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# STRENGTH OF MATERIALS

## MECHANICAL ENGINEERING

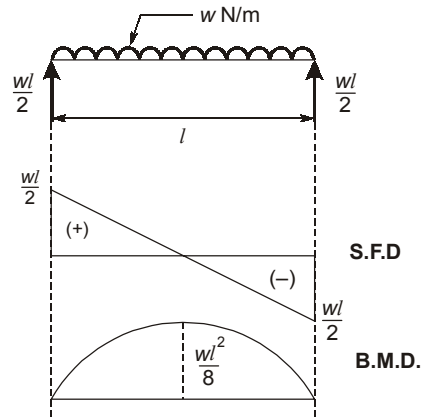
Date of Test : 22/04/2022

### ANSWER KEY > Strength of Materials

1. (d)	7. (a)	13. (c)	19. (c)	25. (a)
2. (a)	8. (b)	14. (b)	20. (a)	26. (b)
3. (c)	9. (c)	15. (b)	21. (b)	27. (a)
4. (b)	10. (c)	16. (d)	22. (b)	28. (b)
5. (d)	11. (d)	17. (a)	23. (a)	29. (c)
6. (d)	12. (c)	18. (b)	24. (b)	30. (a)

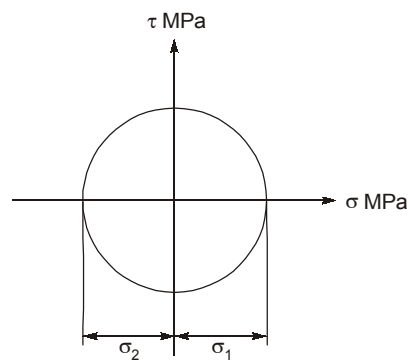
## DETAILED EXPLANATIONS

1. (d)



2. (a)

It is a case of pure shear stress,



$$\sigma_1 = +400 \text{ MPa}$$

$$\sigma_2 = -400 \text{ MPa}$$

$$\tau = 0$$

3. (c)

As per maximum shear stress theory,

$$\begin{aligned} \text{Absolute } \tau_{\max} &= \text{Max of } \left[ \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_1}{2}, \frac{\sigma_2}{2} \right] \\ &= \frac{\sigma_{yt}}{2} \quad (\sigma_{yt} = \text{yield point stress}) \\ &= \frac{80}{2} = \frac{\sigma_{yt}}{2} \\ \therefore \sigma_{yt} &= 80 \text{ MPa} \end{aligned}$$

4. (b)

$\frac{T}{J} = \frac{\tau}{R}$ , Here  $T$  and  $\tau$  are same, so  $\frac{J}{R}$  should be same i.e. polar section modulus will be same.

5. (d)

$$\text{Elongation due to self weight} = \frac{\gamma L^2}{2E}$$

$\gamma$ -(specific weight).

6. (d)

Toughness is the ability of material to absorb the energy upto failure point i.e. toughness is the total area under stress-strain curve.

7. (a)

$$\text{Circumferential stress} = \frac{Pd_i}{2t}$$

$$P = 6 \text{ MPa}$$

$$d_i = 600 \text{ mm}$$

$$t = 10 \text{ mm}$$

$$\begin{aligned} \sigma_c &= \frac{6 \times 600}{2 \times 10} = 180 \text{ MPa} \\ &= 180 \times 1000 \text{ kPa} \\ &= 18 \times 10^4 \text{ kPa} \end{aligned}$$

8. (b)

$$M = 3000 \text{ Nm}$$

$$T = 4000 \text{ Nm}$$

$$\begin{aligned} M_e &= \frac{1}{2} [M + \sqrt{M^2 + T^2}] \\ &= \frac{1}{2} [3000 + \sqrt{(3000)^2 + (4000)^2}] \\ &= 8000 \times \frac{1}{2} = 4000 \text{ Nm} \end{aligned}$$

9. (c)

$$\sigma_x = 10 \text{ MPa}$$

$$\sigma_y = 7 \text{ MPa}$$

$$\tau_{xy} = 2 \text{ MPa}$$

$$\max\left(\frac{\sigma_1}{2}, \frac{\sigma_2}{2}, \frac{\sigma_1 - \sigma_2}{2}\right) = \frac{S_{yt}}{2N}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{17}{2} \pm \frac{\sqrt{3^2 + 4 \times 4}}{2} = \frac{17 \pm 5}{2}$$

$$\sigma_1 = 11 \text{ MPa} \qquad \sigma_2 = 6 \text{ MPa}$$

$$\Rightarrow \frac{\sigma_1}{2} = \frac{S_{yt}}{2N}$$

$$\Rightarrow \sigma_1 = \frac{S_{yt}}{N} = \frac{18}{N}$$

$$\Rightarrow 11 = \frac{18}{N}$$

$$\Rightarrow N = 1.636$$

10. (c)

Given data:  $d = 2 \text{ m}$ ,  $t = 40 \text{ mm}$ ,  $P = 3.5 \text{ kPa}$ ,  $F = 200 \text{ kN}$ 

$$E = 2G(1 + \mu)$$

$$\frac{200}{2 \times 80} = 1 + \mu$$

$$\mu = 0.25$$

$$\sigma_h = \frac{Pd}{2t}$$

$$= \frac{3.5 \times 2000}{2 \times 40} \times 10^3 = 87.5 \text{ kPa}$$

$$\sigma'_l = \frac{Pd}{4t} = 0.04375 \text{ MPa} = 43.75 \text{ kPa}$$

$$\sigma''_l = \frac{F}{\pi dt} = 795.77 \text{ KPa}$$

$$\Rightarrow \epsilon_h = \frac{1}{E} [\sigma_h - \mu(\sigma'_l + \sigma''_l)] = -6.12 \times 10^{-7}$$

11. (d)

We know that strain energy,

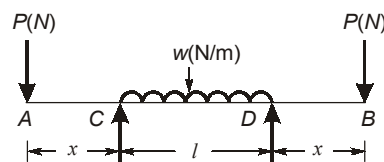
$$U = \frac{P^2 L}{2AE}$$

It is obvious from the above equation that strain energy is proportional to the square of load applied. We know that sum of squares of two number is less than the square of their sum.

$$[(P_1 + P_2)^2 > P_1^2 + P_2^2]$$

Thus  $U > U_1 + U_2$

12. (c)



From the symmetry of the figure,

$$R_C = R_D = P + \frac{wl}{2}$$

Bending moment at mid point,

$$= -\frac{wl}{2} \times \frac{l}{4} + R_C \times \frac{l}{2} - P \left( x + \frac{l}{2} \right) = 0$$

gives

$$x = \frac{wl^2}{8P}$$

13. (c)

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \text{ or } \tau = \frac{GR\theta}{L}$$

$$\tau \propto \frac{1}{L}$$

14. (b)

$$\text{Area} = 50 \times 110 = 5500 \text{ mm}^2$$

$$P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$$

$$\text{Young's modulus} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$\text{Stress} = \frac{F}{A} = \frac{30 \times 10^3}{5500} = 5.4545 \text{ N/mm}^2$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{0.625}{1500} = 4.16 \times 10^{-4}$$

$$E = \frac{5.4545}{4.16 \times 10^{-4}} = 13090.90 \text{ N/mm}^2$$

15. (b)

$$A = \frac{\pi}{4} \times 80^2 = 5026.55 \text{ mm}^2$$

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 80^4$$

$$I = 2010619.30 \text{ mm}^4$$

Direct compressive stress due to horizontal component,

$$\sigma_a = \frac{15 \cos 30^\circ \times 10^3}{A} = \frac{15000 \times \cos 30^\circ}{5026.55}$$

$$\sigma_a = 2.584 \text{ MPa (comp.)}$$

Bending stress at point B due to vertical component of loads,

$$\sigma_b = \frac{My}{I} \text{ (Comp. at point B)}$$

$$\sigma_b = \frac{(15 \times 10^3 \sin 30^\circ) \times 0.5 \times 40 \times 10^{-3}}{2010619.30 \times 10^{-12}}$$

Bending stress at point B,  $\sigma_b = 74.604 \text{ MPa}$  (Comp.)

Total stress at point B,  $\sigma_{total} = \sigma_b + \sigma_a = 74.604 + 2.584$

$$\sigma_{total} = 77.188 \text{ MPa} \text{ (Comp.)}$$

16. (d)

$$M = EI \frac{d^2 y}{dx^2}$$

$$\Rightarrow M = EI \frac{d^2}{dx^2} \left( \frac{-2W_0 L^4}{\pi^4 EI} \sin \left( \frac{\pi x}{L} \right) \right)$$

$$\Rightarrow M = \frac{-2W_0 L^4}{\pi^4} \times \left[ \frac{d^2}{dx^2} \left( \sin \left( \frac{\pi x}{L} \right) \right) \right]$$

$$\Rightarrow M = \frac{2W_0 L^2}{\pi^2} \sin \left( \frac{\pi x}{L} \right)$$

$$\Rightarrow \text{Shear force} = EI \frac{d^3 y}{dx^3} = \frac{2W_0 L}{\pi} \cos \left( \frac{\pi x}{L} \right)$$

$$\Rightarrow \text{Load} = -EI \frac{d^4 x}{dx^4} = +2W_0 \sin \left( \frac{\pi x}{L} \right)$$

17. (a)

Shear force is varying linearly between AC, therefore, there is uniformly distributed load between AC.

$$R_A = 81 \text{ kN}$$

Uniformly distributed load between A and C

$$= \frac{81 - (-19)}{5} = 20 \text{ kN/m}$$

$$\text{Vertical load at } D = 59 - 19 = 40 \text{ kN}$$

$$R_E = 59 \text{ kN}$$

Now,

$$\frac{81}{a} = \frac{19}{5-a}$$

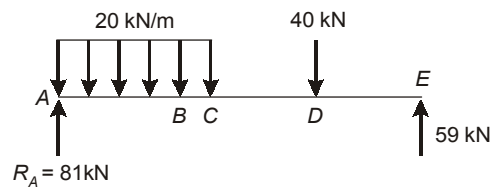
⇒

$$a = \frac{405}{100} = 4.05$$

Bending moment is maximum where shear force changes sign.

$$\text{Maximum Bending Moment} = \frac{1}{2} \times 81 \times 4.05 = 164.025 \text{ kN-m}$$

**Alternate :**



For maximum bending moment,

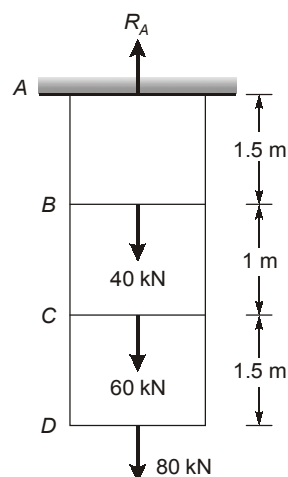
$$\text{S.F.} = 0$$

$$81 = 20x$$

$$x = 4.05$$

$$\begin{aligned} \text{Maximum Bending Moment} &= 81 \times 4.05 - \frac{20 \times 4.05 \times 4.05}{2} \\ &= 164.025 \text{ kN-m} \end{aligned}$$

18. (b)



$$R_A = 40 + 60 + 80 = 180 \text{ kN}$$

$$\text{Force } P_1 \text{ on portion } AB = 180 \text{ kN (tensile)}$$

$$\text{Force } P_2 \text{ on portion } BC = 180 - 40 = 140 \text{ kN (tensile)}$$

$$\text{Force } P_3 \text{ on portion } CD = 80 \text{ kN (tensile)}$$

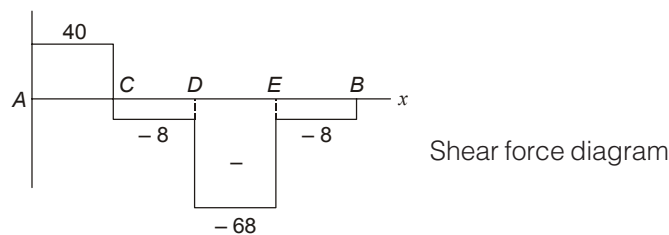
$$\Delta = \frac{1}{AE}(P_1L_1 + P_2L_2 + P_3L_3)$$

$$\begin{aligned}\Delta &= \frac{1}{1200 \times 2.05 \times 10^5} \times [180 \times 1500 + 140 \times 1000 + 80 \times 1500] \times 10^3 \\ &= \frac{530000 \times 10^3}{1200 \times 2.05 \times 10^5} = 2.15 \text{ mm}\end{aligned}$$

19. (c)

$$\begin{aligned}\Sigma M_B &= 0 \\ R_A \times 4.5 &= 48 \times 3 + 60 \times 1.5 - 60 \times 0.9 \\ R_A &= 40 \text{ kN (upward)} \\ R_B &= 48 - 40 = 8 \text{ kN (upward)}\end{aligned}$$

Making shear force diagram



Maximum shear force = 68 kN

20. (a)

$$\begin{aligned}\text{Weight of water} &= \rho Vg = 1000 \times \frac{\pi}{4} \times d^2 \times l \times 9.81 \\ &= 1000 \times \frac{\pi}{4} \times (0.5)^2 \times 10 \times 9.81 = 19261.89 \text{ N}\end{aligned}$$

And this is uniformly distributed,

$$\begin{aligned}\Rightarrow M_{\text{maximum}} &= \frac{wl^2}{8} = \frac{19261.89 \times 10 \times 1000}{8} \text{ N-mm} \\ \sigma_{\text{maximum}} &= \frac{M_{\text{max}}}{Z} = \frac{19261.89 \times 10 \times 1000}{8 \times \pi \times d^2 \times t} \times 4 \\ &= \frac{19261.89 \times 10 \times 1000 \times 4}{8 \times \pi \times (500)^2 \times 25} = 4.91 \text{ MPa}\end{aligned}$$

21. (b)

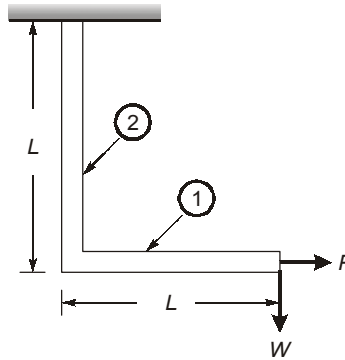
We know that,

$$\frac{\Delta V}{V} = \frac{3\sigma}{E}[1-2\mu]$$

$$\begin{aligned}\therefore \frac{\Delta V}{200 \times 100 \times 50} &= \frac{3 \times 15}{200 \times 1000} [1 - (2 \times 0.3)] \\ \Delta V &= 90 \text{ mm}^3\end{aligned}$$

22. (b)

For finding the vertical deflection assuming a dummy vertical load  $W$ .



$$(S.E)_1 = \int_0^L \frac{(Wx)^2 dx}{2EI} = \frac{W^2 L^3}{6EI}$$

$$(S.E)_2 = \int_0^L \frac{(WL - Px)^2 dx}{2EI}$$

$$= \int_0^L \frac{(W^2 L^2 + P^2 x^2 - 2WLx) dx}{2EI} = \frac{W^2 L^3}{2EI} + \frac{P^2 L^3}{6EI} - \frac{2WPL^3}{4EI}$$

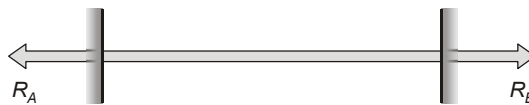
$$(S.E)_{\text{Total}} = \frac{W^2 L^3}{6EI} + \frac{W^2 L^3}{2EI} + \frac{P^2 L^3}{6EI} - \frac{2WPL^3}{4EI}$$

$$\delta_A = \frac{(S.E)}{\partial W} = \frac{2WL^3}{6EI} + \frac{2WL^3}{2EI} + 0 - \frac{2PL^3}{4EI}$$

Putting  $W = 0$ ,

$$\delta_A = \left( \frac{PL^3}{2EI} \right) = \frac{PL^3}{2EI} \text{ (downward)}$$

23. (a)



$$R_A = R_B = \text{Initial tension } (P_1) + \text{Tension produced due to thermal contraction } (P_2)$$

$$\frac{P_2 l}{AE} = \alpha \Delta T l$$

$$\Rightarrow P_2 = \alpha \Delta T A E$$

$$= 19 \times 10^{-6} \times 25 \times \frac{\pi}{4} \times (1.6)^2 \times 100 \times 10^3 = 95.504 \text{ N}$$

$$R_A = P_1 + P_2 = 245.504 \text{ N}$$

$$\text{Normal stress} = \frac{R_A}{A} = \frac{245.504}{\frac{\pi}{4} \times (1.6)^2} = 122.104 \text{ MPa}$$

$$\text{Maximum shear stress} = \frac{\sigma}{2} = 61.05 \text{ MPa}$$



24. (b)

According to given conditions,

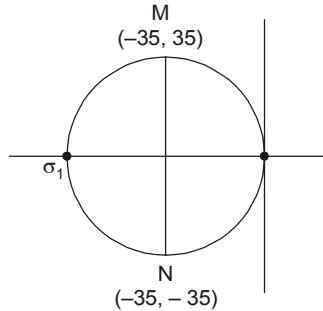
$$(\sigma_n)_{\text{at } \theta = 45^\circ} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta = 0$$

$$0 = -35 + \tau_{xy}$$

⇒

$$\tau_{xy} = 35 \text{ MPa}$$

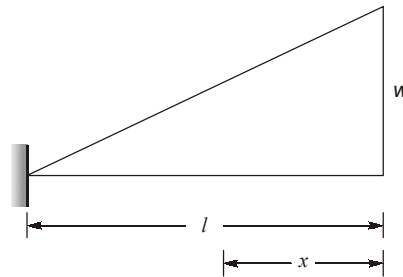
Using Mohr's circle,



Radius at Mohr's circle = 35 MPa

$\sigma_1$  = Major principal stress =  $2R = 70$  MPa (Compressive)

25. (a)



$$\text{Maximum shear force} = \frac{wl}{2} = 37.5 \text{ kN}$$

$$\Rightarrow w = \frac{37.5 \times 2}{l} = 37.5 \text{ kN/m}$$

Bending moment at,  $x = l$

$$= \frac{wl}{2} \times \frac{2}{3} \times l = \frac{wl^2}{3}$$

$$= \frac{37.5 \times 10^3}{10^3} \times \frac{(2)^2 \times 10^6}{3} = 50 \times 10^6 \text{ N-mm}$$

26. (b)

$$J_s = \frac{\pi}{32} \times (50)^4 = 613592 \text{ mm}^4$$

$$J_b = \frac{\pi}{32} \times (75^4 - 50^4) = 2492719 \text{ mm}^4$$

$$T_s = \frac{G_s J_s \theta}{l} \quad \text{and} \quad T_b = \frac{G_b J_b \theta}{l}$$

$$\text{The total torque, } T = T_s + T_b = (G_s J_s + G_b J_b) \frac{\theta}{l}$$

$$\theta = \frac{Tl}{G_s J_s + G_b J_b} = \frac{(800 \times 10^3)(1.5 \times 10^3)}{(8 \times 10^4 \times 613592) + (4 \times 10^4 \times 2492719)}$$

$$= 0.008065 \text{ radian} = 0.462^\circ$$

27. (a)

$$\begin{aligned} \text{Total strain energy} &= \frac{1}{2} P_1 \delta_1 + \frac{1}{2} P_2 \delta_2 + \frac{1}{2} P_3 \delta_3 \\ &= \frac{1}{2} \times P \times \frac{PL_1}{A_1 E} + \frac{1}{2} P \times \frac{PL_2}{A_2 E} + \frac{1}{2} \times P \times \frac{PL_3}{A_3 E} \\ &= \frac{P^2}{2E} \left[ \frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right] \\ &= \frac{(10 \times 1000)^2}{2 \times 200 \times 10^9} \left[ \frac{100 \times 4 \times 1000}{\pi \times (30)^2} + \frac{120 \times 4 \times 1000}{\pi \times (20)^2} + \frac{80 \times 4 \times 1000}{\pi \times (10)^2} \right] \\ &= 0.3855 \text{ N-m} \end{aligned}$$

28. (b)

$$T_1 = \frac{T_0 \times \frac{3L}{4}}{L} = \frac{3T_0}{4}$$

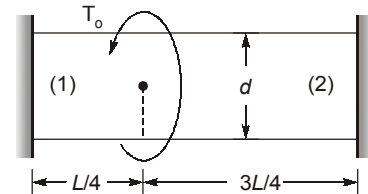
$$T_2 = \frac{T_0 \times \frac{L}{4}}{L} = \frac{T_0}{4}$$

$$\text{Maximum shear stress} = \frac{16 \times \frac{3T_0}{4}}{\pi d^3} = \frac{12T_0}{\pi d^3}$$

$$\text{At, } r = 0$$

$$\text{Shear stress} = 0$$

$$\text{Difference} = \frac{12T_0}{\pi d^3} - 0 = \frac{12T_0}{\pi d^3}$$



29. (c)

$$\text{Force perpendicular to beam} = P \cos 30^\circ$$

$$\text{Axial force in beam} = P \sin 30^\circ \text{ (compressing)}$$

$$\text{Direct stress, } \sigma_d = \frac{-P \sin 30^\circ}{A} = \frac{20 \times 10^3 \times 0.5}{480 \times 10^{-4}} = 0.2083 \text{ MPa}$$

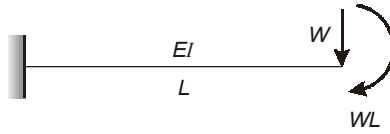
$$\text{Maximum bending stress} = \frac{WL}{4} / Z = \frac{\frac{P \cos 30^\circ}{4} \times \frac{3}{\cos 30^\circ}}{24 \times 10^{-4}} = 6.25 \text{ MPa}$$

Maximum compressive stress in the beam

$$= 0.2083 + 6.25 = 6.4583 \text{ MPa}$$

30. (a)

The given cantilever beam can be modified in to a beam as shown below



$$\begin{aligned} \text{Deflection at, } Q &= \frac{WL^3}{3EI} + \frac{WL \times L^2}{2EI} \\ &= \frac{2WL^3 + 3WL^3}{6EI} = \frac{5WL^3}{6EI} \end{aligned}$$

$$\begin{aligned} \text{Slope at, } Q &= \frac{WL^2}{2EI} + \frac{WL \times L}{EI} \\ &= \frac{WL^2 + 2WL^2}{2EI} = \frac{3WL^2}{2EI} \end{aligned}$$

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