CLASS TEST

S.No. : 01PT_CE_A+B_080522

MADE EASY
India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Lucknow | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | E-mail: info@madeeasy.in | Ph: 011-45124612

STRENGTH OF MATERIALS

CIVIL ENGINEERING

Date of Test : 10/05/2022

ANSWER KEY

1. (b)	7. (c)	13. (b)	19. (a)	25. (c)
2. (b)	8. (d)	14. (b)	20. (b)	26. (b)
3. (a)	9. (c)	15. (b)	21. (a)	27. (d)
4. (c)	10. (a)	16. (d)	22. (d)	28. (b)
5. (d)	11. (d)	17. (c)	23. (d)	29. (c)
6. (c)	12. (b)	18. (d)	24. (a)	30. (b)

DETAILED EXPLANATIONS

1. (b)

Let, the stress developed on each side is σ.

Strain along one side due to $\sigma = \frac{\sigma}{E}(1-2\mu)$

Strain along one side due to temperature rise = αT

As cube is restrained from all sides, therefore, both strains should cancel out each other i.e. algebric sum of both strains should be zero.

$$\Rightarrow \quad \frac{\sigma}{E}(1-2\mu) = \alpha T$$

$$\Rightarrow \quad \sigma = \frac{E\alpha T}{1-2\mu}$$

2. (b)

$$\because \quad \text{Stiffness, } k \propto \frac{1}{\text{Number of coils } (n)}$$

$$\Rightarrow \quad k_1 n_1 = k_2 n_2$$

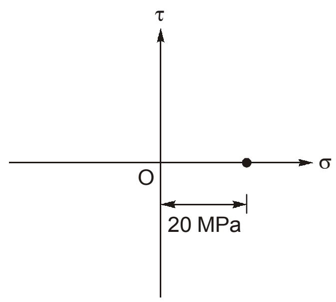
$$\Rightarrow \quad k_1 \times 25 = k_2 \times 20$$

$$\therefore \quad k_2 = 1.25\, k_1$$

3. (a)

This is the case of hydrostatic loading and in this case Mohr's circle results in a point.

$\therefore$ Diameter of resulting Mohr's circle = 0 MPa



4. (c)

A couple anywhere in the beam will cause equal and opposite support reactions in the beam. So the SFD will be rectangular or uniform throughout the beam.

6. (c)

For beam of uniform strength, maximum bending stress remains constant throughout.

7. (c)

$$\sigma_x = 60 \text{ N/mm}^2$$

$$\sigma_y = -20 \text{ N/mm}^2$$

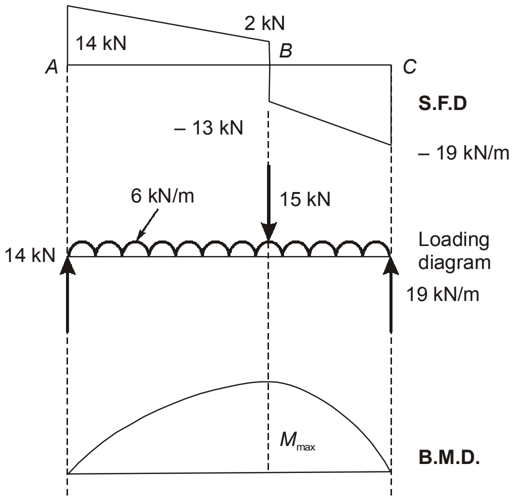
$$\tau = 40 \text{ N/mm}^2$$

$$\text{Normal stress, } \sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau \sin 2\theta$$

$$= 60 \cos^2 60^\circ - 20 \sin^2 60^\circ + 40 \sin 120^\circ$$

$$= 34.64 \text{ N/mm}^2$$

8. (d)



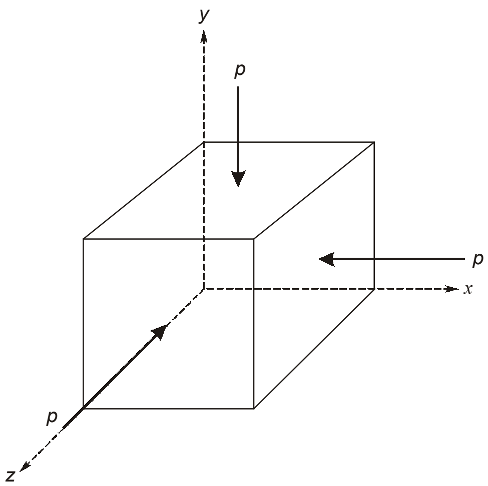
$$M_B - M_A = (BM)_B$$
$=$ Area of shear force diagram from A to B since $M_A = 0$, due to simply supported beam.

$$= \frac{1}{2}(14+2)\times 2 = 16\,\text{kN-m}$$

10. (a)
Since slope at $B = 0$, therefore beam AB will act as fixed beam.

$$\therefore \quad \frac{wL^2}{12} = \frac{wa^2}{2} \Rightarrow a = \frac{L}{\sqrt{6}}$$

11. (d)



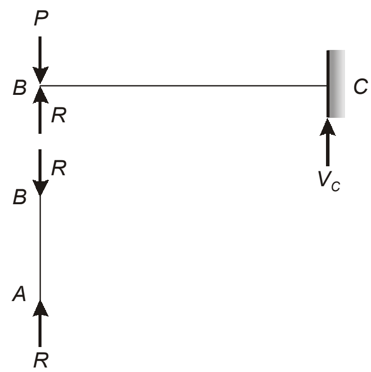
$\because$ Pressure is a compressive stress

$$\therefore \quad \varepsilon_x = -\frac{200}{200\times 10^3} - \frac{1}{4}\left(\frac{-200}{200\times 10^3}\right) - \frac{1}{4}\left(\frac{-200}{200\times 10^3}\right)$$
$$= -5\times 10^{-4}\ \text{mm/mm}$$

$$\therefore \quad \text{Elongation, } \Delta_x = \varepsilon_x L_x = -5\times 10^{-4}\times 50$$
$$= -0.025\ \text{mm} = -2.5\times 10^{-2}\ \text{mm}$$

12. (b)

Let the reaction at support A be R.



$$\text{Deflection at } A \text{ in beam } BC = \text{Compression in column } AB$$

$$\frac{(P-R)L^3}{3EI} = \frac{RL}{AE}$$

$$\frac{(P-R)L^2}{3I} = \frac{R}{A}$$

$$\frac{PL^2}{3I} = \frac{R}{A} + \frac{RL^2}{3I}$$

$$\frac{PL^2}{3I} = R\left[\frac{3I + AL^2}{3IA}\right]$$

$$R = \frac{PAL^2}{3I + AL^2} = \frac{P}{1+\left(\frac{3I}{AL^2}\right)}$$

13. (b)

$$\text{Total load on beam, } w = 10 + 20 = 30 \text{ kN/m}$$

$$\text{Maximum shear force, } F = \frac{wL}{2} = \frac{30 \times 8}{2} = 120 \text{ kN}$$

$$\text{Average shear stress, } \tau_{avg} = \frac{F}{A} = \frac{120 \times 10^3}{\frac{1}{2} \times 200 \times 300} = 4 \text{ N/mm}^2$$

$$\text{Maximum shear stress, } \tau_{max} = \frac{3}{2}\tau_{avg} = \frac{3}{2} \times 4 = 6 \text{ N/mm}^2$$

14. (b)

$$\because \quad \frac{G\theta}{L} = \frac{T}{J}$$

$$\Rightarrow \quad \theta = \frac{TL}{GJ} = \frac{T}{(GJ/L)} = \frac{100}{10{,}000} = 0.01 \text{ rad}$$

$$\text{Also, torsional strain energy} = \frac{1}{2} \times T\theta$$

$$= \frac{1}{2} \times 100 \times 0.01 = 0.5 \text{ kN-m} \neq 1 \text{ kN-m}$$

15. (b)

$$M = 3.5 \text{ kNm}; \; T = 5 \text{ kNm}; \; d = 80 \text{ mm}$$

$$\sigma_1 = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] \times 10^6 = \frac{16}{\pi (80)^3}\left[3.5 + \sqrt{(3.5)^2 + 5^2}\right] \times 10^6$$

$$= 95.5 \text{ N/mm}^2$$

$$\sigma_2 = \frac{16}{\pi d^3}\left[M - \sqrt{M^2 + T^2}\right] \times 10^6 = -25.89 \text{ N/mm}^2$$

$$\text{Maximum strain, } \varepsilon = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{1}{E}[95.5 + 0.28 \times 25.89] = \frac{102.75}{E}$$

If 'σ' be the stress producing the same maximum strain then,

$$\frac{\sigma}{E} = \frac{102.75}{E}$$

$$\Rightarrow \quad \sigma = 102.75 \text{ N/mm}^2$$

16. (d)

$$M_1 = \frac{PL}{4} = \frac{4 \times 4}{4} = 4 \text{ kNm}$$

$$M_2 = \frac{wl^2}{8} = \frac{1 \times 4^2}{8} = \frac{1 \times 16}{8} = 2 \text{ kNm}$$

Given, same area, same depth, so, by pure bending formula,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \quad \sigma \propto M$$

$$\frac{\sigma_2}{\sigma_1} = \frac{M_2}{M_1} = 0.5$$

18. (d)

Shear force at A = 400 N

Bending moment at A = 240 Nm

Torque at A = 100 Nm

19. (a)

For the triangular portion, let the load per unit metre is w.

$$\therefore \quad \frac{1}{2} \times w \times 2 = 60$$

$$\Rightarrow \quad w = 60 \text{ kN/m}$$

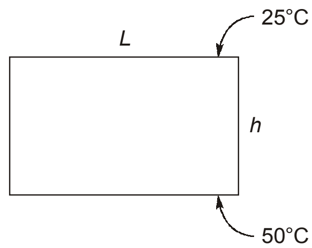
Now,

$$\sum M_A = 0$$

$$\Rightarrow 20 \times 5 - R_D \times 4 + \frac{1}{2} \times 60 \times 2\left(1 + \frac{2}{3} \times 2\right) = 0$$

$$\Rightarrow \quad R_D = 60 \text{ kN}$$

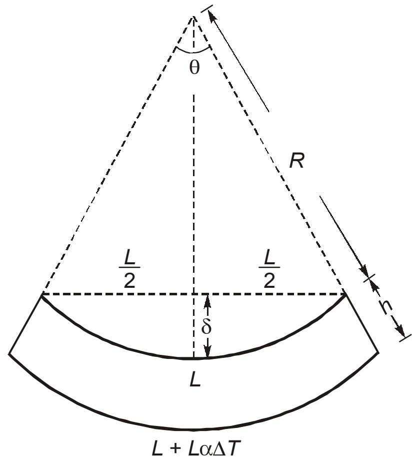
20. (b)



Let top fibre is assume to be reference.

$\therefore$ The change in temperature between top and bottom fibre,

$$\Delta T = 25°C$$



From the property of circle

$$(2R-\delta)\delta = \frac{L}{2}\cdot\frac{L}{2}$$

$$2R\delta - \cancel{\delta^2}^{\,0} = \frac{L^2}{4}$$

$$\therefore \quad R = \frac{L^2}{8\delta}$$

Now,
$$\text{arc}\,(L) = R\theta$$
$$\text{arc}\,(L + L\alpha\Delta T) = (R+h)\,\theta$$

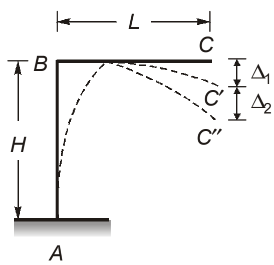
$$\therefore \quad \frac{L+L\alpha\Delta T}{L} = \frac{(R+h)\theta}{R\theta}$$

$$1 + \alpha\Delta T = 1+\frac{h}{R}$$

$$\therefore \quad h = R\alpha\Delta T = \frac{L^2}{8\delta}\alpha\,\Delta T = 0.1875 \text{ m}$$
$$= 187.5 \text{ mm}$$

21. (a)

The beam will deflect as



Vertical deflection at C, $\Delta = \Delta_1 + \Delta_2$

Δ_1 = Deflection due to moment in BC

$$\Delta_1 = \frac{ML^2}{2EI} = \frac{\mu L^2}{2EI}$$

Δ_2 = Deflection due to moment in AB

$$\Delta_2 = \frac{MH}{EI} \times L = \frac{\mu LH}{EI}$$

$$\Delta = \Delta_1 + \Delta_2$$

$$\Rightarrow \quad \Delta = \frac{\mu L^2}{2EI} + \frac{\mu LH}{EI}$$

$$\Rightarrow \quad \Delta = \frac{\mu L}{EI}\left(\frac{L}{2} + H\right)$$

22. (d)

Deflection at free end, $\delta_1 = \dfrac{WL^3}{3EI_1}$

Here, $I_1 = \dfrac{bd^3}{12}$

With doubling of depth and width,

$$I_2 = \frac{(2b) \times (2d)^3}{12} = 16\, I_1$$

$$\delta \propto \frac{1}{I}$$

$$\therefore \quad \delta_2 = \delta_1 \times \frac{I_1}{I_2}$$

$$\Rightarrow \quad \delta_2 = \frac{\delta_1}{16}$$

δ_2 as a percentage of δ_1 $= \dfrac{1}{16} \times 100 = 6.25\%$

23. (d)

Let the total length of spring before cutting be l.

Short piece, $l_1 = \dfrac{l}{4}$

Long piece, $l_2 = \dfrac{3l}{4}$

Since, $K = \dfrac{Gd^4}{64R^3 n}$

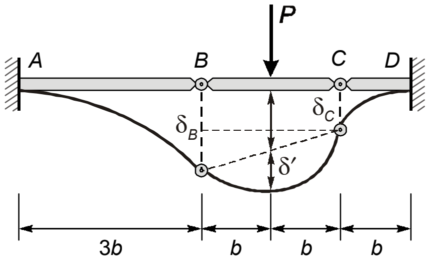
or $K \propto \dfrac{1}{n}$ or $K \propto \dfrac{1}{\text{Length}}$

$$\therefore \quad \frac{K_1}{K} = \frac{l}{l_1} = \frac{l}{(l/4)}$$

or $\dfrac{K_1}{K} = 4$

$$\Rightarrow \quad K_1 = 4K$$

24. (a)



Free body diagram of given beam is

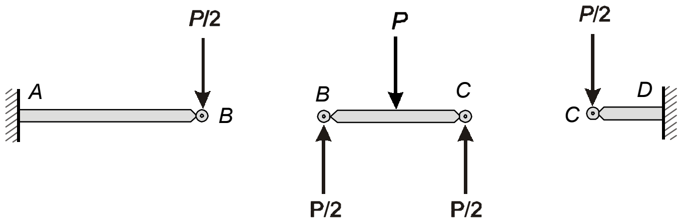


Fig. 1 Fig. 2 Fig. 3

The deflection of point under load P,

$$\delta = \delta_c + \frac{\delta_B - \delta_C}{2} + \delta'$$

$$\delta = \frac{1}{2}(\delta_B + \delta_C) + \delta' \qquad \text{.....(i)}$$

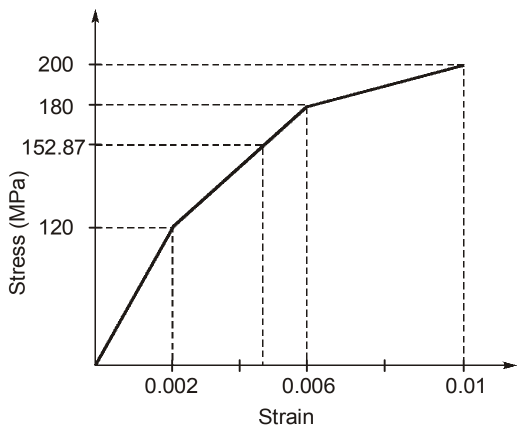
δ_B is deflection of point B in Fig. 1
δ_C is deflection of point C in Fig. 3
δ' is deflection of point under load P in Fig. 2

$$\delta_B = \frac{wL^3}{3EI} = \left(\frac{P}{2}\right)(3b)^3\left(\frac{1}{3EI}\right) = \frac{9Pb^3}{2EI}$$

$$\delta_C = \frac{wL^3}{3EI} = \left(\frac{P}{2}\right)(b)^3\left(\frac{1}{3EI}\right) = \frac{Pb^3}{6EI}$$

From (i),
$$\delta = \frac{1}{2}(\delta_B + \delta_C) + \frac{P(2b)^3}{48EI} = \frac{5Pb^3}{2EI}$$

25. (c)



$$\text{Stress in the bar} = \left(\frac{P}{A}\right) = \left[\frac{300 \times 10^3}{\frac{\pi}{4} \times (50)^2}\right] = 152.78 \text{ N/ mm}^2$$

$$152.78 = 120 + \frac{(180-120)}{(0.004)} \times d$$
$$32.78 = 15000 \times d$$
$$d = 2.185 \times 10^{-3}$$
$$\text{Total strain in bar} = 0.002 + 0.002185 = 4.185 \times 10^{-3}$$
$$\text{Total elongation} = (4.185 \times 10^{-3} \times 2 \times 10^{3}) \text{ mm} = 8.37 \text{ mm}$$

26. (b)

For $\theta = 0°$: $\varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

For $\theta = 40°$

$$\varepsilon_B = \varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_B = 1496 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$;

Then simplify and rearrange:

$$0.41318\,\varepsilon_y + 0.49240\phi_{xy} = 850.49 \times 10^{-6} \qquad .....(i)$$

For $\theta = 140°$

$$\varepsilon_C = \varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\phi_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_C = -39.44 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$;

Then simplify and rearrange:

$$0.41318\,\varepsilon_y - 0.49240\phi_{xy} = -684.95 \times 10^{-6} \qquad .....(ii)$$

Solve equation (i) and (ii)

$$\varepsilon_y = 200.3 \times 10^{-6} \text{ and } \phi_{xy} = 1559.2 \times 10^{-6}$$

Hooke's Law $\quad \sigma_x = \frac{E}{1-\mu^2}\left(\varepsilon_x + \mu\varepsilon_y\right) = 91.6 \text{ MPa}$

27. (d)

$$\text{Stress tensor} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$\sigma_{xx} = 10 \qquad \sigma_{yy} = 20 \qquad \sigma_{zz} = 10 \qquad \tau_{xz} = 5 = \tau_{zx}$

$$\sigma_1/\sigma_2 = \left(\frac{\sigma_{xx} + \sigma_{zz}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \tau_{xz}^2}$$
$$= \left(\frac{10+10}{2}\right) \pm \sqrt{0+5^2}$$
$$= 10 \pm 5$$

$\sigma_1 = 15 \qquad \sigma_2 = 5 \qquad \sigma_{yy} = \sigma_3 = 20 \qquad$ [As there is no shear stress in the $X-Y$ or $Y-Z$ plane]

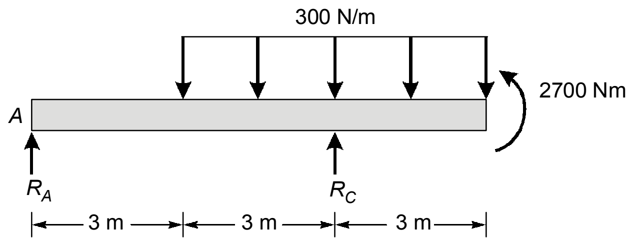
$$\sigma_y^2 \geq \frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$
$$\geq \frac{1}{2}[100 + 225 + 25]$$
$$\sigma_y^2 \geq \frac{350}{2}$$
$$\sigma_y \geq 13.23 \text{ MPa}$$

28. (b)

$$\frac{1}{2}w\left(\frac{L}{2}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right)-\frac{wL}{4}\left(\frac{a}{2}\right) = 0$$

$$\Rightarrow \quad a = \frac{L}{3}$$

29. (c)



$$\sum M_A = 0$$

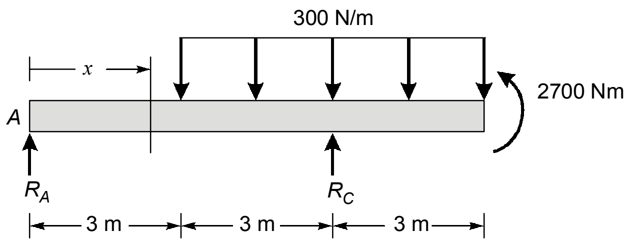
$$\Rightarrow \quad 300\,(6)\,(3+3) - R_C\,(6) - 2700 = 0$$

$$\Rightarrow \quad R_C = 1350 \text{ N}$$

$$\sum F_y = 0$$

$$\Rightarrow \quad R_A + R_C = 300\,(6)$$

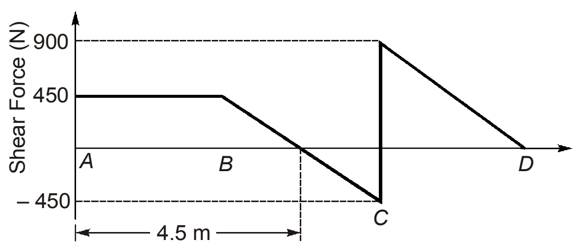
$$\Rightarrow \quad R_A = 450 \text{ N}$$



$V = 450$ N $\quad 0 \le x < 3$m .....(iii)

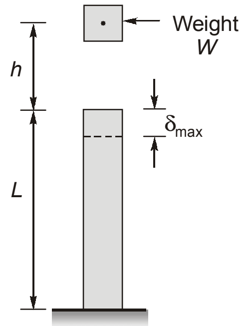
$V = [450 - 300\,(x-3)]$ N $\quad 3 \text{ m} \le x < 6$ m .....(iv)

$V = [450 - 300\,(x-3) + 1350]$ N $\quad 6 \text{ m} \le x < 9$ m .....(v)



30. (b)

Work done by falling weight is equal to strain energy of the bar



The falling of weight is a case of sudden impact, therefore, the workdone by weight is equal to the product of load applied and displacement.

$$\Rightarrow \qquad W(h+\delta_{max}) = \frac{\sigma^2_{max}}{E} \times AL$$

Here,

$$\sigma_{max} = 150 \text{ MPa}$$
$$W = 25 \text{ N}$$

δ_{max} = maximum displacement corresponding to maximum stress σ_{max} at the time of impact

$$= \frac{\sigma_{max} \cdot L}{E}$$

$$\Rightarrow \qquad W\left(h+\frac{\sigma_{max} \cdot L}{E}\right) = \frac{\sigma^2_{max}}{E} \times AL$$

Here,

$$\sigma_{max} = 150 \text{ MPa}$$
$$W = 25 \text{ N}$$
$$L = 1000 \text{ mm}$$
$$E = 200000 \text{ MPa}$$
$$A = \frac{\pi}{4} \times 10^2 = 78.54 \text{ mm}^2$$

putting values in equation,

$$\Rightarrow \qquad h = \frac{150^2 \times 78.54 \times 1000}{2,00,000 \times 25} - \frac{150 \times 1000}{2,00,000}$$

$$\Rightarrow \qquad h = 352 \text{ mm}$$

■■■■