CLASS TEST

S.No. : 01PT_ME_A+B_080522

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STRENGTH OF MATERIALS

MECHANICAL ENGINEERING

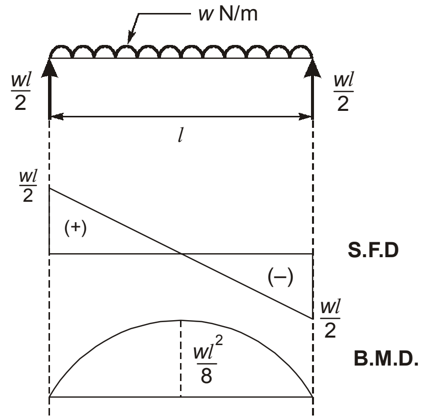
Date of Test : 10/05/2022

ANSWER KEY

1.	(d)	7.	(a)	13.	(c)	19.	(c)	25.	(a)
2.	(a)	8.	(b)	14.	(b)	20.	(a)	26.	(b)
3.	(c)	9.	(c)	15.	(b)	21.	(b)	27.	(a)
4.	(b)	10.	(c)	16.	(d)	22.	(b)	28.	(b)
5.	(d)	11.	(d)	17.	(a)	23.	(a)	29.	(c)
6.	(d)	12.	(c)	18.	(b)	24.	(b)	30.	(a)

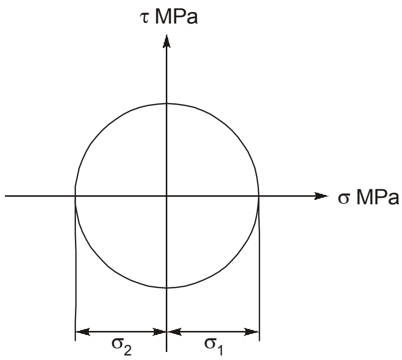
DETAILED EXPLANATIONS

1. (d)



2. (a)

It is a case of pure shear stress,



$$\sigma_1 = +400 \text{ MPa}$$
$$\sigma_2 = -400 \text{ MPa}$$
$$\tau = 0$$

3. (c)

As per maximum shear stress theory,

$$\text{Absolute } \tau_{max} = \text{Max of } \left[\frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_1}{2}, \frac{\sigma_2}{2}\right]$$
$$= \frac{\sigma_{yt}}{2} \left(\sigma_{yt} = \text{yield point stress}\right)$$
$$= \frac{80}{2} = \frac{\sigma_{yt}}{2}$$
$$\therefore \quad \sigma_{yt} = 80 \text{ MPa}$$

4. (b)

$\frac{T}{J} = \frac{\tau}{R}$, Here T and τ are same, so $\frac{J}{R}$ should be same i.e. polar section modulus will be same.

5. (d)

Elongation due to self weight $= \frac{\gamma L^2}{2E}$ γ-(specific weight).

6. (d)

Toughness is the ability of material to absorb the energy upto facture point i.e. toughness is the total area under stress-strain curve.

7. (a)

$$\text{Circumferential stress} = \frac{Pd_i}{2t}$$

$$P = 6 \text{ MPa}$$
$$d_i = 600 \text{ mm}$$
$$t = 10 \text{ mm}$$

$$\sigma_c = \frac{6\times 600}{2\times 10} = 180 \text{ MPa}$$
$$= 180 \times 1000 \text{ kPa}$$
$$= 18 \times 10^4 \text{ kPa}$$

8. (b)

$$M = 3000 \text{ Nm}$$
$$T = 4000 \text{ Nm}$$
$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$
$$= \frac{1}{2}\left[3000 + \sqrt{(3000)^2 + (4000)^2}\right]$$
$$= 8000 \times \frac{1}{2} = 4000 \text{ Nm}$$

9. (c)

$$\sigma_x = 10 \text{ MPa}$$
$$\sigma_y = 7 \text{ MPa}$$
$$\tau_{xy} = 2 \text{ MPa}$$

$$\max\left(\frac{\sigma_1}{2}, \frac{\sigma_2}{2}, \frac{\sigma_1 - \sigma_2}{2}\right) = \frac{S_{yt}}{2N}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{17}{2} \pm \frac{\sqrt{3^2 + 4\times 4}}{2} = \frac{17 \pm 5}{2}$$

$$\sigma_1 = 11 \text{ MPa} \qquad \sigma_2 = 6 \text{ MPa}$$

$$\Rightarrow \quad \frac{\sigma_1}{2} = \frac{S_{yt}}{2N}$$

$$\Rightarrow \quad \sigma_1 = \frac{S_{yt}}{N} = \frac{18}{N}$$

$$\Rightarrow \quad 11 = \frac{18}{N}$$

$$\Rightarrow \quad N = 1.636$$

10. (c)
Given data: $d = 2$ m , $t = 40$ mm, $P = 3.5$ kPa, $F = 200$ kN

$$E = 2G(1 + \mu)$$

$$\frac{200}{2 \times 80} = 1 + \mu$$

$$\mu = 0.25$$

$$\sigma_h = \frac{Pd}{2t}$$

$$= \frac{3.5 \times 2000}{2 \times 40} \times 10^3 = 87.5 \text{ kPa}$$

$$\sigma_l' = \frac{Pd}{4t} = 0.04375 \text{ MPa} = 43.75 \text{ kPa}$$

$$\sigma_l'' = \frac{F}{\pi dt} = 795.77 \text{ KPa}$$

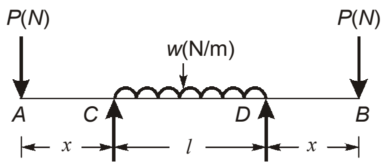
$$\Rightarrow \quad \varepsilon_h = \frac{1}{E}\left[\sigma_h - \mu(\sigma_l' + \sigma_l'')\right] = -6.12 \times 10^{-7}$$

11. (d)
We know that strain energy,

$$U = \frac{P^2 L}{2AE}$$

It is obvious from the above equation that strain energy is proportional to the square of load applied. We know that sum of squares of two number is less than the square of their sum.
$[(P_1 + P_2)^2 > P_1^2 + P_2^2]$
Thus $U > U_1 + U_2$

12. (c)



From the symmetry of the figure,

$$R_C = R_D = P + \frac{wl}{2}$$

Bending moment at mid point,

$$= -\frac{wl}{2} \times \frac{l}{4} + R_C \times \frac{l}{2} - P\left(x + \frac{l}{2}\right) = 0$$

gives $$x = \frac{wl^2}{8P}$$

13. (c)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \text{ or } \tau = \frac{GR\theta}{L}$$

$$\tau \propto \frac{1}{L}$$

14. (b)

$$\text{Area} = 50 \times 110 = 5500 \text{ mm}^2$$

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$\text{Young's modulus} = \frac{\text{Longitudnal stress}}{\text{Longitudnal strain}}$$

$$\text{Stress} = \frac{F}{A} = \frac{30 \times 10^3}{5500} = 5.4545 \text{ N/mm}^2$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{0.625}{1500} = 4.16 \times 10^{-4}$$

$$E = \frac{5.4545}{4.16 \times 10^{-4}} = 13090.90 \text{ N/mm}^2$$

15. (b)

$$A = \frac{\pi}{4} \times 80^2 = 5026.55 \text{ mm}^2$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 80^4$$

$$I = 2010619.30 \text{ mm}^4$$

Direct compressive stress due to horizontal component,

$$\sigma_a = \frac{15 \cos 30° \times 10^3}{A} = \frac{15000 \times \cos 30°}{5026.55}$$

$$\sigma_a = 2.584 \text{ MPa (comp.)}$$

Bending stress at point B due to vertical component of loads,

$$\sigma_b = \frac{My}{I} \text{(Comp. at point } B)$$

$$\sigma_b = \frac{(15 \times 10^3 \sin 30°) \times 0.5 \times 40 \times 10^{-3}}{2010619.30 \times 10^{-12}}$$

Bending stress at point B, $\sigma_b = 74.604$ MPa (Comp.)

Total stress at point B, $\sigma_{total} = \sigma_b + \sigma_a = 74.604 + 2.584$

$$\sigma_{\text{total}} = 77.188 \text{ MPa} \quad \text{(Comp.)}$$

16. (d)

$$M = EI \frac{d^2 y}{dx^2}$$

$$\Rightarrow \quad M = EI \frac{d^2}{dx^2}\left(\frac{-2W_0 L^4}{\pi^4 EI} \sin\left(\frac{\pi x}{L}\right)\right)$$

$$\Rightarrow \quad M = \frac{-2W_0 L^4}{\pi^4} \times \left[\frac{d^2}{dx^2}\left(\sin\left(\frac{\pi x}{L}\right)\right)\right]$$

$$\Rightarrow \quad M = \frac{2W_0 L^2}{\pi^2} \sin\left(\frac{\pi x}{L}\right)$$

$$\Rightarrow \quad \text{Shear force} = EI \frac{d^3 y}{dx^3} = \frac{2W_0 L}{\pi} \cos\left(\frac{\pi x}{L}\right)$$

$$\Rightarrow \quad \text{Load} = -EI \frac{d^4 x}{dx^4} = +2W_0 \sin\left(\frac{\pi x}{L}\right)$$

17. (a)

Shear force is varying linearly between AC, therefore, there is uniformly distributed load between AC.

$$R_A = 81 \text{ kN}$$

Uniformly distributed load between A and C

$$= \frac{81-(-19)}{5} = 20 \text{ kN/m}$$

$$\text{Vertical load at } D = 59 - 19 = 40 \text{ kN}$$

$$R_E = 59 \text{ kN}$$

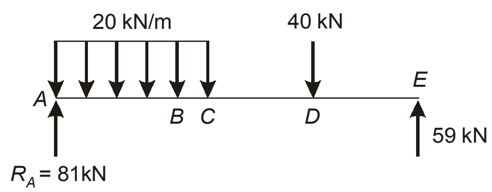
Now,
$$\frac{81}{a} = \frac{19}{5-a}$$

$$\Rightarrow \quad a = \frac{405}{100} = 4.05$$

Bending moment is maximum where shear force changes sign.

$$\text{Maximum Bending Moment} = \frac{1}{2} \times 81 \times 4.05 = 164.025 \text{ kN-m}$$

Alternate :



For maximum bending moment,

$$\text{S.F.} = 0$$

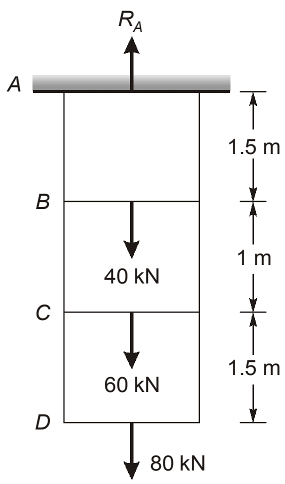
$$81 = 20x$$

$$x = 4.05$$

$$\text{Maximum Bending Moment} = 81 \times 4.05 - \frac{20 \times 4.05 \times 4.05}{2}$$

$$= 164.025 \text{ kN-m}$$

18. (b)



$$R_A = 40 + 60 + 80 = 180 \text{ kN}$$

Force P_1 on portion AB = 180 kN (tensile)

Force P_2 on portion BC = 180 – 40 = 140 kN (tensile)

Force P_3 on portion CD = 80 kN (tensile)

$$\Delta = \frac{1}{AE}(P_1L_1 + P_2L_2 + P_3L_3)$$

$$\Delta = \frac{1}{1200 \times 2.05 \times 10^5} \times [180 \times 1500 + 140 \times 1000 + 80 \times 1500] \times 10^3$$

$$= \frac{530000 \times 10^3}{1200 \times 2.05 \times 10^5} = 2.15 \text{ mm}$$

19. (c)

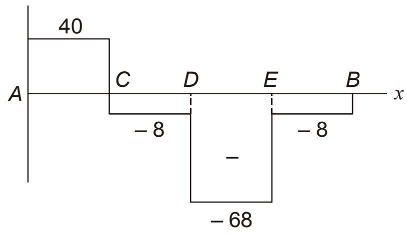
$$\Sigma M_B = 0$$

$$R_A \times 4.5 = 48 \times 3 + 60 \times 1.5 - 60 \times 0.9$$

$$R_A = 40 \text{ kN (upward)}$$

$$R_B = 48 - 40 = 8 \text{ kN (upward)}$$

Making shear force diagram



Shear force diagram

Maximum shear force = 68 kN

20. (a)

$$\text{Weight of water} = \rho Vg = 1000 \times \frac{\pi}{4} \times d^2 \times l \times 9.81$$

$$= 1000 \times \frac{\pi}{4} \times (0.5)^2 \times 10 \times 9.81 = 19261.89\ N$$

And this is uniformly distributed,

$$\Rightarrow \quad M_{maximum} = \frac{wl^2}{8} = \frac{19261.89 \times 10 \times 1000}{8} \text{ N-mm}$$

$$\sigma_{maximum} = \frac{M_{max}}{Z} = \frac{19261.89 \times 10 \times 1000}{8 \times \pi \times d^2 \times t} \times 4$$

$$= \frac{19261.89 \times 10 \times 1000 \times 4}{8 \times \pi \times (500)^2 \times 25} = 4.91 \text{ MPa}$$

21. (b)

We know that,

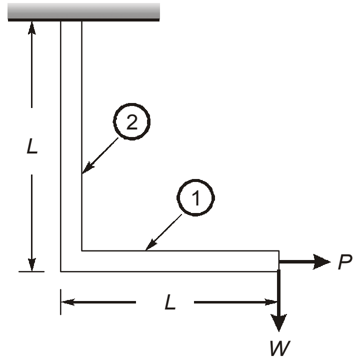
$$\frac{\Delta V}{V} = \frac{3\sigma}{E}[1 - 2\mu]$$

$$\therefore \quad \frac{\Delta V}{200 \times 100 \times 50} = \frac{3 \times 15}{200 \times 1000}[1 - (2 \times 0.3)]$$

$$\Delta V = 90 \text{ mm}^3$$

22. (b)

For finding the vertical deflection assuming a dummy vertical load W.



$$(S.E)_1 = \int_0^L \frac{(Wx)^2 dx}{2EI} = \frac{W^2L^3}{6EI}$$

$$(S.E)_2 = \int_0^L \frac{(WL - Px)^2 dx}{2EI}$$

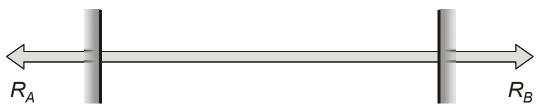
$$= \int_0^L \frac{(W^2L^2 + P^2x^2 - 2WLx)\,dx}{2EI} = \frac{W^2L^3}{2EI} + \frac{P^2L^3}{6EI} - \frac{2WPL^3}{4EI}$$

$$(S.E)_{\text{Total}} = \frac{W^2L^3}{6EI} + \frac{W^2L^3}{2EI} + \frac{P^2L^3}{6EI} - \frac{2WPL^3}{4EI}$$

$$\delta_A = \frac{(S.E)}{\partial W} = \frac{2WL^3}{6EI} + \frac{2WL^3}{2EI} + 0 - \frac{2PL^3}{4EI}$$

Putting $W = 0$, $$\delta_A = \left(\frac{PL^3}{2EI}\right) = \frac{PL^3}{2EI} \text{(downward)}$$

23. (a)



$R_A = R_B$ = Initial tension (P_1) + Tension produced due to thermal contraction (P_2)

$$\frac{P_2 l}{AE} = \alpha \Delta T l$$

$$\Rightarrow \quad P_2 = \alpha \Delta TAE$$

$$= 19 \times 10^{-6} \times 25 \times \frac{\pi}{4} \times (1.6)^2 \times 100 \times 10^3 = 95.504 \text{ N}$$

$$R_A = P_1 + P_2 = 245.504 \text{ N}$$

$$\text{Normal stress} = \frac{R_A}{A} = \frac{245.504}{\frac{\pi}{4} \times (1.6)^2} = 122.104 \text{ MPa}$$

$$\text{Maximum shear stress} = \frac{\sigma}{2} = 61.05 \text{ MPa}$$

24. (b)

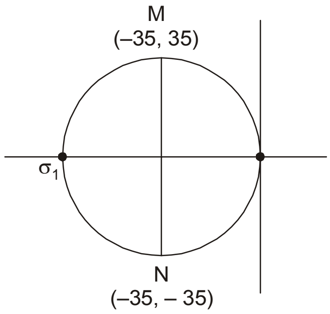
According to given conditions,

$$(\sigma_n)_{\text{at }\theta = 45^\circ} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta = 0$$

$$0 = -35 + \tau_{xy}$$

$$\Rightarrow \quad \tau_{xy} = 35 \text{ MPa}$$

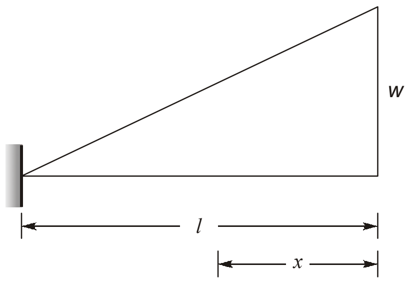
Using Mohr's circle,



Radious at Mohr's circle = 35 MPa

σ_1 = Major principal stress = $2R$ = 70 MPa (Compressive)

25. (a)



$$\text{Maximum shear force} = \frac{wl}{2} = 37.5 \text{ kN}$$

$$\Rightarrow \quad w = \frac{37.5 \times 2}{2} = 37.5 \text{kN/m}$$

$$\text{Bending moment at}, x = l$$

$$= \frac{wl}{2} \times \frac{2}{3} \times l = \frac{wl^2}{3}$$

$$= \frac{37.5 \times 10^3}{10^3} \times \frac{(2)^2 \times 10^6}{3} = 50 \times 10^6 \text{ N-mm}$$

26. (b)

$$J_s = \frac{\pi}{32} \times (50)^4 = 613592 \text{ mm}^4$$

$$J_b = \frac{\pi}{32} \times \left(75^4 - 50^4\right) = 2492719 \text{ mm}^4$$

$$T_s = \frac{G_s J_s \theta}{l} \quad \text{and} \quad T_b = \frac{G_b J_b \theta}{l}$$

$$\text{The total torque, } T = T_s + T_b = (G_s J_s + G_b J_b)\frac{\theta}{l}$$

$$\theta = \frac{Tl}{G_sJ_s + G_bJ_b} = \frac{(800\times10^3)(1.5\times10^3)}{(8\times10^4\times613592)+(4\times10^4\times2492719)}$$

$$= 0.008065 \text{ radian} = 0.462°$$

27. (a)

$$\text{Total strain energy} = \frac{1}{2}P_1\delta_1 + \frac{1}{2}P_2\delta_2 + \frac{1}{2}P_3\delta_3$$

$$= \frac{1}{2}\times P\times\frac{PL_1}{A_1E} + \frac{1}{2}P\times\frac{PL_2}{A_2E} + \frac{1}{2}\times P\times\frac{PL_3}{A_3E}$$

$$= \frac{P^2}{2E}\left[\frac{L_1}{A_1}+\frac{L_2}{A_2}+\frac{L_3}{A_3}\right]$$

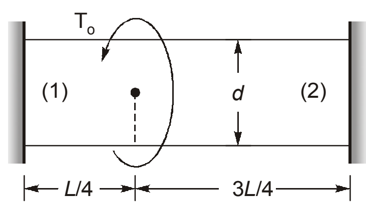
$$= \frac{(10\times1000)^2}{2\times200\times10^9}\left[\frac{100\times4\times1000}{\pi\times(30)^2}+\frac{120\times4\times1000}{\pi\times(20)^2}+\frac{80\times4\times1000}{\pi\times(10)^2}\right]$$

$$= 0.3855 \text{ N-m}$$

28. (b)

$$T_1 = \frac{T_0\times\frac{3L}{4}}{L} = \frac{3T_0}{4}$$

$$T_2 = \frac{T_0\times\frac{L}{4}}{L} = \frac{T_0}{4}$$



$$\text{Maximum shear stress} = \frac{16\times\frac{3T_0}{4}}{\pi d^3} = \frac{12T_0}{\pi d^3}$$

At, $r = 0$

Shear stress $= 0$

$$\text{Difference} = \frac{12T_0}{\pi d^3} - 0 = \frac{12T_0}{\pi d^3}$$

29. (c)

Force perpendicular to beam = $P\cos 30°$

Axial force in beam = $P\sin 30°$ (compressing)

$$\text{Direct stress, } \sigma_d = \frac{-P\sin30°}{A} = \frac{20\times10^3\times0.5}{480\times10^{-4}} = 0.2083 \text{ MPa}$$

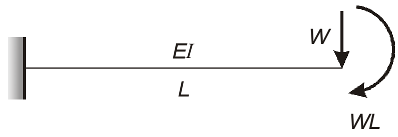
$$\text{Maximum bending stress} = \frac{WL}{4}/Z = \frac{\frac{P\cos30°}{4}\times\frac{3}{\cos30°}}{24\times10^{-4}} = 6.25 \text{ MPa}$$

Maximum compressive stress in the beam

$$= 0.2083 + 6.25 = 6.4583 \text{ MPa}$$

30. (a)

The given cantilever beam can be modified in to a beam as shown below



$$\text{Deflection at, } Q = \frac{WL^3}{3EI} + \frac{WL \times L^2}{2EI}$$

$$= \frac{2WL^3 + 3WL^3}{6EI} = \frac{5WL^3}{6EI}$$

$$\text{Slope at, } Q = \frac{WL^2}{2EI} + \frac{WL \times L}{EI}$$

$$= \frac{WL^2 + 2WL^2}{2EI} = \frac{3WL^2}{2EI}$$

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