CLASS TEST

S.N. : 01_SK_A+B_090422

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NETWORK THEORY

EC + EE

Date of Test : 09/04/2022

ANSWER KEY

1.	(c)	7.	(b)	13.	(c)	19.	(c)	25.	(c)
2.	(d)	8.	(d)	14.	(c)	20.	(a)	26.	(b)
3.	(b)	9.	(c)	15.	(c)	21.	(c)	27.	(b)
4.	(c)	10.	(d)	16.	(d)	22.	(c)	28.	(a)
5.	(a)	11.	(c)	17.	(a)	23.	(a)	29.	(c)
6.	(a)	12.	(c)	18.	(c)	24.	(b)	30.	(c)

DETAILED EXPLANATIONS

1. (c)

Since total power absorbed or delivered in the circuit

$\Rightarrow \quad \Sigma P = 0;$

then $-30 \times 6 + 6 \times 12 + 3\,V_0 + 28 + 28 \times 2 - 3 \times 10 = 0$

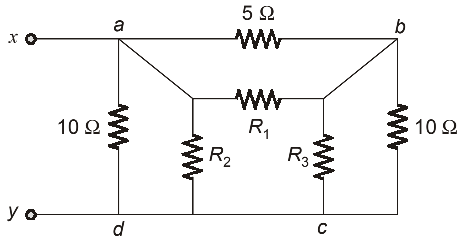
$72 + 84 + 3\,V_0 = 210;$

or $3\,V_0 = 54$

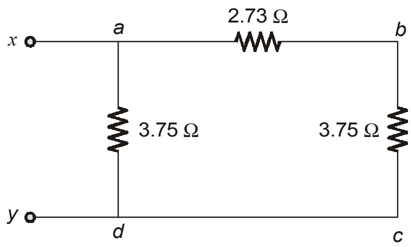
$\Rightarrow \quad V_0 = 18\text{ V}$

2. (d)

Converting Y circuit to Δ circuit, the circuit is as shown below,



Further simplifying the circuit we get,



$$R_1 = 2 + 2 + \frac{2 \times 2}{2 + 0} = 6\ \Omega$$

Due to symmetry of the network inside star network,

$$R_1 = R_2 = R_3 = 6\ \Omega$$

$$R_{ab} = \frac{5R_1}{5 + R_1} = 2.73\ \Omega$$

$$R_{bc} = \frac{10R_3}{10 + R_3} = 3.75\ \Omega$$

$$R_{ad} = \frac{10R_2}{10 + R_2} = 3.75\ \Omega$$

$$R_{x-y} = R_{ad} \,\|\, (R_{ab} + R_{bc})$$

$$= 2.375\ \Omega$$

3. (b)

For the given circuit,

$$\text{time constant, } \tau = \frac{L}{R_{eq}}$$

$$R_{eq} = (5 \,||\, 20) + 1 = 5\ \Omega$$

$$L = 2\text{ H};$$

$$\tau = \frac{2}{5}\text{sec}$$

$\therefore$ $$i(t) = i(0)e^{-t/\tau}$$

$$i(t) = 10e^{-2.5t}\text{ A}$$

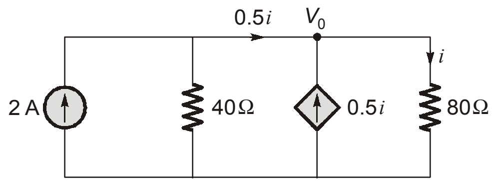
For $v(t)$; $$v(t) = 20\, i_0$$ (i_0 is current in 20 Ω resistor)

Where $$i_0 = \frac{5}{5+20}[-i(t)] = -2\, e^{-2.5t}\text{ A}$$

$$v(t) = 20 \times (-2\, e^{-2.5t}) = -40\, e^{-2.5\,t}\text{ V for } t > 0$$

4. (c)

Before $t = 0$, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in figure after transforming the voltage source.



From circuit,

$$0.5\, i = 2 - \frac{V_0}{40} \quad \text{...(i)}$$

or, $$i = \frac{V_0}{80} \quad \text{...(ii)}$$

Hence, from equation (i);

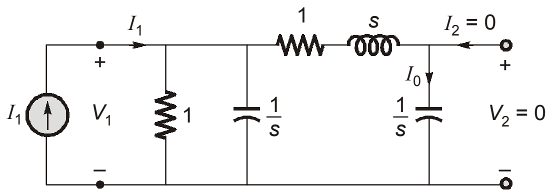
$$0.5 \times \frac{V_0}{80} = 2 - \frac{V_0}{40}$$

or $$V_0 = \frac{320}{5} = 64\text{ V}$$

or $$i = \frac{V_0}{80} = 0.8\text{ A}$$

5. (a)

Consider the circuit given below.



For Z_{21} ;

$$I_0 = \frac{\left(1 \| \frac{1}{s}\right) \times I_1}{\left(1 \| \frac{1}{s}\right) + 1 + s + \frac{1}{s}} = \frac{\frac{1}{s+1} \times I_1}{\frac{1}{s+1} + 1 + s + \frac{1}{s}}$$

$$I_0 = \frac{\frac{s}{s+1}}{\frac{s}{s+1} + s^2 + s + 1} I_1 = \frac{s}{s^3 + 2s^2 + 3s + 1} I_1$$

$$\therefore \quad V_2 = \frac{1}{s} \times I_0 = \frac{I_1}{s^3 + 2s^2 + 3s + 1}$$

$$Z_{21} = \frac{V_2}{I_1} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

6. (a)

Applying KVL,

$$V_{xy} - j\omega L_2\, i_2 - j\omega\, M\, i_1 = 0$$

as $\quad i_2 = 0$

then, $\quad V_{xy} - j\omega\, M\, i_1 = 0 \quad$ ...(i)

applying KVL in loop 1

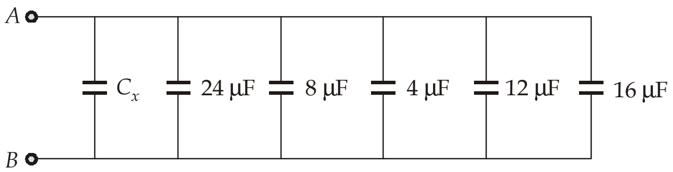
$$V_1 = j\omega L_1\, i_1 \quad \text{...(ii)}$$

Using value of i_1 from equation (ii) in equation (i)

$$V_{xy} = j\omega M\left[\frac{V_1}{j\omega L_1}\right] = \frac{MV_1}{L_1}$$

7. (b)

The circuit can be redrawn as,



$$\therefore \quad C_{eq} = C_x + [24 + 8 + 4 + 12 + 16]\ \mu\text{F}$$

$$C_{eq} = C_x + 64\ \mu\text{F} \quad \text{...(i)}$$

$\because$ Energy stored in capacitor is

$$E = \frac{1}{2}CV^2$$

$$536 = \frac{1}{2}C_{eq} \times (4)^2$$

or $$C_{eq} = \frac{536 \times 2}{(4)^2} = 67\ \mu\text{F}$$

From equation (i), $$C_x = 67 - 64 = 3\ \mu\text{F}$$

8. (d)

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{212.2}{\sqrt{2}} = \frac{150 \times \sqrt{2}}{\sqrt{2}} = 150.04 \approx 150\ \text{V}$$

and $\quad V_R = 120\ \text{V} \quad$ (given)

$$\therefore \quad V_L = \sqrt{150^2 - 120^2} = 90\ \text{V}$$

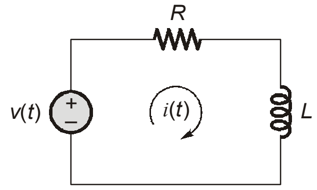
$$\because \quad |I| = \frac{V_R}{R} = \frac{120}{1 \times 10^3} = 0.12\ \text{A}$$

$$\therefore \quad |V_L| = |I| \times \omega L$$

or $$L = \frac{|V_L|}{\omega \times |I|} = \frac{90}{500 \times 0.12} = 1.5\ \text{H}$$

9. (c)

The circuit can be redrawn as,



Applying KVL, we get,

$$v(t) = L\frac{di}{dt} + R \cdot i$$

Since, the unit step voltage has been given i.e.

$$v(t) = u(t)$$

or $$v(t) - iR = L\frac{di}{dt}$$

$$(v(t) - iR)dt = Ldi \qquad \text{...(i)}$$

or $$\frac{1}{L}dt = \frac{di}{v(t) - iR}$$

Integrating both sides, we get,

$$\frac{1}{L}\int_0^t dt = \int_{I_o}^{I} \frac{di}{1 - iR} \qquad \text{...(ii)}$$

On solving equation (ii), we get,

or, $$i(t) = \frac{1}{R}(1 - e^{-R/L\,t}) + I_o(-e^{-R/L\,t})$$

at $t = \tau = \frac{L}{R}$, $$i(t = \tau) = \frac{0.632\text{ V}}{R} + 0.368\, I_o$$

10. (d)

For maximum power to be transferred,

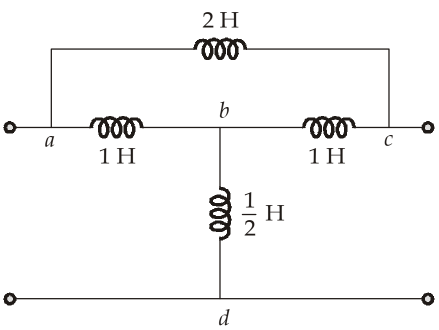
$$Z_L = Z_s^*$$

Here, $Z_s = (2 - j4)\Omega$

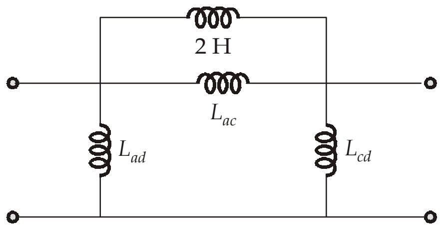
$\therefore$ $$Z_s^* = (2 - j4)^* = (2 + j4)\ \Omega$$

11. (c)

The given circuit can be drawn as,



Converting 'Y' 'acd' to 'Δ', we get

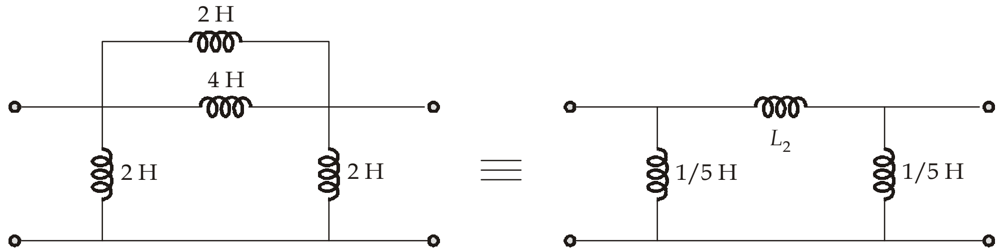


Here, $$L_{cd} = \frac{1\times\frac{1}{2}+1\times\frac{1}{2}+1\times 1}{1} = 2\text{ H}$$

$$L_{ad} = \frac{1\times\frac{1}{2}+1\times\frac{1}{2}+1\times 1}{1} = 2\text{ H}$$

$$L_{ac} = \frac{1\times 1+1\times\frac{1}{2}+1\times\frac{1}{2}}{\frac{1}{2}} = 4\text{ H}$$

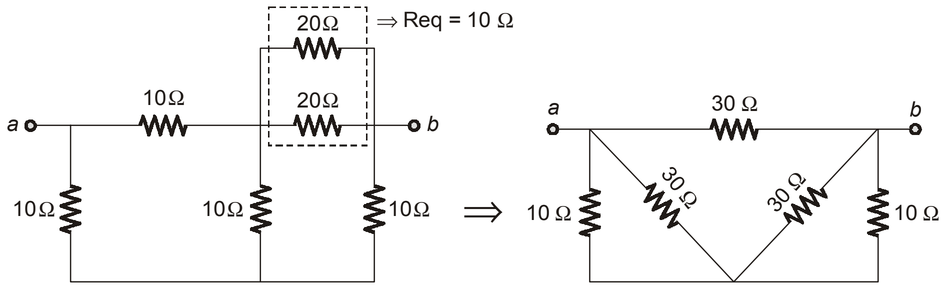
∴ The circuit can be redrawn as



where, $$L_2 = \frac{2\times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\text{ H}$$

12. (c)

To find R_{Th}, consider the below circuit

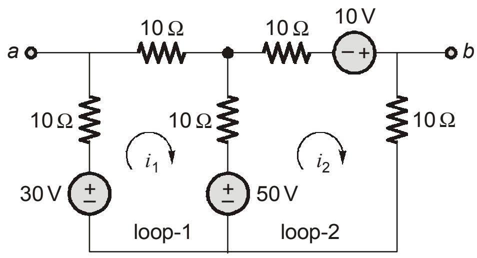


Transforming the Y-sub network to a Δ network.

then $$R_{Th} = R_{ab} = [30 \,||\, (7.5 + 7.5)] = 10\ \Omega$$

To find V_{Th}, we transform the 20 V and 5 V sources.

We obtain the circuit shown as below



Applying KVL in loop-1 we get

$$-30 + 50 + 30\, i_1 - 10\, i_2 = 0$$

or $$3\, i_1 - i_2 = -2 \qquad ...(i)$$

Apply KVL in loop-2, we get

$$-50 - 10 + 30\, i_2 - 10\, i_1 = 0$$

or $$-i_1 + 3\, i_2 = 6 \qquad ...(ii)$$

From equation (i) and (ii), we get

$$i_1 = 0\text{ A},$$
$$i_2 = 2\text{ A}$$

Now: applying KVL to the output loop,

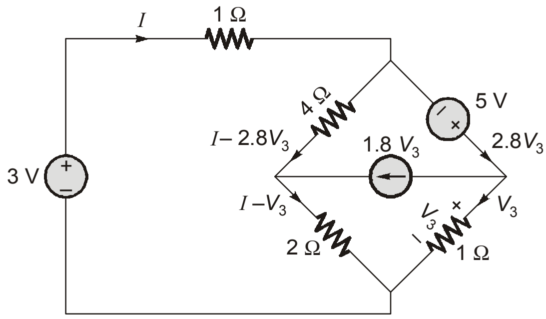
$$-V_{ab} - 10\, i_1 + 30 - 10\, i_2 = 0$$

$$\Rightarrow \quad -V_{ab} + 30 - 10 \times 2 = 0$$

$$V_{Th} = V_{ab} = 10\text{ V}$$

13. (c)

Showing the corresponding currents in all the branches, the circuit is shown as below



Now we apply KVL in outer loop

$$-3 + I(1) - 5 + V_3 = 0$$

$$I + V_3 = 8 \qquad ...(i)$$

Applying KVL in bridge,

$$-5 + V_3 - 2\,(I - V_3) - 4\,(I - 2.8V_3) = 0$$

$$14.2\, V_3 - 6\, I = 5$$

$$I = 5.37\text{ A}$$

$$V_3 = 2.62\text{ V}$$

14. (c)

Applying KCL at node

$$\frac{V_0}{4} + 10 + 2V_0 = 0;$$

Solving we get,

$$V_0 = -4.444\text{ V}$$

Current through the controlled source

$$i = 2V_0 = -8.888\text{A}$$

and the voltage across it,

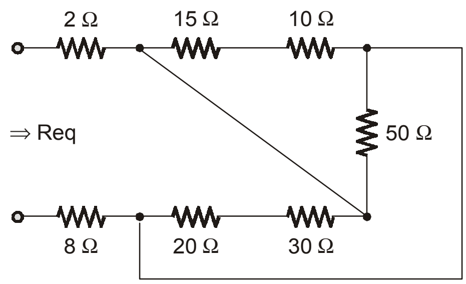
$$V = (6+4)\times\frac{V_0}{4} = -11.111 \text{ volts}$$

Hence, power absorbed by dependent source

$$P = (-8.888)(-11.111) = 98.75 \text{ W}$$

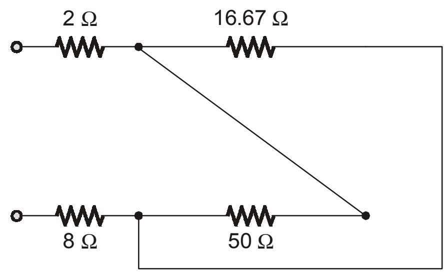
15. (c)

1st combine 10 Ω and 40 Ω resistors and redrawing the circuit.



We now have,

$$(10\ \Omega + 15\ \Omega)\ ||\ 50\ \Omega = 16.67\ \Omega$$



$$\text{equivalent resistance} = 2\Omega + (50\Omega \,||\, 16.67\Omega) + 8\Omega$$

$$= 22.5\ \Omega$$

16. (d)

The voltage across inductor is,

$$v_L = L\frac{di_L}{dt} = L\frac{di_s(t)}{dt}$$

Current across capacitor is given by,

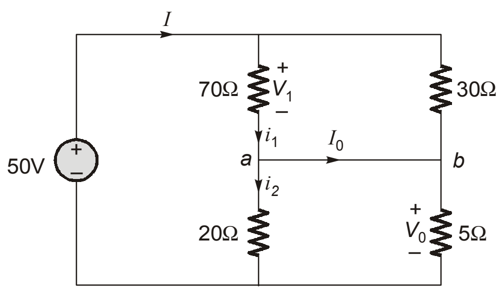
$$i_c = C\frac{dv_c}{dt}$$

$$v_c = 3v_L$$

$$\Rightarrow \quad i_c = 3C\frac{dV_L}{dt} = 3C.L\frac{d^2 i_s(t)}{dt^2} = -9.6 \sin 4t \text{ A}$$

17. (a)

∵ From, the circuit;



We can get ; $I = \dfrac{50}{(70\|30)+(20\|5)} = \dfrac{50}{(21+4)} = 2A$

$\because$ $V_1 = 21 \times I = 42$ V, & $V_0 = 4I = 8$V

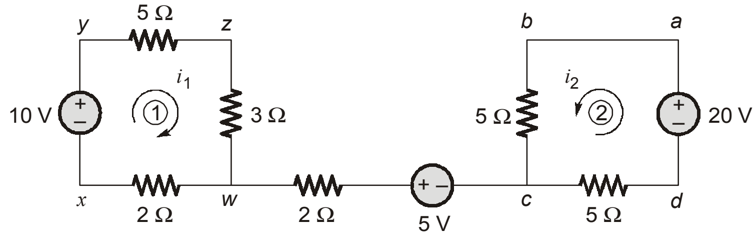
$i_1 = \dfrac{V_1}{70} = 0.6$ A; $i_2 = \dfrac{V_0}{20} = 0.4$ A

Now, KCL at node '*a*', $i_1 = i_2 + I_0$; $I_0 = 0.6 - 0.4 = 0.2$ A

Hence, $V_0 = 8$ V, $I_0 = 0.2$ A

18. (c)

The circuit can be redrawn as



The current in the loop 1 is given by,

$$i_1 = \frac{10}{5+3+2} = 1\text{ A} \quad \text{...(i)}$$

$\therefore$

$$V_{w-x} = 2 \times 1 = 2\text{ V}$$

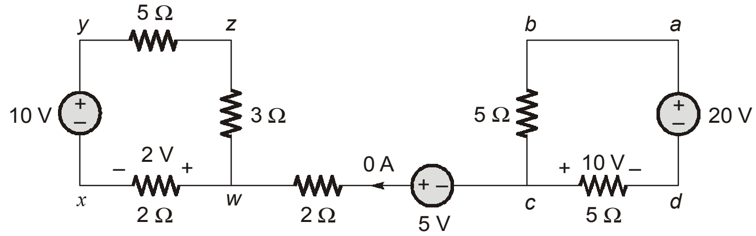
$$V_{x-w} = -V_{w-x} = -2\text{ V}$$

Similarly the current in loop 2 is given by,

$$i_2 = \frac{20}{5+5} = 2\text{ A} \quad \text{...(ii)}$$

$$V_{c-d} = 5 \times 2 = 10\text{ V}$$

Thus, the circuit can be redrawn with respective polarities of the voltage drops as,



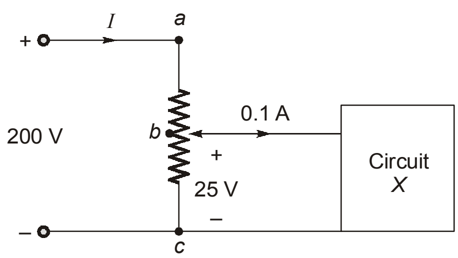
By inspecting the respective polarities, we get,

$$V_{x-d} = V_{x-w} + V_{w-c} + V_{c-d}$$

$$= -2 + 5 + 10 = 13\text{ V}$$

19. (c)

By redrawing the circuit, we get,



Let us assume the tap position at 'b'

$\because$ $\quad R_{ac} = 500\,\Omega$ (given)

Let, $\quad R_{bc} = x\ \Omega$

$\therefore$ $\quad R_{ab} = (500 - x)\Omega$

Also, $\quad V_{ab} = 200 - 25 = 175$ V

The circuit current, $\quad I = \dfrac{25}{x} + 0.1$

We can also write, $\quad V_{ab} = I \times (500 - x) = \left(\dfrac{25}{x} + 0.1\right)(500 - x)$

or $\quad 0.1x^2 + 150x - 12500 = 0$

By solving, we get,

$$x = 79.16\,\Omega \quad \text{and} \quad -1579.16\,\Omega$$

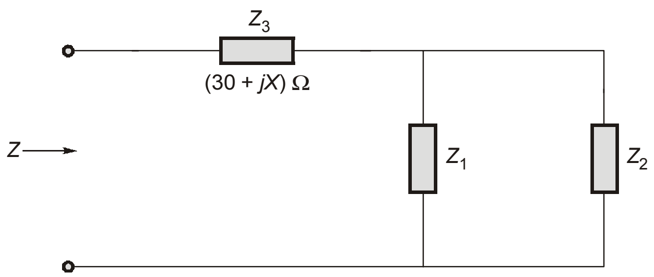
By considering $x = 79.16\ \Omega$,

$$I = \frac{25\text{ V}}{x} + 0.1\text{ A} = 0.416\text{ A}$$

$$\text{Total power supplied} = 200\text{ V} \times I = 200\text{ V} \times 0.416\text{ A} = 83.2\text{ W}$$

20. (a)

The circuit can be redrawn as,



Total impedance of this circuit, $\quad Z = Z_3 + (Z_1 \,||\, Z_2)$

$$= (30 + jX) + \left[\frac{(20 + j10)(10 - j30)}{20 + j10 + 10 - j30}\right]$$

$$= (30 + jX) + \frac{200 - j600 + j100 + 300}{30 - j20}$$

$$= (30 + jX) + \left(\frac{500 - j500}{30 - j20}\right)$$

Using factorization, we get,

$$= 30 + jX + \left[\frac{500(1 - j)(30 + j20)}{(30)^2 + (20)^2}\right]$$

$$= 30 + jX + \frac{5}{13}(50 - j10) = \left(30 + \frac{5}{13} \times 50\right) + j\left(X - \frac{5}{13} \times 10\right)$$

At the resonant condition, the imaginary part of the total impedance expression is zero.

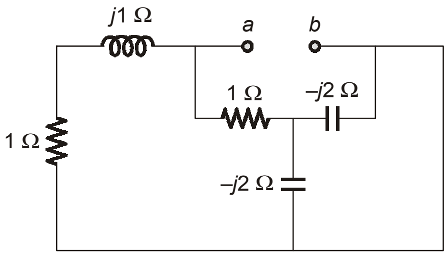
$\therefore$ $\quad X - \dfrac{5}{13} \times 10 = 0$

or, $\quad X = \dfrac{50}{13} = 3.85\ \Omega$

21. (c)

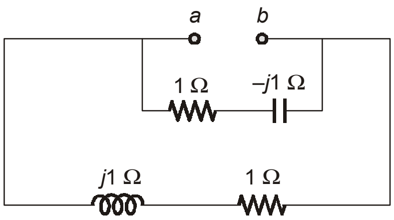
In order to determine Z_L for maximum transfer, we find Thevenin's equivalent impedance Z_{Th} across the terminals of Z_L. For this, the voltage source should be replaced by a short circuit and element values should be expressed in terms of their respective impedances.

Thus, by redrawing the circuit, we get,



As the two capacitors are connected is parallel, the equivalent impedance of their combination is

$$(-j2\ \Omega \parallel -j2\ \Omega) = -j1\ \Omega$$



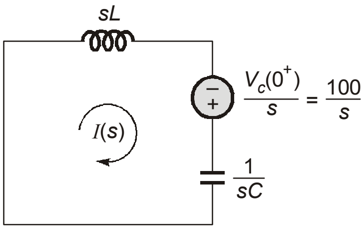
Therefore,
$$Z_{Th} = \frac{(1+j1)(1-j1)}{1+j1+1-j1} = \frac{2}{2} = 1\Omega$$

For maximum power transfer,
$$Z_L = Z_{Th}^{*} = 1\Omega$$

22. (c)

$$i_L(0^-) = i_L(0^+) = 0\text{ A}$$
$$v_c(0^-) = v_c(0^+) = 100\text{ V}$$

The circuit can be redrawn in s-domain as,



or
$$I(s) = \frac{\left(\frac{100}{s}\right)}{\left(sL + \frac{1}{sC}\right)} = \frac{100}{L}\left(\frac{1}{s^2 + \frac{1}{LC}}\right)$$

$$= 100\sqrt{\frac{C}{L}}\left(\frac{\frac{1}{\sqrt{LC}}}{s^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}\right)$$

Taking inverse Laplace transform of the above equation, we get,

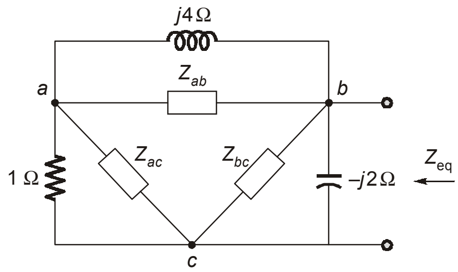
$$i(t) = 100\sqrt{\frac{C}{L}}\sin\frac{1}{\sqrt{LC}}t$$

Now by putting the values of L and C, we get,

$$i(t) = 100\sqrt{\frac{10\times10^{-6}}{1\times10^{-3}}}\sin\frac{t}{\sqrt{10\times1\times10^{-9}}} = (10\sin10^4 t)\text{ A}$$

23. (a)

We apply a Y to Δ transformation,



$$Z_{ab} = \frac{2-j2+j4}{j2} = \frac{j2+2}{j2} = (1-j)\ \Omega$$

$$Z_{ac} = \frac{2+j2}{2} = (1+j)\Omega$$

$$Z_{bc} = \frac{2+j2}{-j1} = (-2+j2)\Omega$$

$\therefore$ $j4\,||\,Z_{ab} = j4\,||\,(1-j) = (1.6 - j0.8)\ \Omega$

$\therefore$ $1\,||\,Z_{ac} = 1\,||\,(1+j) = (0.6 + j0.2)\ \Omega$

$\therefore$ $(j4\,||\,Z_{ab}) + (1\,||\,Z_{ac}) = (2.2 - j0.6)\ \Omega$

$$\frac{1}{Z_{eq}} = \frac{1}{-j2} + \frac{1}{-2+j2} + \frac{1}{2.2-j0.6}$$

$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.404\angle 64.66^\circ$$

$$Z_{eq} = 2.473\angle{-64.66^\circ}\ \Omega = (1.06 - j2.23)\ \Omega$$

24. (b)

Let V_0 represent the voltage across the current source and then we apply KCL at that node,

$$4 + \frac{240 - V_0}{50} = \frac{V_0}{-j20} + \frac{V_0}{40+j30}$$

$$(0.36 + j0.38)\,V_0 = 88$$

$$V_0 = \frac{88}{0.36 + j0.38} = 168.13\angle{-46.55^\circ}\text{ V}$$

Now current through inductor,

$$I_1 = \frac{V_0}{40+j30} = 3.363\angle{-83.42^\circ}\text{ A}$$

Total power delivered in inductor,

$$S = V_0 I_1^* = 168.13\angle{-46.55^\circ} \times 3.633\angle 83.42^\circ$$

$$= (452.34 + j339.25)$$

reactive power in inductor $=$ 339.25 VAR

25. (c)

For $t < 0$, $V_R = \dfrac{2R_1R_2}{R_1+R_2}$,

$\therefore$ $V_R(0^+) = 10 = \dfrac{2R_1R_2}{R_1+R_2}$

$$\therefore \quad R_1 \parallel R_2 = 5\ \Omega \quad \text{...(i)}$$

$$i_L(0) = \frac{2R_1}{R_1 + R_2} A$$

For $t > 0$; $\quad i_L(t) = i_L(0) \times e^{-t/\tau_1} \quad$ [Where $\tau_1 = L/R_2 = 1/50R_2$]

$$i_L(t) = \frac{2R_1}{R_1 + R_2} e^{-50R_2 t}$$

$$\therefore \quad V_R(t) = \frac{2R_1 R_2}{R_1 + R_2} e^{-50R_2 t}$$

$$V_R(1\text{ ms}) = 5 = 10e^{-50R_2 \times 10^{-3}}$$

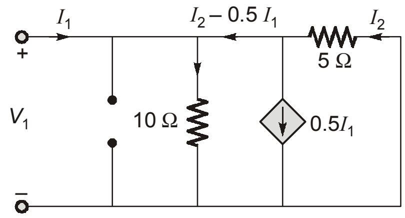
$$5 = 10e^{-0.05R_2}$$

$$0.05\,R_2 = 0.6931,$$

$$R_2 = 13.863\ \Omega$$

26. (b)

Short circuiting V_2 and redrawing the circuit



$$V_2 = 0\text{ V}$$

$$V_1 = 10(I_1 - 0.5I_1 + I_2)$$

$$V_1 = 5I_1 + 10I_2$$

$$V_1 = -5I_2$$

$$Y_{21} = \frac{I_2}{V_1} = \frac{-1}{5} = -0.2\ \mho$$

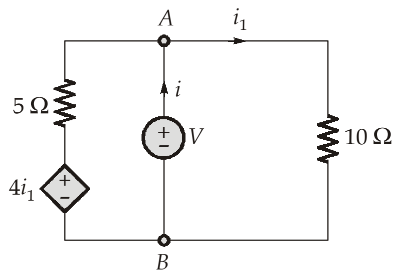
$$V_1 = 5I_1 - 10 \times \frac{V_1}{5}$$

$$3V_1 = 5I_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{3}{5} = 0.6\ \mho$$

27. (b)

For maximum power transfer, let us calculate the Thevenin's equivalent resistance,



Using KCL at node A, we get,

$$\frac{V - 4i_1}{5} + \frac{V}{10} = i$$

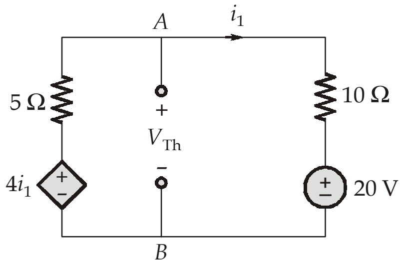
$$2V - 8i_1 + V = 10i$$

$$2V - 8\left(\frac{V}{10}\right) + V = 10i$$

$$(20 + 10 - 8)V = 100i$$

or $$R_{Th} = \frac{V}{i} = \frac{100}{22}\,\Omega = 4.545\,\Omega$$

Finding V_{Th}:



Using KCL at node A, we get,

$$\frac{V_{Th} - 4i_1}{5} + \frac{V_{Th} - 20}{10} = 0$$

$$2V_{Th} + V_{Th} - 20 = 8i_1$$

$$3V_{Th} = 8\left(\frac{V_{Th} - 20}{10}\right) + 20$$

$$3V_{Th} - 20 = \frac{8V_{Th}}{10} - \frac{160}{10}$$

$$3V_{Th} - 0.8V_{Th} = 4$$

$$V_{Th} = \frac{4}{2.2} = 1.818\text{ V}$$

$\therefore$ Maximum power transferred will be,

$$P = \frac{V_{Th}^2}{4R_{Th}} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(1.818)^2}{4 \times 4.545} = 181.81\text{ mW}$$

28. (a)

At steady state,

$$I = \frac{E}{R} = 1\text{ A}$$

When switch moves from position 'a' to 'b', the tapped energy in L starts discharging through 'C'.

$\therefore$ By KVL in the circuit

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

or, $$sLI(s) - Li_L(0^+) + \frac{1}{Cs}I(s) + \frac{v_c(0^+)}{s} = 0$$

$\because$ $$i_L(0^-) = i_L(0^+) = \frac{E}{R} = 1\text{ A}$$

and $$v_c(0^-) = v_c(0^+) = 0\text{ V}$$

Thus,

$$\frac{1}{Cs}I(s) + sLI(s) + \left(-\frac{E}{R}\right) = 0$$

or,
$$I(s) = \frac{Es/R}{L\left[s^2 + \frac{1}{LC}\right]}$$

taking inverse Laplace transform of the above equation, we get,

$$i(t) = \frac{1}{L}\cdot\frac{E}{R}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

By putting the values of parameters, we get,

$$i(t) = \cos t \text{ A}$$

29. (c)

In case of charging,

$$v_c(t) = v(0^+)[1 - e^{-t/\tau}]$$

$$20 = 50\,[1 - e^{-5/\tau}]$$

$$\frac{20}{50} - 1 = -e^{-5/\tau}$$

$$-\frac{5}{\tau} = -0.510$$

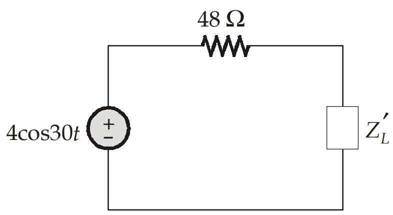
or, $\tau = 9.788$ sec

$\because$ $\tau = RC = R \times 50 \times 10^{-6} = 9.788$ sec

$\therefore$ $R = 195.76$ kΩ

30. (c)

Referring the secondary side, towards the primary, we get,



Where, $Z_L = 8 - \frac{j}{\omega C} = (8 - j4)\ \Omega$ and $n = \frac{1}{3}$

$\because$ $Z_L' = \frac{Z_L}{n^2} = 9Z_L = (72 - j36)\ \Omega$

$\therefore$ $I_1 = \frac{4\angle 0°}{48 + 72 - j36} = \frac{4\angle 0°}{125.28\angle -16.70°}$

$I_1 = 0.0319\angle 16.70°$

$\therefore$ $P_{8\,\Omega} = \left|\frac{I_1^2}{2}\right| \times 72 = 0.5088 \times 10^{-3} \times 72$

$P_{8\,\Omega} = 36.63$ mW

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