S.No. : 01 GH1_ME_D_030519
Engineering Mathematics

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CLASS TEST 2019-2020

MECHANICAL ENGINEERING

Subject : Engineering Mathematics

Date of test : 03/05/2019

Answer Key

1. (b)	7. (d)	13. (a)	19. (b)	25. (b)
2. (a)	8. (a)	14. (a)	20. (c)	26. (c)
3. (b)	9. (a)	15. (c)	21. (a)	27. (c)
4. (b)	10. (a)	16. (d)	22. (a)	28. (d)
5. (a)	11. (d)	17. (c)	23. (c)	29. (a)
6. (d)	12. (c)	18. (a)	24. (c)	30. (c)

DETAILED EXPLANATIONS

1. (b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}_{3\times 3}$$

$$\text{order of matrix} = 3$$
$$\text{Rank} = 2$$

$\therefore$ dimension of null space of $A = 3 - 2 = 1$.

2. (a)

$$f(z) = 1+(1-z)+(1-z)^2+----=\frac{1}{1-(1-z)} = \frac{1}{1-1+z} = \frac{1}{z}$$

3. (b)

$$f(x) = -2+6x-4x^2+0.5x^3$$
$$f'(x) = 6-8x+1.5x^2$$
$$x_{ini} = 0$$

By Newton Raphson Method,
$$x_1 = x_{ini} - \frac{f(x_{ini})}{f'(x_{ini})} = 0 - \frac{-2}{6}$$

$\Rightarrow$
$$x_1 = \frac{1}{3}$$

$\therefore$
$$\Delta x = x_1 - x_{ini} = \frac{1}{3}$$

4. (b)

$$u = f(x-cy)$$
$$\frac{\partial u}{\partial x} = f'(x-cy)(1)$$
$$\frac{\partial u}{\partial y} = f'(x-cy)(-c) = -c\cdot f'(x-cy) = -c\cdot\frac{\partial u}{\partial x}$$

$\therefore$
$$\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0$$

5. (a)

$$\text{Curl of vector} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y^3)\ \frac{\partial}{\partial z}(3z^2)\right] - j\left[\frac{\partial}{\partial x}(y^3)\ \frac{\partial}{\partial z}(2x^2)\right] + k\left[\frac{\partial}{\partial x}(3z^2)\ \frac{\partial}{\partial y}(2x^2)\right]$$

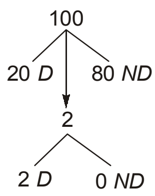
$$= i[3y^2 - 6z] - j[0] + k[0 + 0]$$

At, $x = 1$, $y = 1$ and $z = 1$

$$\text{Curl} = i(3 \times 1^2 - 6 \times 1) = -3i$$

6. (d)

Problem can be solved by hypergeometric distribution



$$p(X = 2) = \frac{20C_2 \times 80C_0}{100C_2} = \frac{19}{495}$$

7. (d)

Let $$z = a + bi$$

Since z is shown inside the unit circle in I quadrant, a and b are both +ve and $0 < \sqrt{a^2 + b^2} < 1$

Now $$\frac{1}{z} = \frac{1}{a + bi}$$

$$\frac{a - bi}{a^2 + b^2} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$$

Since a, $b > 0$,

$$\frac{a}{\sqrt{a^2 + b^2}} > 0$$

$$\frac{-b}{a^2 + b^2} < 0$$

So $\frac{1}{z}$ is in IV quadrant.

$$\left|\frac{1}{z}\right| = \sqrt{\left(\frac{a}{a^2 + b^2}\right)^2 + \left(\frac{-b}{a^2 + b^2}\right)^2}$$

$$= \sqrt{\frac{1}{a^2 + b^2}} = \frac{1}{\sqrt{a^2 + b^2}}$$

Since $$0 < \sqrt{a^2 + b^2} < 1$$

$$\frac{1}{\sqrt{a^2 + b^2}} > 1$$

So $\frac{1}{z}$ is outside the unit circle is IV quadrant.

8. (a)

$$\frac{d^2y}{dx^2} + y = 0$$

$$D^2 + 1 = 0$$

$$D = \pm i = 0 \pm 1i$$

∴ General solution is

$$\begin{aligned} y &= e^{0x}[C_1 \cos(1 \times x) + C_2 \sin(1 \times x)] \\ &= C_1 \cos x + C_2 \sin x \\ &= P \cos x + Q \sin x \end{aligned}$$

where P and Q are some constants.

9. (a)

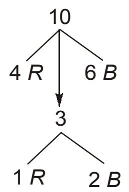
10. (a)

$$L(\cos \omega t) = \frac{s}{s^2 + \omega^2}$$

11. (d)

The problem can be represented by the following diagram.

$$\text{p(1R and 2B)} = \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{60}{120} = \frac{1}{2}$$



12. (c)

Given equation are

$$\begin{aligned} x + 2y + z &= 6 \\ 2x + y + 2z &= 6 \\ x + y + z &= 5 \end{aligned}$$

Given system can be written as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 5 \end{bmatrix}$$

Augmented matrix is $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 2 & 1 & 2 & 6 \\ 1 & 1 & 1 & 5 \end{array}\right]$

By gauss elimination

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 2 & 1 & 2 & 6 \\ 1 & 1 & 1 & 5 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & -3 & 0 & -6 \\ 0 & -1 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{R_3 - \frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & -3 & 0 & -6 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$r(A) = 2$$
$$r(A \mid B) = 3$$

Since the rank of coefficient matrix is 2 and rank of argument matrix is 3, which is not equal. Hence system has no solution i.e. system is inconsistent.

13. (a)

Putting

$$f'(x) = 6x^2 - 6x - 36 = 0$$

$\Rightarrow$ $$x^2 - x - 6 = 0$$

$\Rightarrow$ $$x = 3 \text{ or } -2$$

Now $$f''(x) = 12x - 6$$

and $$f''(3) = 30 > 0 \text{ (minima)}$$

and $$f''(-2) = -30 < 0 \text{ (maxima)}$$

Hence maxima is at $x = -2$ only.

14. (a)

A. $$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$\Rightarrow$ $$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log y = \log x + \log c = \log cx$$

$$y = cx \quad \text{... Equation of straight line.}$$

B. $$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{y} = \frac{-dx}{x} \Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\log y = -\log x + \log c$$

$$\log y + \log x = \log c$$

$$\log yx = \log c$$

$$yx = c$$

$$y = c/x \quad \text{... Equation of hyperbola.}$$

C. $\dfrac{dy}{dx} = \dfrac{x}{y}$, $y\,dy = x\,dx$

$$\Rightarrow \int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} - \frac{x^2}{2} = \frac{c^2}{2} \rightarrow \text{const}$$

$$y^2 - x^2 = c^2$$

$$\frac{y^2}{c^2} - \frac{x^2}{c^2} = 1 \qquad \text{... Equation of hyperbola.}$$

D. $\dfrac{dy}{dx} = \dfrac{-x}{y} \Rightarrow \int y\,dy = -\int x\,dx$

$$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{c^2}{2}$$

$$\frac{y^2}{2} + \frac{x^2}{2} = \frac{c^2}{2}$$

$$x^2 + y^2 = c^2 \qquad \text{... Equation of a circle}$$

15. (c)

$$f(x) = 2x^3 - 3x^2 \text{ in } [-1, 2]$$
$$f'(x) = 6x^2 - 6x$$
$$f'(x) = 0$$
$$6x^2 - 6x = 0$$
$$6x(x-1) = 0$$
$$x = 0, 1$$
$$f''(x) = 12x - 6$$
$$f''(0) = -6 \quad \text{Max}$$
$$f''(1) = 6 \quad \text{Min}$$

$x = -1 \quad f(-1) = -5$ G. Min.
$x = 2 \quad f(2) = 4$
$x = 0 \quad f(0) = 0$
$x = 1 \quad f(1) = -1$

G. Minima is -5 at $x = 1$.

16. (d)

$$\text{Trace} = \text{Sum of eigen values}$$
$$1 + a = 6$$
$$\Rightarrow \quad \boldsymbol{a = 5}$$

Determinant = Product of eigen values

$$(a - 4b) = -7$$
$$5 - 4b = -7$$
$$-4b = -12$$
$$\Rightarrow \quad \boldsymbol{b = 3}$$
$$\therefore \quad a = 5,\ b = 3$$

17. (c)

From Newton– Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad \text{... (i)}$$

Given function is, $f(x) = x^3 + 3x - 7$

and $f'(x) = 3x^2 + 3$

Putting $x_0 = 1,$

$$f(x_0) = f(1) = (1)^3 + 3 \times (1) - 7 = -3$$

$$f'(x_0) = f'(1) = 3 \times (1)^2 + 3 = 6$$

Substituting x_0, $f(x_0)$ and $f'(x_0)$ values into (i) we get,

$\therefore$ $$x_1 = 1 - \left(\frac{-3}{6}\right) \times 1 = 1.5$$

18. (a)

Eigen values are

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$\therefore$ $$\lambda = \pm i$$

to find eigen vector,

$\lambda = +i$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ $$-i\, x_1 - x_2 = 0 \text{ and } x_1 - ix_2 = 0$$

clearly, $$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -j \end{bmatrix} \text{and} \begin{bmatrix} j \\ 1 \end{bmatrix}, \text{ satisfy}$$

$$\lambda = -i \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$ix_1 - x_2 = 0 \text{ and } x_1 + ix_2 = 0$$

clearly,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} j \\ -1 \end{bmatrix} \text{and} \begin{bmatrix} 1 \\ j \end{bmatrix}, \text{ satisfy}$$

Thus, the two eigen value of the given matrix are $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$.

19. (b)

Let $$I = \int_o^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \quad \text{...(i)}$$

Since $\int_o^a f(x)dx = \int_o^a f(a-x)dx$

$$I = \int_o^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \quad \text{...(ii)}$$

(i) + (ii) ⇒ $$2I = \int_o^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

⇒ $$2I = \int_o^a dx$$

⇒ $2I = a$

⇒ $I = a/2$

20. (c)

We need absolute maximum of
$f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6]
First find local maximum if any by putting $f'(x) = 0$.
i.e. $f'(x) = 3x^2 - 18x + 24 = 0$
i.e. $x^2 - 6x + 8 = 0$
$x = 2, 4$
Now $f''(x) = 6x - 18$
$f''(2) = 12 - 18 = -6 < 0$(So $x = 2$ is a point of local maximum)
and $f''(4) = 24 - 18 = +6 > 0$(So $x = 4$ is a point of local minimum)
Now tabulate the values of f at end point of interval and at local maximum point, to find absolute maximum in given range, as shown below:

x	$f(x)$
1	21
2	25
6	41

Clearly the absolute maxima is at $x = 6$
and absolute maximum value is 41.

21. (a)

$$AB^T = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} 7 & 8 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$$

22. (a)

$$f(t) = L^{-1}\left[\frac{1}{s^2(s+1)}\right]$$

$$\frac{1}{s^2(s+1)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}$$

$$\frac{1}{s^2(s+1)} = \frac{As(s+1)+B(s+1)+C(s^2)}{s^2(s+1)}$$

Matching coefficient of s^2, s and constant in numerator we get,

$$A + C = 0 \quad \text{... (i)}$$
$$A + B = 0 \quad \text{... (ii)}$$
$$B = 1 \quad \text{... (iii)}$$

Solving we get $A = -1$, $B = 1$, $C = 1$

So, $$f(t) = L^{-1}\left[\frac{-1}{s}+\frac{1}{s^2}+\frac{1}{s+1}\right]$$

$$= -1 + t + e^{-t} = t - 1 + e^{-t}$$

23. (c)

Pole, $z = 2$ lies inside $|z| = 3$

$$\text{Res } f(z) = \lim_{z\to 2}(z-2)\frac{z^2-2z+3}{z-2}$$

$z = 2$, $$= 8 - 4 + 3 = 7$$

By Cauche residue theorem

$$I = 2\pi i(7) = 14\pi i$$

24. (c)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x = 2$, $$f(x_0) = 2+\sqrt{2}-3 = \sqrt{2}-1$$

$$f'(x) = 1+\frac{1}{2\sqrt{x}}$$

$$f'(x_0) = 1+\frac{1}{2\sqrt{2}}$$

Then, $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2-\frac{\sqrt{2}-1}{1+\frac{1}{2\sqrt{2}}}$$

$\Rightarrow$ $$x_1 = 1.694$$

25. (b)

$$f(x) = x^3 - 3x^2 - 24x + 100 \qquad x \in [-3, 3]$$
$$f'(x) = 3x^2 - 6x - 24$$
$$f'(x) = 0 \quad \text{at } x = 4, -2$$

Critical points are {–3, –2, 3}

$$f(-3) = -27 - 27 + 72 + 100 = 118$$
$$f(-2) = -8 - 12 + 48 + 100 = 128$$
$$f(3) = 27 - 27 - 72 + 100 = 28$$

Hence $f(x)$ has minimum value at $x = 3$ which is 28.

26. (c)

$$\frac{dy}{dt} = -5y$$
$$\int \frac{dy}{y} = -\int 5dt$$
$$\ln y = -5t + C$$

at $t = 0$

$$y = 2$$
$$\ln 2 = C$$

So, $\ln y = -5t + \ln 2$

$$\ln \frac{y}{2} = -5t$$
$$\frac{y}{2} = e^{-5t}$$
$$y = 2e^{-5t}$$

at $t = 3$

$$y = 2e^{-15}$$

27. (c)

$$x(z) = \frac{1-2z}{z(z-1)(z-2)}$$

poles are $z = 0$, $z = 1$ and $z = 2$

Residue at $z = 0$

$$\text{residue} = \text{value of } \frac{1-2z}{(z-1)(z-2)} \text{ at } z = 0$$
$$= \frac{1-2\times 0}{(0-1)(0-2)} = \frac{1}{2}$$

Residue at $z = 1$

$$\text{residue} = \text{value of } \frac{1-2z}{z(z-2)} \text{ at } z = 1$$
$$= \frac{1-2\times 1}{1(1-2)} = 1$$

Residue at $z = 2$

$$\text{residue} = \text{value of } \frac{1-2z}{z(z-1)} \text{ at } z = 2$$

$$= \frac{1-2\times 2}{2(2-1)} = -\frac{3}{2}$$

$\therefore$ The residues at its poles are $\frac{1}{2}, 1$ and $-\frac{3}{2}$.

28. (d)

$P(\text{A wins}) = p(6$ in first throw by $A) + p(A$ not 6, B not 6, A 6$) + \ldots$

$$= \frac{1}{6} + \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6} + \cdots$$

$$= \frac{1}{6}\left(1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \cdots\right) = \frac{1}{6}\cdot\frac{1}{1-\left(\frac{5}{6}\right)^2} = \frac{6}{11}$$

29. (a)

Space headway,

$$S = 60\,t - 60\,t^2$$

$$\frac{dS}{dt} = 60 - 120t = 0$$

$$t = 0.5 \text{ hr} = 30 \text{ minutes}$$

$$\frac{d^2S}{dt^2} = -120 \times 0 \text{ (Maxima)}$$

$\therefore$ Maximum space head

$$S_{max} = 60 \times 0.5 - 60 \times (0.5)^2 = 15 \text{ km}$$

30. (c)

$$\frac{d^2y}{dx^2} = y$$

$\Rightarrow$ $D^2y = y$ $(\therefore\ d/dx = D)$

$$(D^2 - 1)y = 0$$

$$D^2 - 1 = 0$$

$$D = \pm 1$$

$$y = C_1\,e^x + C_2\,e^{-x}$$

Given point passes through origin

$\Rightarrow$ $0 = C_1 + C_2$

$C_1 = -C_2$ ...(i)

Also, point passes through (ln 2, 3/4)

$\Rightarrow$ $\frac{3}{4} = C_1\,e^{\ln 2} + C_2\,e^{-\ln 2}$

$$\frac{3}{4} = 2C_1 + \frac{C_2}{2}$$

$\Rightarrow$ $C_2 + 4C_1 = 1.5$ ...(ii)

From (i) $C_1 = -C_2$, putting in (ii), we get

$\Rightarrow$ $-3C_2 = 1.5$

$C_2 = -0.5$

$\therefore$ $C_1 = 0.5$

$\Rightarrow$ $y = 0.5\,(e^x - e^{-x})$

$$y = \frac{e^x - e^{-x}}{2}$$

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