CLASS TEST

S.No. : 01 SK_AB_240322

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SIGNALS AND SYSTEMS

EC + EE

Date of Test : 24/03/2022

ANSWER KEY

1.	(d)	7.	(d)	13.	(b)	19.	(a)	25.	(b)
2.	(b)	8.	(b)	14.	(b)	20.	(d)	26.	(a)
3.	(a)	9.	(c)	15.	(b)	21.	(c)	27.	(d)
4.	(a)	10.	(b)	16.	(c)	22.	(d)	28.	(c)
5.	(a)	11.	(c)	17.	(a)	23.	(d)	29.	(b)
6.	(c)	12.	(a)	18.	(a)	24.	(c)	30.	(b)

DETAILED EXPLANATIONS

1. (d)

$$2^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-2z^{-1}}$$

$\therefore$ ROC $\Rightarrow |z| > 2$

and $$(4)^n u[-n-1] \overset{z}{\longleftrightarrow} \frac{1}{1-4z^{-1}}$$

$\therefore$ ROC $\Rightarrow |z| < 4$

$\therefore$ for the signal to converge

ROC $\Rightarrow 2 < |z| < 4$

2. (b)

$$y[n] = x[n] * \delta[n-2] = x[n-2]$$

$$\therefore \quad y[2] = x[0] = 1$$

3. (a)

$$\because \quad j\frac{d}{dt}[F(j\omega)] \longleftrightarrow t[f(t)]$$

thus at $t = 0$ the answer will be zero.

4. (a)

$$X(e^{j\omega}) = e^{j\omega} + e^{-j\omega} + 2(e^{2j\omega} - e^{-2j\omega}) + 3(e^{3j\omega} + e^{-3j\omega})$$

$$= 2\cos\omega + 4j\sin(2\omega) + 2\cos 3\omega = 2\cos(\pi) + 4j\sin(2\pi) + 6\cos(3\pi)$$

$$= -2 + 0 - 6 = -8$$

$$|Xe^{j\pi}| = 8$$

5. (a)

$$Re\{x(t)\} = \frac{x(t) + x^*(t)}{2}$$

$\therefore$ The Fourier coefficient of $x^*(t)$ are

$$b_K = \frac{1}{T}\int_T x^*(t)e^{-jK\frac{2\pi}{T}t}\,dt$$

Taking conjugate on both sides

$$b_K^* = \frac{1}{T}\int_T x(t)e^{-j(-K)\frac{2\pi}{T}t}\,dt$$

$$\therefore \quad a_{-K} = b_K^*$$

$\therefore$ Fourier series Coefficient of $Re\{x(t)\} = \dfrac{a_K + a_{-K}^*}{2}$

6. (c)

From the given figure of $x(t)$ and $y(t)$, we get conclude that,

$$x(0) = y(5)$$

at $t = 5$ sec,
$$y(5) = x(-5a + 20) = x(0)$$

$$\therefore \quad -5a + 20 = 0$$

$$\therefore \quad a = 4$$

7. (d)

Given, signal $x(t)$ has energy 'E'

for
$$ax(t) \overset{E}{\longleftrightarrow} a^2E$$

$$\therefore \quad ax(bt + c) \overset{E}{\longleftrightarrow} \frac{a^2E}{b}$$

From the given signal $a = 2$, $b = 5$ and hence

$\therefore$ The energy of signal $2x(5t - 6)$ is

$$E = \frac{(2)^2 \times 10}{5} = 8 \text{ J}$$

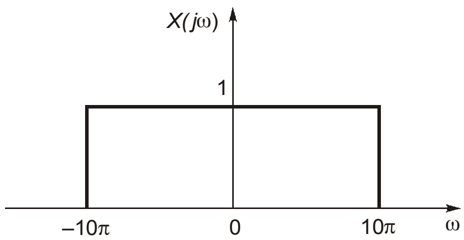
8. (b)

Given,
$$x(t) = \frac{\sin(10\pi t)}{\pi t}$$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 \; ; & |\omega| \le 10\pi \\ 0 \; ; & |\omega| > 10\pi \end{cases}$$

or



$\therefore$ The maximum frequency 'ω_m' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore \quad T_s < \frac{1}{10}$$

9. (c)

Given,
$$H(z) = \frac{z}{z - 0.2} = \frac{1}{1 - 0.2z^{-1}} \qquad \text{ROC} : |z| > 0.2$$

Since the ROC : $|z| > 0.2$, which includes unit circle.

$\therefore$ The impulse response will be stable.

10. (b)

For sequence
$$X_1[n] \overset{Z}{\longleftrightarrow} X_1[z] \;\; ; \text{ROC} = R$$

For sequence
$$X_2[n] = X_1[-n] \overset{Z}{\longleftrightarrow} X_1[1/z] \;\; ; \text{ROC} = 1/R$$

$\therefore$ ROC's are reciprocal of each other.

11. (c)

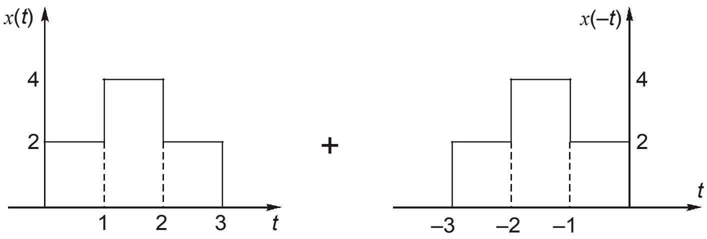
$$x(t) \xleftrightarrow{L.T} X(s) = \frac{s+3}{(s+3)^2+4}$$

$$y(t) \Rightarrow \int x(\tau)d\tau \Rightarrow \frac{s+3}{s[(s+3)^2+4]}$$

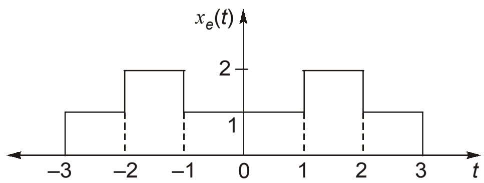
12. (a)

$$x_e(t) = \frac{x(t)+x(-t)}{2}$$

∴ $x(t) =$



∴ $x_e(t) =$



13. (b)

$$Y(e^{j\omega}) = X(e^{j\omega}) * X(e^{j\omega-\pi/2})$$

∴ In time domain

$$Y(e^{j\omega}) = 2\pi x_1[n]\cdot x_2[n]$$

now, If $X(e^{j\omega}) \longleftrightarrow n\left(\frac{3}{4}\right)^{|n|}$

Then $X(e^{j\omega-\pi/2}) \longleftrightarrow n\cdot e^{j\pi/2}\left(\frac{3}{4}\right)^{|n|}$

∴ $y[n] \longleftrightarrow 2\pi n^2 e^{j\pi n/2}\left(\frac{3}{4}\right)^{2|n|}$

14. (b)

$$X(z) = \frac{z}{z-1} \qquad |z| > 1$$

$$Y(z) = \frac{2z}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3}$$

$$H(z) = \frac{2(z-1)}{z-\frac{1}{3}} \qquad |z| > \frac{1}{3}$$

$$X'(z) = \frac{z}{z-\frac{1}{2}} \qquad |z| > \frac{1}{2}$$

$$Y'(z) = H(z)\cdot X'(z)$$

$$= \frac{2z(z-1)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \qquad |z| > \frac{1}{2}$$

Taking inverse z transform

$$y[n] = \left[-6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{3}\right)^n\right]u[n]$$

$k_1 = -6, \quad k_2 = 8$

so, $\quad k_1 + k_2 = 2$

15. (b)

If $x[n]$ is real

$$\text{odd}[x[n]] \xrightarrow{FT} jIm[X(e^{j\omega})]$$

$$\therefore \quad \text{odd}[x[n]] = F^{-1}\left[\frac{1}{2}\left(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}\right)\right]$$

$$= \frac{1}{2}\left[\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right]$$

$$\because \quad \text{odd}\,[x[n]] = \frac{x[n]-x[-n]}{2}$$

Since, $\quad x[n] = 0$ for $n > 0$

$$x[n] = 2\,\text{odd}[x[n]]$$

$$= \delta[n+1] - \delta[n+2] \text{ for } n < 0$$

using parshevel's theorem

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|X\left(e^{j\omega}\right)\right|^2 d\omega = \sum_{n=-\infty}^{\infty}|x[n]|^2$$

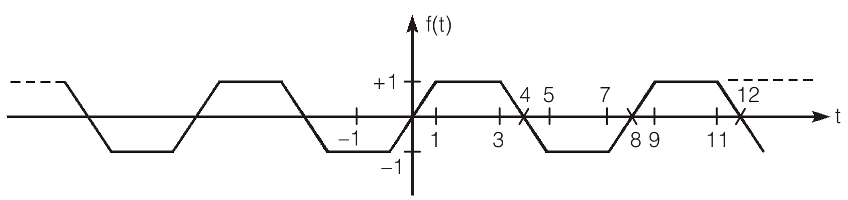
$$(x[0])^2 - 2 = 3$$

$$x[0] = \pm 1$$

$$\because \quad x[0] > 0$$

$$\therefore \quad x[n] = \delta[n] + \delta[n+1] - \delta[n+2]$$

16. (c)



The signal f(t) has hidden, odd and half-wave symmetry.

So,
$$a_0 \neq 0$$
$$a_n = 0; \quad \forall n$$
$$b_n \neq 0; \; n = 1, 3, 5$$

Therefore, non zero Fourier series coefficients are

a_0 and b_n, $n = 1, 3, 5,$............

17. (a)

As,
$$x(t)\cos^2 t = x(t)\left[\frac{1}{2}+\frac{1}{2}\cos 2t\right]$$
$$= \frac{1}{2}x(t)+\frac{1}{2}\cos 2t$$

Now,
$$g(t) = x(t).\cos^2 t * \frac{\sin t}{\pi t}$$
$$= x(t)\left[\frac{1+\cos 2t}{2}\right] * \frac{\sin t}{\pi t}$$

$\therefore$
$$G(j\omega) = \left[\frac{1}{2}X(j\omega)+\frac{1}{4}X(\omega-2)+\frac{1}{4}X(\omega+2)\right]\times \text{rect}\left(\frac{\omega}{2}\right)$$

Thus the given solution will be

$\therefore$
$$G(j\omega) = \frac{1}{2}X(j\omega)$$

or
$$g(t) = \frac{1}{2}x(t)$$

Thus to get the desired result
$$h(t) = \frac{1}{2}\delta(t)$$

18. (a)

$$x[n] = \delta[n]$$
$$X(e^{j\omega}) = 1$$
$$\frac{dX(e^{j\omega})}{d\omega} = 0$$

$\therefore$
$$Y(e^{j\omega}) = e^{-j\omega}X(e^{j\omega})$$
$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega}.e^{j\omega n}d\omega = \frac{\sin\pi(n-1)}{\pi(n-1)}$$

19. (a)

$\because$
$$x^*(t) \xleftrightarrow{F} X^*(-j\omega)$$

and
$$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{F} \frac{X(j\omega)}{j\omega}+\pi X(0)\delta(\omega)$$

20. (d)

$$H(j\omega) = \frac{1+2e^{-j\omega}}{1+\dfrac{1}{2e^{-j\omega}}} = \frac{1+2e^{-j\omega}}{2e^{-j\omega}+1}\cdot 2e^{-j\omega}$$

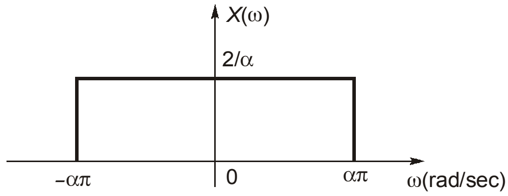
$$|H(j\omega)| = 2$$

21. (c)

We know that,

Energy, $$E = \int_{-\infty}^{\infty} x^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$$

Now, $X(\omega) =$



$$E = \frac{1}{2\pi}\int_{-\alpha\pi}^{\alpha\pi}\left|\frac{2}{\alpha}\right|^2 d\omega$$

$$0.5 = \frac{1}{2\pi}\times\frac{4}{\alpha^2}[2\alpha\pi]$$

$$0.5 = \frac{4}{\alpha}$$

$\therefore$ $$\alpha = 8$$

22. (d)

Given, $$x[n] = \cos\frac{\pi}{4}n + \sin\left(\frac{\pi}{3}n+\frac{1}{2}\right)$$

Let $$x[n] = x_1[n] + x_2[n]$$

Let N_1 be the period of $x_1[n]$.

$$\frac{\pi/4}{2\pi} = \frac{m}{N_1}$$

$\Rightarrow$ $$N_1 = 8$$

Let N_2 be the period of $x_2[n]$.

$$\frac{\pi/3}{2\pi} = \frac{m}{N_2}$$

$\Rightarrow$ $$N_2 = 6$$

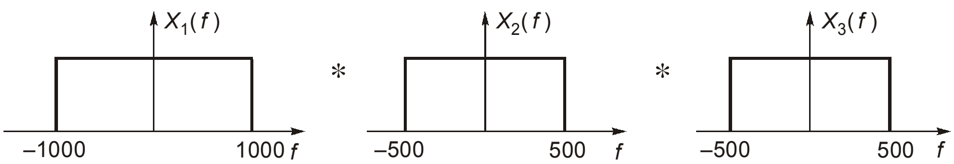
Overall time period, $$N = \text{LCM}(N_1, N_2) = \text{LCM}(6, 8)$$

$\therefore$ $$N = 24$$

23. (d)

We know that,

For $$X(f) = X_1(f) * X_2(f) * X_3(f)$$



Sampling frequency, $$f_s = 2(1000 + 500 + 500)$$

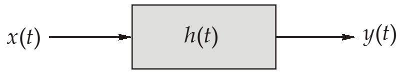
$$f_s = 4000 \text{ samples/sec}$$

24. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3+j\omega}$$

and output, $$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$



We know that, $$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3+j\omega} - \frac{1}{4+j\omega} = \frac{1}{(3+j\omega)(4+j\omega)}$$

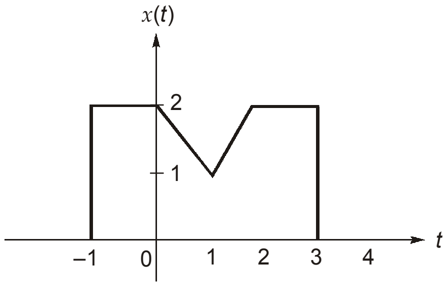
$\therefore$ $$X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4+j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t}u(t)$$

25. (b)

Given,



By the definition of Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot e^{j\omega t}\,d\omega$$

at $t = 0$,

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,d\omega$$

$\therefore$ $$\int_{-\infty}^{\infty} X(j\omega)\,d\omega = 2\pi\,x(0) = 2\pi(2) = 4\pi \approx 12.57$$

26. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!}z^{-n}$$

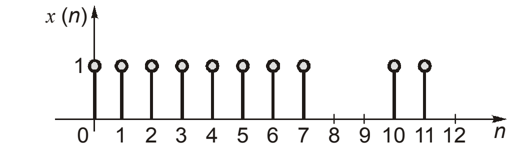
$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2z}\right)^n}{n!}$$

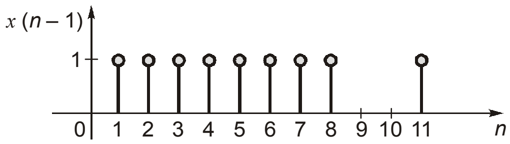
$$X(z) = 1 + \frac{\frac{1}{2z}}{1!} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \ldots$$

$$X(z) = e^{1/2z}$$

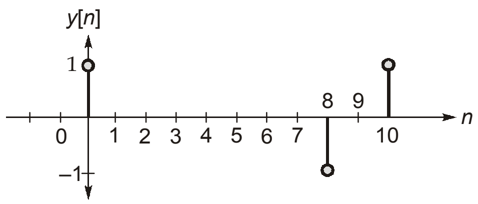
$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

27. (d)





Subtracting the two signal, we get



$\Rightarrow N = 10$

28. (c)

We know that the Laplace transform of

$$\cos(at)\,u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \quad \cos(\pi t)\,u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

$$= \cos\pi t[u(t) - u(t-1)]$$

$$= \cos(\pi t)u(t) - \cos\pi t\, u(t-1)$$

$$= \cos\pi t u(t) - \cos\pi(t - 1 + 1)u(t-1)$$

$$= \cos\pi t\, u(t) - \cos[\pi(t-1) + \pi]\, u(t-1)$$

$$x(t) = \cos(\pi t)\, u(t) + \cos[\pi(t-1)]\, u(t-1)$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{se^{-s}}{s^2 + \pi^2} \qquad [\because x(t - t_0) = X(s)\cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

29. (b)

Given signals, $x(t) = \sin\omega_0 t$

$h(t) = \text{sgn}\, t$

from the multiplication property of Fourier transform,

$$x(t)\,h(t) = \frac{1}{2\pi}[X(\omega) * H(\omega)]$$

Fourier transform of $x(t)$ is $X(\omega)$,

$$X(\omega) = \frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$$

Fourier transform of $h(t)$ is $H(\omega)$,

$$H(\omega) = \frac{2}{j\omega}$$

$$\therefore \quad x(t)\,h(t) \xleftrightarrow{\text{FT}} \frac{1}{2\pi}\left[\frac{\pi}{j}(\delta(\omega-\omega_0)-\delta(\omega-\omega_0)) * \frac{2}{j\omega}\right]$$

$$\xleftrightarrow{\text{FT}} \frac{1}{2\pi}\left[\left[\frac{\pi}{j}\times\frac{2}{j(\omega-\omega_0)}\right]-\left[\frac{\pi}{j}\times\frac{2}{j(\omega+\omega_0)}\right]\right]$$

$$(\because X(\omega)*\delta(\omega-\omega_0) = X(\omega-\omega_0))$$

$$\xleftrightarrow{\text{FT}} \left[\frac{-1}{\omega-\omega_0}+\frac{1}{\omega+\omega_0}\right] = \frac{-\omega-\omega_0+\omega-\omega_0}{\omega^2-\omega_0^2}$$

$$\therefore \quad x(t)\,h(t) \xleftrightarrow{\text{FT}} \frac{-2\omega_0}{\omega^2-\omega_0^2}$$

30. (b)

Given,

$$X(z) = \frac{1}{1-2.5z^{-1}+z^{-2}} = \frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)}$$

$$\frac{1}{(z^{-1}-2)\left(z^{-1}-\frac{1}{2}\right)} = \frac{A}{z^{-1}-2}+\frac{B}{z^{-1}-\frac{1}{2}}$$

$$\therefore \quad A = \frac{1}{2-\frac{1}{2}} = \frac{1}{3/2} = \frac{2}{3}$$

$$B = \frac{1}{\frac{1}{2}-2} = -\frac{2}{3}$$

$$\therefore \quad X(z) = \frac{\frac{2}{3}}{z^{-1}-2}+\frac{-\frac{2}{3}}{z^{-1}-\frac{1}{2}}$$

$$= \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}}+\frac{\frac{4}{3}}{1-2z^{-1}}$$

Given $X(z)$ is a causal system, the ROC is right of the right most pole.

$\therefore |z| > 2$

hence,

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n]+\frac{4}{3}(2)^n u[n]$$

$$\therefore \quad x(0) = -\frac{1}{3}+\frac{4}{3} = \frac{3}{3} = 1$$