpha = \pi$

$\therefore$ $e = 2R$

30. (c)



$$E_S = 200 \text{ kN/mm}^2$$
$$E_C = 105 \text{ kN/mm}^2$$
D_C = diameter of copper bar = 100 mm

D_S = diameter of steel bar = 50 mm

$$R = \frac{P}{2} = 200 \text{ kN}$$

$\therefore$ Vertical displacement of point B

$$= \frac{P.L_C}{A_C.E_C} + \frac{(P/2).L_S}{A_S.E_S}$$

$$= \frac{400\times10^3(\text{N})\times8.8\times1000(\text{mm})}{\frac{\pi}{4}.(100)^2 \text{mm}^2\times105\times10^3\frac{\text{N}}{\text{mm}^2}} + \frac{200\times10^3(\text{N})\times0.8\times1000(\text{mm})}{\frac{\pi}{4}.(50)^2 \text{mm}^2\times200\times10^3\frac{\text{N}}{\text{mm}^2}}$$

$$= 4.675 \text{ mm}$$

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