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CLASS TEST 2019-2020 → ELECTRONICS ENGINEERING

Subject : Signal & System

Date of test : 21/04/2019

— *Answer Key* —

1.	(d)	7.	(a)	13.	(d)	19.	(b)	25.	(a)
2.	(d)	8.	(b)	14.	(c)	20.	(a)	26.	(a)
3.	(b)	9.	(a)	15.	(a)	21.	(b)	27.	(a)
4.	(a)	10.	(a)	16.	(a)	22.	(a)	28.	(a)
5.	(a)	11.	(b)	17.	(a)	23.	(c)	29.	(d)
6.	(a)	12.	(a)	18.	(b)	24.	(c)	30.	(d)

Detailed Explanations

1. **(d)**

$$2^n u[n] \xleftrightarrow{z} \frac{1}{1-2z^{-1}}$$

$\therefore$ $\qquad\qquad$ ROC $\Rightarrow |z| > 2$

and $\qquad (4)^n u[-n-1] \xleftrightarrow{z} \dfrac{1}{1-4z^{-1}}$

$\therefore$ $\qquad\qquad$ ROC $\Rightarrow |z| < 4$

$\therefore$ $\quad$ for the signal to converge

$\qquad\qquad$ ROC $\Rightarrow 2 < |z| < 4$

2. **(d)**

Since the transfer function has one pole on the R.H.S thus the system is unstable, so the final value theorem is not applicable in this case.

3. **(b)**

$$y[n] = x[n] * \delta[n-2] = x[n-2]$$
$\therefore \qquad y[2] = x[0] = 1$

4. **(a)**

$\because \qquad\qquad j\dfrac{d}{dt}[F(j\omega)] \longleftrightarrow t[f(t)]$

thus at $t = 0$ the answer will be zero.

6. **(a)**

$\because \qquad\qquad X[k] = W_N^{n_0 k}$

for $N = 4$, $\qquad\qquad X[k] = W_N^{n_0 k}$ for $k = 0, 1, 2, 3$

taking IDFT, we get, $\qquad x[n] = \delta[n-n_0]$

$\therefore \qquad\qquad$ energy $= \Sigma |x[n]|^2 = 1$

7. **(a)**

$$X(e^{j\omega}) = e^{j\omega} + e^{-j\omega} + 2(e^{2j\omega} - e^{-2j\omega}) + 3(e^{3j\omega} + e^{-3j\omega})$$

$$= 2\cos\omega + 4j\sin(2\omega) + 2\cos3\omega = 2\cos(\pi) + 4j\sin(2\pi) + 6\cos(3\pi)$$

$$= -2 + 0 - 6 = -8$$

$$|Xe^{j\pi}| = 8$$

8. **(b)**

Hilbert transform will shift the signal by $\dfrac{-\pi}{2}$ thus $x(t) = e^{j(2\pi ft - \pi/2)} = -je^{j2\pi ft}$

9. **(a)**

$\because$ For causal system degree of numerator < degree of denominator. (degree is highest power of z)

10. **(a)**

$$Re\{x(t)\} = \frac{x(t) + x^*(t)}{2}$$

∴ The Fourier coefficient of $x^*(t)$ are

$$b_K = \frac{1}{T}\int_T x^*(t)e^{-jK\frac{2\pi}{T}t}\,dt$$
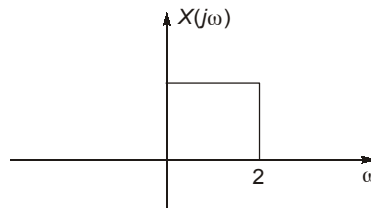
Taking conjugate on both sides

$$b_K^* = \frac{1}{T}\int_T x(t)e^{-j(-K)\frac{2\pi}{T}t}\,dt$$

∴ $$a_{-K} = b_K^*$$

∴ Fourier series Coefficient of $Re\{x(t)\} = \dfrac{a_K + a_{-K}^*}{2}$

11. **(b)**

The signal can be represented as

Since the signal is neither even nor odd symmetric in frequency domain thus it will be a complex signal in time domain.

12. **(a)**

$$\begin{aligned} Y(z) &= X(z) \cdot H(z) = 2[z^3 + z^2 - 3z^{-1} + 5z^{-3}] \cdot [2(z^{-2})] \\ &= 2[z + 2z^{-1} - 3z^{-3} + 5z^{-5}] \end{aligned}$$

∵ $$z^{-2} = 0$$
thus $$Y[2] = 0$$

13. **(d)**

1. $x(t) = 0$ for $t < 0$ thus the signal is causal

∵ $x(t)$ is real thus $X(j\omega)$ is an even symmetric signal

now $$Re\{X(j\omega)\} \xleftrightarrow{\ F.T\ } even\{x(t)\}$$

∴ $$|t|e^{-|t|} = even\{x(t)\}$$

∴ $$\frac{x(t) + x(-t)}{2} = |t|e^{-|t|}$$

∴ $\qquad x(t) = 2t\,e^{-|t|}$

{∵ right sided $x(-t) = 0$}

$\qquad x(t) = 2t\,e^{-t}\,u(t)$

14. **(c)**

$$x(t) \xleftarrow{\ L.T\ } X(s) = \frac{s+3}{(s+3)^2+4}$$

$$y(t) \Rightarrow \int x(\tau)d\tau \Rightarrow \frac{s+3}{s[(s+3)^2+4]}$$

15. **(a)**

$$\frac{\sin\left(\dfrac{3}{2}\omega\right)}{\sin\left(\dfrac{\omega}{2}\right)} \xleftarrow{\ F.T\ } x_1[n] = \begin{cases} 1 & -1 < n < 1 \\ 0 & \text{otherwise} \end{cases}$$

now $\qquad \displaystyle\sum_{k=-\infty}^{n} x[n] \xleftarrow{\ F.T\ } \frac{1}{1-e^{-j\omega}}X_1(e^{j\omega}) + 3\pi\delta(\omega)$

$$2\pi\delta(\omega) \xleftarrow{\ F.T\ } 1$$

∴ $\qquad x[n] = 1 + \displaystyle\sum_{k=-\infty}^{n} x_1[k]$

$$= \begin{cases} 1 & n < -4 \\ n+3 & -1 < n < 1 \\ 4 & n \geq 2 \end{cases}$$

∴ at $n \rightarrow \infty$

$$x[n] = 4$$

16. **(a)**

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

∴ $\qquad x(t) =$

$$\therefore \quad x_e(t) =$$
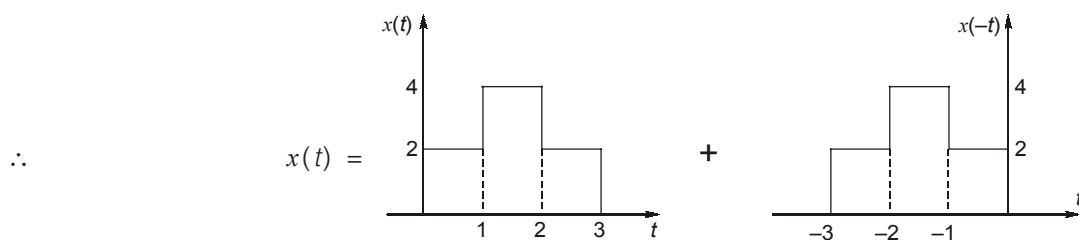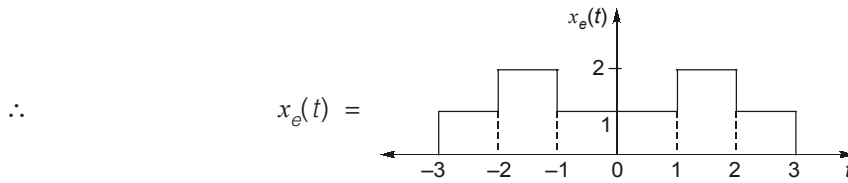
17. **(a)**

Let

$$x'(t) = e^{-2t} u(t)$$

$$X'(j\omega) = \frac{1}{2 + j\omega}$$

now applying time reversal property

$$x(-t) \xrightarrow{\text{FT}} X(-j\omega)$$

$$\therefore \quad e^{2t} u(-t) \xrightarrow{\text{FT}} \frac{1}{2 - j\omega}$$

18. **(b)**

$$Y(e^{j\omega}) = X(e^{j\omega}) * X(e^{j\omega - \pi/2})$$

$\therefore$ In time domain

$$Y(e^{j\omega}) = 2\pi x_1[n] \cdot x_2[n]$$

now, If

$$X(e^{j\omega}) \longleftrightarrow n\left(\frac{3}{4}\right)^{|n|}$$

Then

$$X(e^{j\omega - \pi/2}) \longleftrightarrow n \cdot e^{j\pi/2}\left(\frac{3}{4}\right)^{|n|}$$

$\therefore$

$$y[n] \longleftrightarrow 2\pi n^2 e^{j\pi n/2}\left(\frac{3}{4}\right)^{2|n|}$$

19. **(b)**

$$X(z) = \frac{z}{z - 1} \qquad\qquad |z| > 1$$

$$Y(z) = \frac{2z}{z - \dfrac{1}{3}} \qquad\qquad |z| > \frac{1}{3}$$

$$H(z) = \frac{2(z - 1)}{z - \dfrac{1}{3}} \qquad\qquad |z| > \frac{1}{3}$$

$$X'(z) = \frac{z}{z - \dfrac{1}{2}} \qquad\qquad |z| > \frac{1}{2}$$

$$Y'(z) = H(z) \cdot X'(z)$$

$$= \frac{2z(z-1)}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)} \qquad |z| > \frac{1}{2}$$

Taking inverse z transform

$$y[n] = \left[-6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{3}\right)^n\right]u[n]$$

$k_1 = -6, \quad k_2 = 8$
so, $\qquad\qquad k_1 + k_2 = 2$

20. **(a)**

$$x(t) = \cos\left(2\left(t - \frac{1}{2}\right)\right)$$

$$= \frac{1}{2}\left[e^{j2(t-1/2)} + e^{-j2(t-1/2)}\right]$$

$$= \frac{1}{2}\left[e^{j2t} \cdot e^{-j} + e^{-j2t} \cdot e^{j}\right]$$

According to the given condition the output is

$$= \frac{1}{2}\left[e^{-j}e^{j3t} + e^{j}e^{-j3t}\right] = \frac{1}{2}\left[e^{j3(t-1/3)} + e^{-j3(t-1/3)}\right]$$

$$y(t) = \cos\left[3\left(t - \frac{1}{3}\right)\right]$$

$$\therefore \qquad y\left(\frac{1}{3}\right) = \cos\left[3\left(\frac{1}{3} - \frac{1}{3}\right)\right]$$

$$= \cos 0 = 1$$

21. **(b)**
    If $x[n]$ is real

$$odd[x[n]] \xrightarrow{\quad FT \quad} jIm[X(e^{j\omega})]$$

$$\therefore \qquad odd[x[n]] = F^{-1}\left[\frac{1}{2}\left(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}\right)\right]$$

$$= \frac{1}{2}\left[\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right]$$

$$\because \qquad odd[x[n]] = \frac{x[n] - x[-n]}{2}$$

Since, $\qquad x[n] = 0$ for $n > 0$
$$x[n] = 2\,odd[x[n]]$$
$$= \delta[n+1] - \delta[n+2] \text{ for } n < 0$$

using parshevel's theorem

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|X\left(e^{j\omega}\right)\right|^2 d\omega = \sum_{n=-\infty}^{\infty}\left|x[n]\right|^2$$

$$(x[0])^2 - 2 = 3$$

$$x[0] = \pm 1$$

$$\because \qquad x[0] > 0$$

$$\therefore \qquad x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

22. **(a)**

$$\left|H(j\omega)\right| = \left|3\cos\left(\frac{\omega}{2}\right)\right|; \qquad \angle H(j\omega) = -\omega/2$$

So, $$H(\omega) = 3\cos\left(\frac{\omega}{2}\right)\cdot e^{-j\omega/2} = \frac{3}{2}\left[e^{j\omega/2} + e^{-j\omega/2}\right]\cdot e^{-j\omega/2}$$

$$= \frac{3}{2}\left[1 + e^{-j\omega}\right]$$
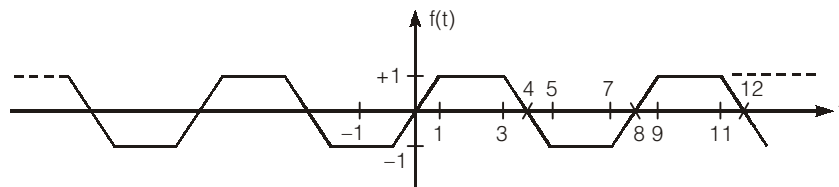
So, $$H(z) = \frac{3}{2}\left[1 + z^{-1}\right]$$

23. **(c)**

Taking Fourier transform of the signal, we obtain

$$H(e^{j\omega}) = \frac{\dfrac{3}{2} - \dfrac{1}{2}e^{-j\omega}}{1 - \dfrac{3}{4}e^{-j\omega} + \dfrac{1}{8}e^{-2j\omega}}$$

$$\therefore \ y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = \frac{3}{2}x[n] - \frac{1}{2}x[n-1]$$

24. **(c)**

The signal f(t) has hidden, odd and half-wave symmetry.

So, $$a_0 \neq 0$$

$$a_n = 0; \ \forall n$$

$$b_n \neq 0; \ n = 1, 3, 5$$

Therefore, non zero Fourier series coefficients are

a_0 and b_n, $n = 1, 3, 5, ...$

25. (a)

As,

$$x(t)\cos^2 t = x(t)\left[\frac{1}{2} + \frac{1}{2}\cos 2t\right]$$

$$= \frac{1}{2}x(t) + \frac{1}{2}\cos 2t$$

Now,

$$g(t) = x(t).\cos^2 t * \frac{\sin t}{\pi t}$$

$$= x(t)\left[\frac{1 + \cos 2t}{2}\right] * \frac{\sin t}{\pi t}$$

$$\therefore \quad G(j\omega) = \left[\frac{1}{2}X(j\omega) + \frac{1}{4}X(\omega - 2) + \frac{1}{4}X(\omega + 2)\right] \times \text{rect}\left(\frac{\omega}{2}\right)$$

Thus the given solution will be

$$\therefore \quad G(j\omega) = \frac{1}{2}X(j\omega)$$

or

$$g(t) = \frac{1}{2}x(t)$$

Thus to get the desired result

$$h(t) = \frac{1}{2}\delta(t)$$

26. (a)

$$x[n] = \delta[n]$$
$$X(e^{j\omega}) = 1$$

$$\frac{dX(e^{j\omega})}{d\omega} = 0$$

$$\therefore \quad Y(e^{j\omega}) = e^{-j\omega} X(e^{j\omega})$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega}.e^{j\omega n}d\omega = \frac{\sin\pi(n-1)}{\pi(n-1)}$$

27. (a)

$$\because \quad x^*(t) \xleftrightarrow{\ F\ } X^*(-j\omega)$$

and

$$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{\ F\ } \frac{X(j\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

28. (a)

$$X(j0) = \int_{-\infty}^{\infty} x(t)dt = 7$$

29. **(d)**

$$H(j\omega) = \frac{1+2e^{-j\omega}}{1+\dfrac{1}{2e^{-j\omega}}} = \frac{1+2e^{-j\omega}}{2e^{-j\omega}+1} \cdot 2e^{-j\omega}$$

$$\left| H(j\omega) \right| = 2$$

30. **(d)**

The impulse response convolves with the input signal and the output can be broken up into shifted versions of input.

■■■■