



MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Bhopal | Hyderabad | Jaipur | Lucknow | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | E-mail: info@madeeasy.in | Ph: 011-45124612

THEORY OF MACHINES

MECHANICAL ENGINEERING

Date of Test : 21/03/2022

ANSWER KEY >

- | | | | | |
|--------|---------|---------|---------|---------|
| 1. (c) | 7. (a) | 13. (b) | 19. (b) | 25. (a) |
| 2. (b) | 8. (d) | 14. (c) | 20. (c) | 26. (b) |
| 3. (c) | 9. (c) | 15. (b) | 21. (b) | 27. (c) |
| 4. (d) | 10. (d) | 16. (b) | 22. (b) | 28. (d) |
| 5. (c) | 11. (b) | 17. (a) | 23. (c) | 29. (a) |
| 6. (b) | 12. (c) | 18. (d) | 24. (a) | 30. (d) |

DETAILED EXPLANATIONS

1. (c)
Because shortest link is coupler.
2. (b)
Path of contact in involute profile is a straight line, which is common normal to the involute profiles of mating gears.
3. (c)
A governor is said to be sensitive if the displacement of the sleeve is high for a given fractional change of speed.
4. (d)
5. (c)
We know that minimum number of teeth required on pinion,

$$\begin{aligned}
 t &= \frac{2A\omega}{G\sqrt{1 + \frac{1}{a}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1} \\
 &= \frac{2 \times 1}{3\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20^\circ} - 1} \\
 &= \frac{2}{0.133} = 15.04 \approx 15
 \end{aligned}$$

6. (b)
7. (a)
Given, $N_1 = 1000$
Coefficient of steadiness = $\frac{N_1 + N_2}{2(N_1 - N_2)}$
 $N_1 \rightarrow$ maximum speed
 $N_2 \rightarrow$ minimum speed
 $50 = \frac{1000 + N_2}{2(1000 - N_2)}$
 $\Rightarrow N_2 = 980.19$
maximum fluctuation of speed = $N_1 - N_2 = 19.80$ rpm

8. (d)
Let energy at A be E unit

$$\begin{aligned}
 E_A &= E \\
 E_B &= E + 50 \\
 E_C &= E + 50 - 20 = E + 30 \\
 E_D &= E + 30 + 10 = E + 40 \\
 E_E &= E + 40 - 30 = E + 10 \\
 E_{\max.} &= E + 50 \\
 E_{\min.} &= E + 10
 \end{aligned}$$

$$\begin{aligned}\Delta E(\text{max. fluctuation of energy}) &= E_{\text{max.}} - E_{\text{min.}} \\ &= (E + 50) - (E) \\ &= 50 \text{ J}\end{aligned}$$

9. (c)

$$\epsilon = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

$$\therefore \frac{1}{40} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1}$$

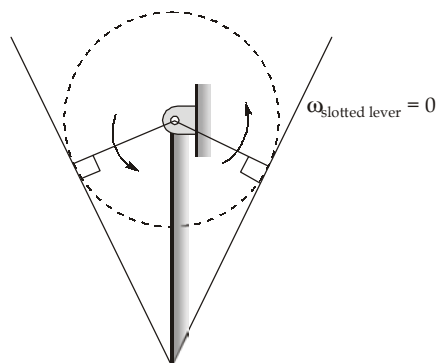
$$\Rightarrow \left(\frac{\omega}{\omega_n}\right)^2 = 41$$

$$\therefore \frac{\omega}{\omega_n} = 6.403$$

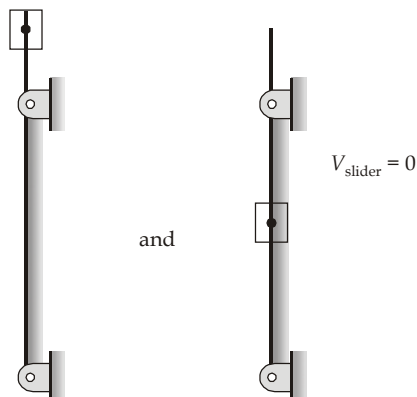
10. (d)

Coriolis component of acceleration will only be zero if either angular velocity of slotted lever is zero or the velocity of slider is zero. The possible 4 conditions are

- Two at the extremes of slotted lever.



- Two when the driving crank and slotted lever are vertical because at that position, velocity of slider will be zero.



11. (b)

Absolute acceleration of B will be vector addition of centripetal acceleration, tangential acceleration due to rotation of link OA and Coriolis acceleration due to sliding motion of block B .

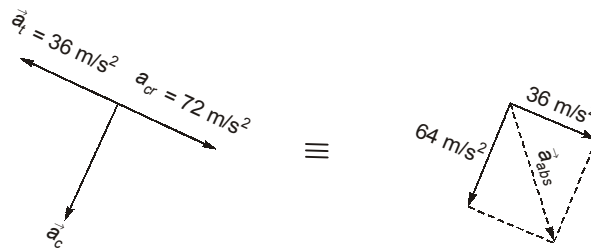
$$\begin{aligned} \vec{a}_c &= \omega^2 r \text{ (towards centre)} \\ &= 4^2 \times 4 = 64 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \vec{a}_t &= r\alpha \text{ (tangential in direction of } \alpha) \\ &= 4 \times 9 = 36 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \text{Coriolis acceleration, } \vec{a}_{cr} &= 2 V_{\text{sliding}} \omega \\ &= 2 \times 9 \times 4 = 72 \text{ m/s}^2 \end{aligned}$$

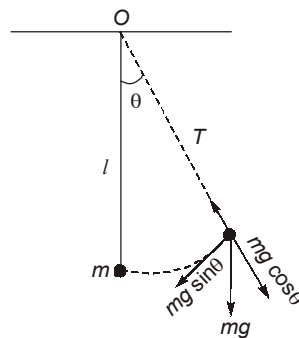
Direction of Coriolis acceleration \rightarrow Rotating direction of sliding velocity which is inwards through 90° in the direction of the angular velocity which is counter clockwise.

Therefore direction of Coriolis acceleration is opposite and collinear to the tangential acceleration.



$$|\vec{a}_{abs}| = \sqrt{64^2 + 36^2} = 73.43 \text{ m/s}^2$$

12. (c)



$$T = -mg \sin \theta$$

$$T = -mgl\theta$$

$$T = I\alpha = -mgl\theta$$

$$I = ml^2$$

$$\alpha = \frac{-g\theta}{l} \tag{... (i)}$$

and we know

$$\alpha = \ddot{\theta} = -\omega_n^2 \theta \tag{... (ii)}$$

Comparing (i) and (ii)

$$\omega_n = \sqrt{\frac{g}{l}}$$

So,

$$\omega_n \propto \sqrt{g}$$

$$g_{\text{Moon}} = \frac{g_{\text{Earth}}}{6}$$

[$\therefore \sin \theta \cong \theta$ as θ is very small]

$$(\omega_n)_{\text{Moon}} = \sqrt{\frac{g_{\text{Moon}}}{l}}$$

$$(\omega_n)_{\text{Moon}} = \frac{1}{\sqrt{6}} \sqrt{\frac{g_{\text{Earth}}}{l}} = \frac{1}{\sqrt{6}} (\omega_n)_{\text{Earth}}$$

$$(\omega_n)_{\text{Moon}} = 0.4082[\omega_n]_{\text{Earth}}$$

13. (b)

In a locomotive, the ratio of the connecting rod length to the crank radius is kept very large in order to minimize the effect of secondary forces.

14. (c)

$$AB = \sqrt{60^2 + 240^2}$$

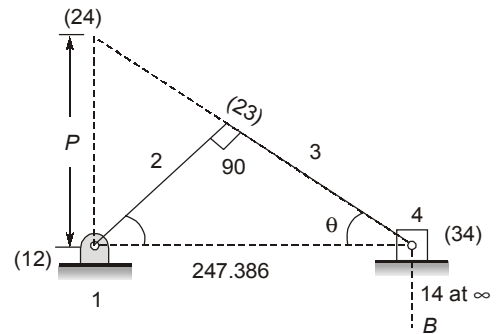
$$AB = 247.386 \text{ mm}$$

$$AB = .247 \text{ m}$$

$$\tan \theta = \frac{60}{240} = \frac{P}{247.386}$$

$$P = 61.8465 \text{ mm}$$

$$P = 0.06184 \text{ m}$$



By Kennedy's theorem,

$$\omega_2(I_{24} I_{12}) = V_4 = \omega_2(I_{24} I_{14})$$

$$\Rightarrow V_4 = 10(0.06184)$$

$$V_4 = 0.618 \text{ m/s}$$

15. (b)

$$X_{max} = \frac{Fo}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$0.04 = \frac{10}{\sqrt{(6250 - 10 \times 25^2)^2 + (C \times 25)^2}}$$

$$\Rightarrow 0.04 = \frac{10}{C \times 25}$$

$$C = 10 \text{ Ns/m}$$

16. (b)

Arm a rotates at 210 rpm clockwise,
 $y = 210$

Gear D is fixed, thus $y + \frac{7x}{3} = 0$

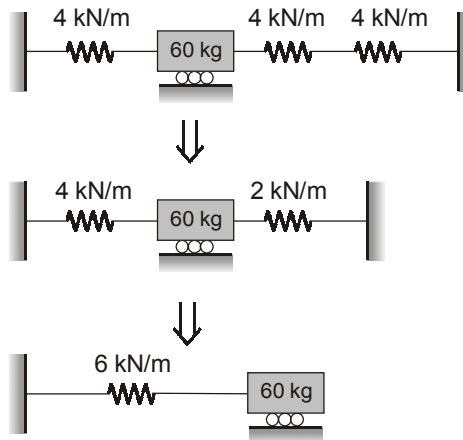
or $210 + \frac{7x}{3}$ or $x = -90$

Speed of A = $y + x$
 $210 - 90 = 120 \text{ rpm (clockwise)}$

Speed of E = $y - \frac{14x}{9} = y - \frac{2x}{7}$
 $= 350 \text{ rpm (clockwise)}$

arm	A	B/C	D	E
O	1	$\frac{-40}{30}$	$\frac{-40x}{30} \times \left(\frac{-70}{40}\right)$	$\frac{-7}{3} \times \frac{40}{60}$
O	x	$\frac{-40x}{30}$	$\frac{7x}{3}$	$\frac{-14x}{9}$
y	$y + x$	$y - \frac{40x}{30}$	$y + \frac{7x}{3}$	$y - \frac{14x}{9}$

17. (a)



$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{6 \times 10^3}{60}}$$

$$\sqrt{100} = 10 \text{ rad/s}$$

18. (d)

Average cutting velocity = $L.N(1 + M)$
 $M \rightarrow$ Quick return ratio
 $L \rightarrow$ Stroke length
 $= 30 \times 250 (1 + 0.6)$
 $= 12 \text{ m/min}$

19. (b)

$$F_{\text{unbalanced force}} = mr\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n} \right]$$

$$= \underbrace{mr\omega^2 \cos\theta}_{\text{Primary unbalanced force}} + \underbrace{mr\omega^2 \frac{\cos 2\theta}{n}}_{\text{Secondary unbalanced force}}$$

20. (c)

Sheared area per hole = πdt
 $= \pi \times 40 \times 15 = 1885 \text{ mm}^2$

Energy required to punch a hole

$$E_1 = 6 \times 1885 = 11310 \text{ Nm}$$

Energy required for punching work/s.

$$= \frac{\text{Energy required}}{\text{hole}} \times \frac{\text{No. of holes}}{S}$$

$$= 11310 \times \frac{30}{60} = 5655 \text{ Nm/s}$$

\therefore Punching takes $\frac{1}{10}$ of a second,

Energy supplied by the motor in $\frac{1}{10}$ s

$$\Rightarrow E_2 = 5655 \times \frac{1}{10} = 565.5 \text{ Nm}$$

Maximum fluctuation of energy of flywheel

$$\Delta E = E_1 - E_2 = 11310 - 565.5 = 10744.5 \text{ Nm}$$

Mean speed,
$$N = \frac{N_1 + N_2}{2} = \frac{160 + 140}{2} = 150 \text{ rpm.}$$

Now, ΔE

$$\Rightarrow 10744.5 = \frac{\pi^2}{900} \times m I^2 N (N_1 - N_2)$$

$$\Rightarrow 10744.5 = \frac{\pi^2}{900} \times m \times 1^2 \times 150(160 - 140)$$

$$\Rightarrow m = 327 \text{ kg}$$

21. (b)

$$r = 51.5 \text{ mm}$$

$$R = 64.2 \text{ mm}$$

$$r_a = 57.5 \text{ mm}$$

$$R_a = 71.2 \text{ mm}$$

$$CR = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

$$= \frac{\left\{ \sqrt{R_a^2 - R^2 \cos^2 \phi} - R \sin \phi \right\} + \left\{ \sqrt{r_a^2 + r^2 \cos^2 \phi} - r \sin \phi \right\}}{\cos \phi}$$

$$= \frac{\pi \left(2 \times \frac{64.2}{20} \right)}{\cos 20^\circ}$$

$$= \frac{\left\{ \sqrt{71.2^2 - 64.2^2 \cos^2 20^\circ} - 64.2 \sin 20^\circ \right\} + \left\{ \sqrt{57.5^2 - 51.5^2 \cos^2 20^\circ} - 51.5 \sin 20^\circ \right\}}{\cos 20^\circ}$$

$$= \frac{15.8568 + 13.4384}{18.95} = 1.545$$

22. (b)

23. (c)

$$T_{\text{mean}} = \frac{1}{\pi} \int_0^\pi T d\theta$$

$$= \frac{1}{\pi} \int_0^\pi (1000 + 400 \sin 2\theta - 500 \cos 2\theta) \times d\theta$$

$$T_{\text{mean}} = 1000 \text{ Nm}$$

At any instant,

$$\Delta T = T - T_{\text{mean}}$$

$$= (1000 + 400 \sin 2\theta - 500 \cos 2\theta) - 1000$$

$$= 400 \sin 2\theta - 500 \cos 2\theta$$

∴ ΔT is zero, when

$$400 \sin 2\theta - 500 \cos 2\theta = 0$$

$$\tan 2\theta = \frac{5}{4}$$

$$2\theta = 51.34^\circ \quad \text{or} \quad 231.34^\circ$$

$$e_{\max} = \int_{25.67}^{115.67} (400 \sin 2\theta - 500 \cos 2\theta) \times d\theta$$

$$e_{\max} = 640.312 \text{ Nm}$$

$$e_{\max} = I k_s \left[\frac{\omega_{\max} + \omega_{\min}}{2} \right]^2$$

$$640.312 = 500 \times .35^2 \times k \times 10\pi \times 10\pi$$

$$k = 0.01059$$

$$\therefore k\% = 1.059\%$$

24. (a)

Energy supplied by the motor is 1s = 5000 Nm

Energy supplied by the motor is 1.5s = 7500 Nm

Energy required/hole = 10000

Energy supplied by flywheel

$$e = 1000 - 7500 = 2500$$

also,
$$e = \frac{1}{2}(\omega_1^2 - \omega_2^2)$$

$$2500 = \frac{1}{2} \times 200 \times 0.5^2 \times \left(\left(\frac{2\pi \times 350}{60} \right)^2 - \omega^2 \right)$$

$$\Rightarrow \omega_2 = 35.25 \text{ rad/s}$$

25. (a)

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} = 2.5, \quad \frac{N_3}{N_4} = \frac{T_4}{T_3} = 4$$

$$\therefore r_1 + r_2 = r_3 + r_4 = 160$$

$$\Rightarrow m_1 T_1 + m_2 T_2 = 320$$

$$\Rightarrow T_1 + T_2 = 100$$

$$T_1 = 28.57, \quad T_2 = 28$$

$$T_2 = 72$$

$$\Rightarrow m_3 T_3 + m_4 T_4 = 320$$

$$T_3 + T_4 = 160$$

$$T_3 = 32$$

$$T_4 = 128$$

$$\text{Exact velocity ratio} = \frac{T_2 T_4}{T_1 T_3} = \frac{72}{28} \times \frac{128}{32} = 10.29$$

26. (b)

$$\frac{x_4}{x_9} = e^{-5 \times \xi \times T_d \times \omega_n} = 0.02$$

Take \ln of both sides

$$5 \times \xi \times \frac{2\pi}{\sqrt{1-\xi^2}} = \ln\left(\frac{1}{0.02}\right)$$

$$\xi = 0.12357$$

$$MF = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$

At resonance, $\frac{\omega}{\omega_n} = 1$

$$MF = \frac{1}{2\xi} = \frac{1}{2 \times 0.12357}$$

$$MF = 4.0463$$

27. (c)

$$KP = \sqrt{R_A^2 - R^2 \cos^2 \phi} - R \sin \phi = 12.65 \text{ mm}$$

$$PL = \sqrt{r_A^2 - r^2 \cos^2 \phi} - r \sin \phi = 11.49 \text{ mm}$$

$$\Rightarrow \text{Arc of contact} = \frac{KP + PL}{\cos \phi} = 25.69 \text{ mm}$$

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\pi m} = 1.635$$

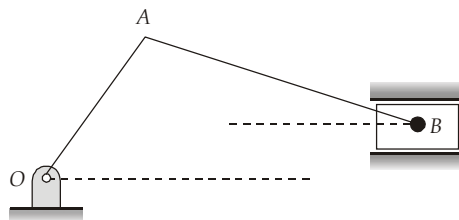
For one pair of teeth in contact, arc of contact

$$= \frac{25.69}{1.635} = 15.712 \text{ mm}$$

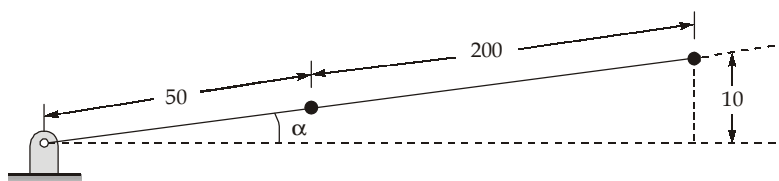
Angle turned by pinion for one pair of teeth in mesh

$$= \frac{15.712}{r} \times \frac{180}{\pi} = \frac{15.712}{50} \times \frac{180}{\pi} = 18.00^\circ$$

28. (d)

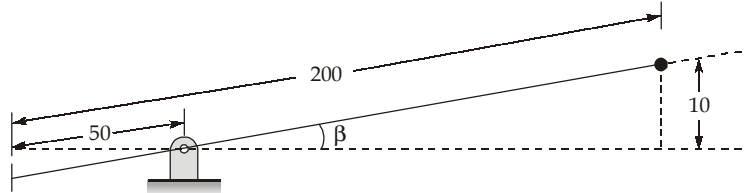


Case I: When the slider is farthest from O,



So,
$$\alpha = \sin^{-1}\left(\frac{10}{50 + 200}\right) = 2.2924^\circ$$

Case II: When the slider is closest to O,



So,
$$\beta = \sin^{-1}\left(\frac{10}{200 - 50}\right) = 3.8225^\circ$$

Angle covered by the crank in backward stroke, $\theta_b = 180 - \alpha + \beta$

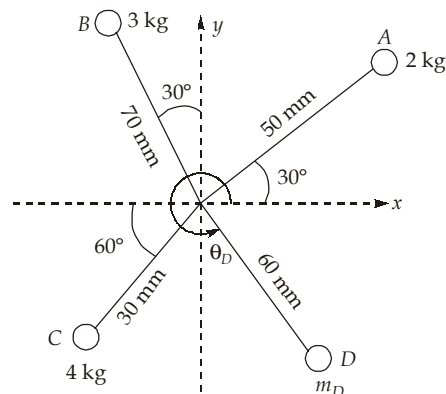
$$\theta_b = 181.5301^\circ$$

So, angle covered by the crank in forward stroke, $\theta_f = 360 - \theta_b$

$$\theta_f = 178.4699^\circ$$

Now,
$$\frac{t_b}{t_f} = \frac{\theta_b}{\theta_f} = \frac{181.5301}{178.4699} = 1.0171$$

29. (a)



$$\Sigma F_{x_{net}} = 0$$

$$\Sigma mrcos\theta = 0$$

$$2 \times 50 \cos 30^\circ + 3 \times 70 \cos 120^\circ + 4 \times 30 \cos 240^\circ + m_D \times 60 \cos \theta_D = 0$$

$$m_D \cos \theta_D = 1.3066$$

...(i)

$$\Sigma F_{y_{net}} = 0$$

$$\Sigma mrsin\theta = 0$$

$$2 \times 50 \sin 30^\circ + 3 \times 70 \sin 120^\circ + 4 \times 30 \sin 240^\circ + m_D \times 60 \sin \theta_D = 0$$

$$m_D \sin \theta_D = -2.1324$$

...(ii)

Using (i) and (ii)

$$m_D = 2.5 \text{ kg}$$

$$\theta_D = 301.49^\circ$$

$$\text{Angle from A} = 301.49 - 30 = 271.49^\circ$$

30. (d)

$$\frac{X_0}{X_N} = e^{\xi \omega_n N T_d}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{40 \times 10^3}{200}} = 14.14 \text{ rad/s}$$

$$50 = e^{0.22 \times 14.14 \times (N T_d)}$$

 \Rightarrow

$$N T_d = 1.26 \text{ s}$$

$$\text{Total time} = 1.26 \text{ s}$$

