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Web: [www.madeeasy.in](http://www.madeeasy.in) | E-mail: [info@madeeasy.in](mailto:info@madeeasy.in) | Ph: 011-45124612**CLASS TEST****CIVIL ENGINEERING****Date of Test : 07/03/2022****ANSWER KEY ➤ Strength of Materials**

- |        |         |         |         |         |
|--------|---------|---------|---------|---------|
| 1. (b) | 7. (c)  | 13. (b) | 19. (a) | 25. (c) |
| 2. (b) | 8. (d)  | 14. (b) | 20. (b) | 26. (b) |
| 3. (a) | 9. (c)  | 15. (b) | 21. (a) | 27. (d) |
| 4. (c) | 10. (a) | 16. (d) | 22. (d) | 28. (b) |
| 5. (d) | 11. (d) | 17. (c) | 23. (d) | 29. (c) |
| 6. (c) | 12. (b) | 18. (d) | 24. (a) | 30. (b) |

## DETAILED EXPLANATIONS

**1. (b)**

Let, the stress developed on each side is  $\sigma$ .

$$\text{Strain along one side due to } \sigma = \frac{\sigma}{E}(1 - 2\mu)$$

Strain along one side due to temperature rise =  $\alpha T$

As cube is restrained from all sides, therefore, both strains should cancel out each other i.e. algebraic sum of both strains should be zero.

$$\Rightarrow \frac{\sigma}{E}(1 - 2\mu) = \alpha T$$

$$\Rightarrow \sigma = \frac{E\alpha T}{1 - 2\mu}$$

**2. (b)**

$$\because \text{Stiffness, } k \propto \frac{1}{\text{Number of coils (n)}}$$

$$\Rightarrow k_1 n_1 = k_2 n_2$$

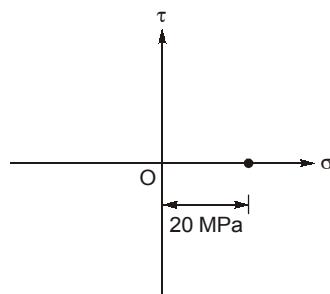
$$\Rightarrow k_1 \times 25 = k_2 \times 20$$

$$\therefore k_2 = 1.25 k_1$$

**3. (a)**

This is the case of hydrostatic loading and in this case Mohr's circle results in a point.

$\therefore$  Diameter of resulting Mohr's circle = 0 MPa



**4. (c)**

A couple anywhere in the beam will cause equal and opposite support reactions in the beam. So the SFD will be rectangular or uniform throughout the beam.

**6. (c)**

For beam of uniform strength, maximum bending stress remains constant throughout.

**7. (c)**

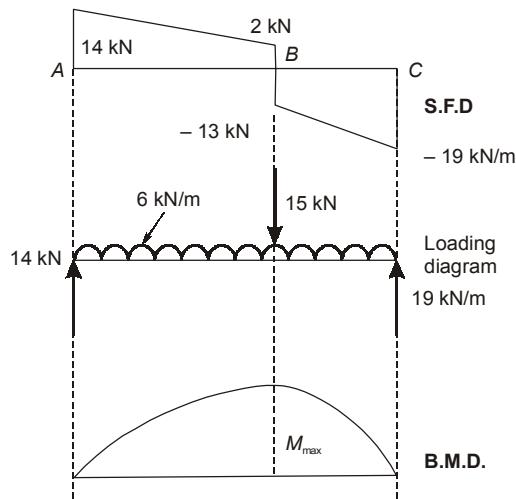
$$\sigma_x = 60 \text{ N/mm}^2$$

$$\sigma_y = -20 \text{ N/mm}^2$$

$$\tau = 40 \text{ N/mm}^2$$

$$\begin{aligned} \text{Normal stress, } \sigma_n &= \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + \tau \sin 2\theta \\ &= 60 \cos^2 60^\circ - 20 \sin^2 60^\circ + 40 \sin 120^\circ \\ &= 34.64 \text{ N/mm}^2 \end{aligned}$$

8. (d)



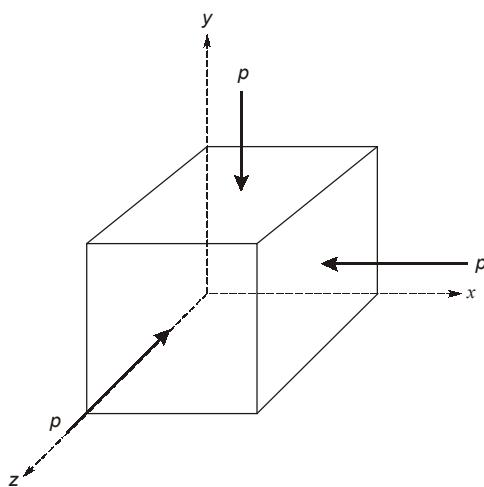
$$\begin{aligned}
 M_B - M_A &= (BM)_B \\
 &= \text{Area of shear force diagram from } A \text{ to } B \text{ since } M_A = 0, \\
 &\quad \text{due to simply supported beam.} \\
 &= \frac{1}{2}(14 + 2) \times 2 = 16 \text{ kN-m}
 \end{aligned}$$

10. (a)

Since slope at  $B = 0$ , therefore beam  $AB$  will act as fixed beam.

$$\therefore \frac{wL^2}{12} = \frac{wa^2}{2} \Rightarrow a = \frac{L}{\sqrt{6}}$$

11. (d)



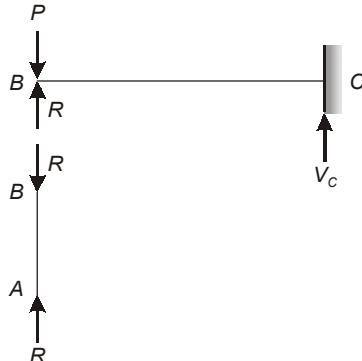
$\therefore$  Pressure is a compressive stress

$$\begin{aligned}
 \therefore \varepsilon_x &= -\frac{200}{200 \times 10^3} - \frac{1}{4}\left(\frac{-200}{200 \times 10^3}\right) - \frac{1}{4}\left(\frac{-200}{200 \times 10^3}\right) \\
 &= -5 \times 10^{-4} \text{ mm/mm}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Elongation, } \Delta_x &= \varepsilon_x L_x = -5 \times 10^{-4} \times 50 \\
 &= -0.025 \text{ mm} = -2.5 \times 10^{-2} \text{ mm}
 \end{aligned}$$

## 12. (b)

Let the reaction at support A be  $R$ .



Deflection at A in beam  $BC$  = Compression in column  $AB$

$$\begin{aligned}\frac{(P-R)L^3}{3EI} &= \frac{RL}{AE} \\ \frac{(P-R)L^2}{3I} &= \frac{R}{A} \\ \frac{PL^2}{3I} &= \frac{R}{A} + \frac{RL^2}{3I} \\ \frac{PL^2}{3I} &= R \left[ \frac{3I + AL^2}{3IA} \right] \\ R &= \frac{PAL^2}{3I + AL^2} = \frac{P}{1 + \left( \frac{3I}{AL^2} \right)}\end{aligned}$$

## 13. (b)

Total load on beam,  $w = 10 + 20 = 30 \text{ kN/m}$

$$\text{Maximum shear force, } F = \frac{wL}{2} = \frac{30 \times 8}{2} = 120 \text{ kN}$$

$$\text{Average shear stress, } \tau_{\text{avg}} = \frac{F}{A} = \frac{120 \times 10^3}{\frac{1}{2} \times 200 \times 300} = 4 \text{ N/mm}^2$$

$$\text{Maximum shear stress, } \tau_{\text{max}} = \frac{3}{2} \tau_{\text{avg}} = \frac{3}{2} \times 4 = 6 \text{ N/mm}^2$$

## 14. (b)

$$\begin{aligned}\because \frac{G\theta}{L} &= \frac{T}{J} \\ \Rightarrow \theta &= \frac{TL}{GJ} = \frac{T}{(GJ/L)} = \frac{100}{10,000} = 0.01 \text{ rad}\end{aligned}$$

$$\begin{aligned}\text{Also, torsional strain energy} &= \frac{1}{2} \times T\theta \\ &= \frac{1}{2} \times 100 \times 0.01 = 0.5 \text{ kN-m} \neq 1 \text{ kN-m}\end{aligned}$$

15. (b)

$$M = 3.5 \text{ kNm}; T = 5 \text{ kNm}; d = 80 \text{ mm}$$

$$\sigma_1 = \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}] \times 10^6 = \frac{16}{\pi(80)^3} [3.5 + \sqrt{(3.5)^2 + 5^2}] \times 10^6 \\ = 95.5 \text{ N/mm}^2$$

$$\sigma_2 = \frac{16}{\pi d^3} [M - \sqrt{M^2 + T^2}] \times 10^6 = -25.89 \text{ N/mm}^2$$

$$\text{Maximum strain, } \epsilon = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} = \frac{1}{E} [95.5 + 0.28 \times 25.89] = \frac{102.75}{E}$$

If 'σ' be the stress producing the same maximum strain then,

$$\frac{\sigma}{E} = \frac{102.75}{E} \\ \Rightarrow \sigma = 102.75 \text{ N/mm}^2$$

16. (d)

$$M_1 = \frac{PL}{4} = \frac{4 \times 4}{4} = 4 \text{ kNm}$$

$$M_2 = \frac{wl^2}{8} = \frac{1 \times 4^2}{8} = \frac{1 \times 16}{8} = 2 \text{ kNm}$$

Given, same area, same depth, so, by pure bending formula,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \\ \Rightarrow \sigma \propto M \\ \frac{\sigma_2}{\sigma_1} = \frac{M_2}{M_1} = 0.5$$

18. (d)

Shear force at A = 400 N

Bending moment at A = 240 Nm

Torque at A = 100 Nm

19. (a)

For the triangular portion, let the load per unit metre is w.

$$\therefore \frac{1}{2} \times w \times 2 = 60 \\ \Rightarrow w = 60 \text{ kN/m}$$

Now,

$$\sum M_A = 0$$

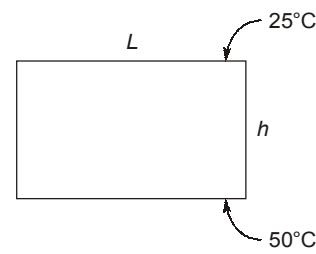
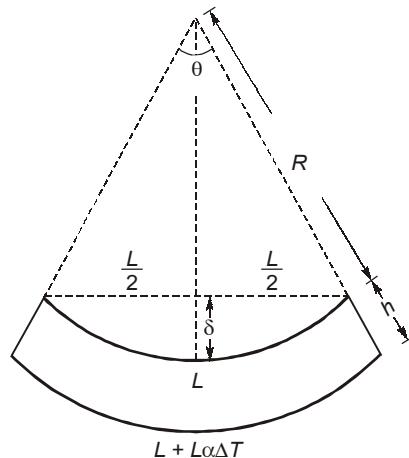
$$\Rightarrow 20 \times 5 - R_D \times 4 + \frac{1}{2} \times 60 \times 2 \left( 1 + \frac{2}{3} \times 2 \right) = 0 \\ \Rightarrow R_D = 60 \text{ kN}$$

## 20. (b)

Let top fibre is assume to be reference.

$\therefore$  The change in temperature between top and bottom fibre,

$$\Delta T = 25^\circ\text{C}$$



From the property of circle

$$(2R - \delta) \theta = \frac{L}{2} \cdot \frac{L}{2}$$

$$2R\delta - \delta^2 = \frac{L^2}{4}$$

$$\therefore R = \frac{L^2}{8\delta}$$

Now,

$$\text{arc}(L) = R\theta$$

$$\text{arc}(L + L\alpha\Delta T) = (R + h)\theta$$

$$\therefore \frac{L + L\alpha\Delta T}{L} = \frac{(R + h)\theta}{R\theta}$$

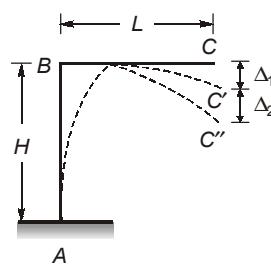
$$1 + \alpha\Delta T = 1 + \frac{h}{R}$$

$$\therefore h = R\alpha\Delta T = \frac{L^2}{8\delta} \alpha \Delta T = 0.1875 \text{ m}$$

$$= 187.5 \text{ mm}$$

## 21. (a)

The beam will deflect as



Vertical deflection at C,  $\Delta = \Delta_1 + \Delta_2$   
 $\Delta_1$  = Deflection due to moment in BC

$$\Delta_1 = \frac{ML^2}{2EI} = \frac{\mu L^2}{2EI}$$

$\Delta_2$  = Deflection due to moment in AB

$$\Delta_2 = \frac{MH}{EI} \times L = \frac{\mu LH}{EI}$$

$$\Delta = \Delta_1 + \Delta_2$$

$$\Rightarrow \Delta = \frac{\mu L^2}{2EI} + \frac{\mu LH}{EI}$$

$$\Rightarrow \Delta = \frac{\mu L}{EI} \left( \frac{L}{2} + H \right)$$

## 22. (d)

$$\text{Deflection at free end, } \delta_1 = \frac{WL^3}{3EI_1}$$

$$\text{Here, } I_1 = \frac{bd^3}{12}$$

With doubling of depth and width,

$$I_2 = \frac{(2b) \times (2d)^3}{12} = 16 I_1$$

$$\delta \propto \frac{1}{I}$$

$$\therefore \delta_2 = \delta_1 \times \frac{I_1}{I_2}$$

$$\Rightarrow \delta_2 = \frac{\delta_1}{16}$$

$$\delta_2 \text{ as a percentage of } \delta_1 = \frac{1}{16} \times 100 = 6.25\%$$

## 23. (d)

Let the total length of spring before cutting be  $l$ .

$$\text{Short piece, } l_1 = \frac{l}{4}$$

$$\text{Long piece, } l_2 = \frac{3l}{4}$$

$$\text{Since, } K = \frac{Gd^4}{64R^3n}$$

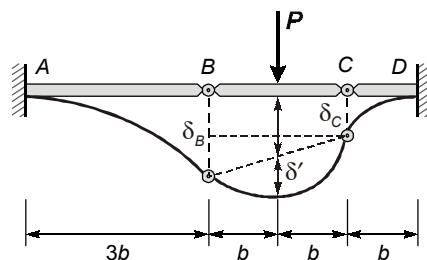
$$\text{or } K \propto \frac{1}{n} \text{ or } K \propto \frac{1}{\text{Length}}$$

$$\therefore \frac{K_1}{K} = \frac{l}{l_1} = \frac{l}{(l/4)}$$

$$\text{or } \frac{K_1}{K} = 4$$

$$\Rightarrow K_1 = 4K$$

24. (a)



Free body diagram of given beam is

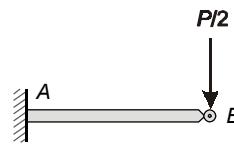


Fig. 1

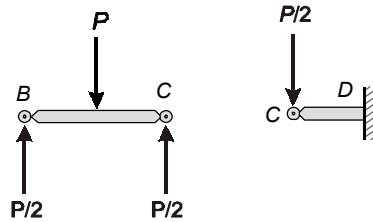


Fig. 2

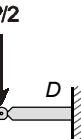


Fig. 3

The deflection of point under load  $P$ ,

$$\delta = \delta_c + \frac{\delta_B - \delta_c}{2} + \delta'$$

$$\delta = \frac{1}{2}(\delta_B + \delta_c) + \delta' \quad \dots\dots(i)$$

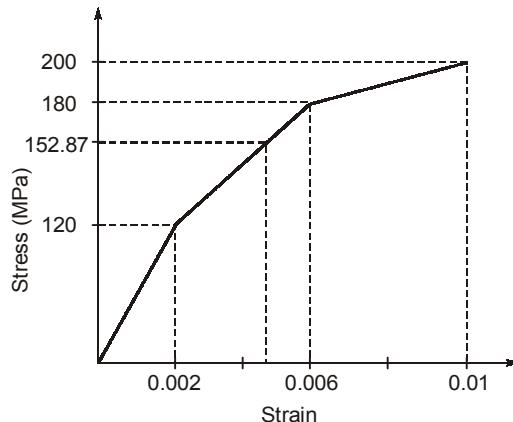
 $\delta_B$  is deflection of point B in Fig. 1 $\delta_c$  is deflection of point C in Fig. 3 $\delta'$  is deflection of point under load  $P$  in Fig. 2

$$\delta_B = \frac{wL^3}{3EI} = \left(\frac{P}{2}\right)(3b)^3 \left(\frac{1}{3EI}\right) = \frac{9Pb^3}{2EI}$$

$$\delta_c = \frac{wL^3}{3EI} = \left(\frac{P}{2}\right)(b)^3 \left(\frac{1}{3EI}\right) = \frac{Pb^3}{6EI}$$

$$\text{From (i), } \delta = \frac{1}{2}(\delta_B + \delta_c) + \frac{P(2b)^3}{48EI} = \frac{5Pb^3}{2EI}$$

25. (c)



$$\text{Stress in the bar} = \left(\frac{P}{A}\right) = \left[\frac{300 \times 10^3}{\frac{\pi}{4} \times (50)^2}\right] = 152.78 \text{ N/mm}^2$$

$$152.78 = 120 + \frac{(180 - 120)}{(0.004)} \times d$$

$$32.78 = 15000 \times d$$

$$d = 2.185 \times 10^{-3}$$

$$\text{Total strain in bar} = 0.002 + 0.002185 = 4.185 \times 10^{-3}$$

$$\text{Total elongation} = (4.185 \times 10^{-3} \times 2 \times 10^3) \text{ mm} = 8.37 \text{ mm}$$

**26. (b)**

For  $\theta = 0^\circ$ :

$$\varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$$

For  $\theta = 40^\circ$

$$\varepsilon_B = \varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\phi_{xy}}{2} \sin 2\theta$$

Substitute

$$\varepsilon_B = 1496 \times 10^{-6} \text{ and } \varepsilon_x = 1100 \times 10^{-6};$$

Then simplify and rearrange:

$$0.41318 \varepsilon_y + 0.49240 \phi_{xy} = 850.49 \times 10^{-6} \quad \dots\dots(i)$$

For  $\theta = 140^\circ$

$$\varepsilon_C = \varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\phi_{xy}}{2} \sin 2\theta$$

Substitute

$$\varepsilon_C = -39.44 \times 10^{-6} \text{ and } \varepsilon_x = 1100 \times 10^{-6};$$

Then simplify and rearrange:

$$0.41318 \varepsilon_y - 0.49240 \phi_{xy} = -684.95 \times 10^{-6} \quad \dots\dots(ii)$$

Solve equation (i) and (ii)

$$\varepsilon_y = 200.3 \times 10^{-6} \text{ and } \phi_{xy} = 1559.2 \times 10^{-6}$$

Hooke's Law

$$\sigma_x = \frac{E}{1-\mu^2} (\varepsilon_x + \mu \varepsilon_y) = 91.6 \text{ MPa}$$

**27. (d)**

$$\text{Stress tensor} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\sigma_{xx} = 10$$

$$\sigma_{yy} = 20$$

$$\sigma_{zz} = 10$$

$$\tau_{xz} = 5 = \tau_{zx}$$

$$\sigma_1/\sigma_2 = \left( \frac{\sigma_{xx} + \sigma_{zz}}{2} \right) \pm \sqrt{\left( \frac{\sigma_{xx} - \sigma_{zz}}{2} \right)^2 + \tau_{xz}^2}$$

$$= \left( \frac{10+10}{2} \right) \pm \sqrt{0+5^2}$$

$$= 10 \pm 5$$

$$\sigma_1 = 15$$

$$\sigma_2 = 5$$

$$\sigma_{yy} = \sigma_3 = 20 \quad [\text{As there is no shear stress in the } X-Y \text{ or } Y-Z \text{ plane}]$$

$$\sigma_y^2 \geq \frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]$$

$$\geq \frac{1}{2} [100 + 225 + 25]$$

$$\sigma_y^2 \geq \frac{350}{2}$$

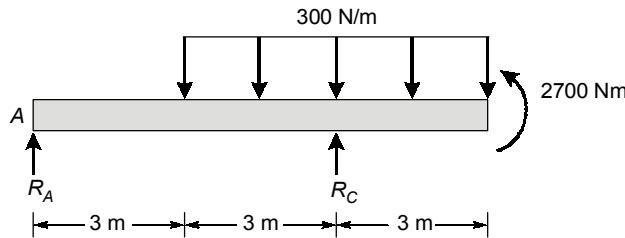
$$\sigma_y \geq 13.23 \text{ MPa}$$

28. (b)

$$\frac{1}{2}w\left(\frac{L}{2}\right)\left(\frac{1}{3}\right)\left(\frac{L}{2}\right) - \frac{wL}{4}\left(\frac{a}{2}\right) = 0$$

$$\Rightarrow a = \frac{L}{3}$$

29. (c)



$$\sum M_A = 0$$

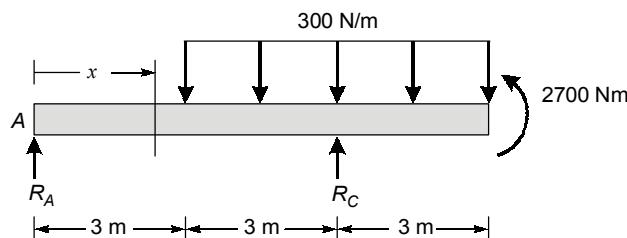
$$\Rightarrow 300(6)(3 + 3) - R_C(6) - 2700 = 0$$

$$\Rightarrow R_C = 1350 \text{ N}$$

$$\sum F_y = 0$$

$$\Rightarrow R_A + R_C = 300(6)$$

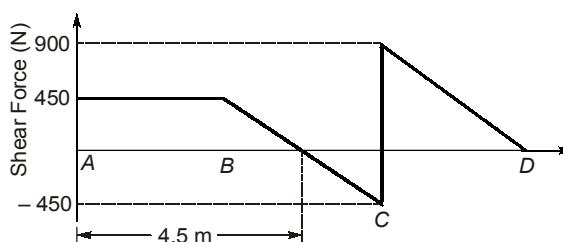
$$\Rightarrow R_A = 450 \text{ N}$$



$$V = 450 \text{ N} \quad 0 \leq x < 3 \text{ m} \quad \dots \dots \text{(iii)}$$

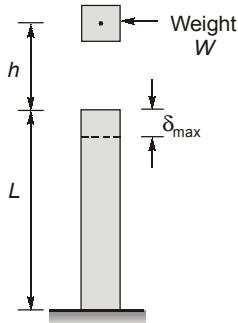
$$V = [450 - 300(x - 3)] \text{ N} \quad 3 \text{ m} \leq x < 6 \text{ m} \quad \dots \dots \text{(iv)}$$

$$V = [450 - 300(x - 3) + 1350] \text{ N} \quad 6 \text{ m} \leq x < 9 \text{ m} \quad \dots \dots \text{(v)}$$



30. (b)

Work done by falling weight is equal to strain energy of the bar



The falling of weight is a case of sudden impact, therefore, the workdone by weight is equal to the product of load applied and displacement.

$$\Rightarrow W(h + \delta_{\max}) = \frac{\sigma_{\max}^2}{E} \times AL$$

Here,  $\sigma_{\max} = 150 \text{ MPa}$   
 $W = 25 \text{ N}$

$\delta_{\max}$  = maximum displacement corresponding to maximum stress  $\sigma_{\max}$  at the time of impact

$$= \frac{\sigma_{\max} \cdot L}{E}$$

$$\Rightarrow W \left( h + \frac{\sigma_{\max} \cdot L}{E} \right) = \frac{\sigma_{\max}^2}{E} \times AL$$

Here,  $\sigma_{\max} = 150 \text{ MPa}$   
 $W = 25 \text{ N}$   
 $L = 1000 \text{ mm}$   
 $E = 200000 \text{ MPa}$

$$A = \frac{\pi}{4} \times 10^2 = 78.54 \text{ mm}^2$$

putting values in equation,

$$\Rightarrow h = \frac{150^2 \times 78.54 \times 1000}{2,00,000 \times 25} - \frac{150 \times 1000}{2,00,000}$$

$$\Rightarrow h = 352 \text{ mm}$$

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