ENGINEERING MATHEMATICS

Date of Test : 02/03/2022

ANSWER KEY ➤

1.	(a)	7.	(a)	13.	(b)	19.	(b)	25.	(c)
2.	(d)	8.	(c)	14.	(a)	20.	(c)	26.	(b)
3.	(a)	9.	(b)	15.	(c)	21.	(d)	27.	(b)
4.	(a)	10.	(a)	16.	(c)	22.	(d)	28.	(b)
5.	(c)	11.	(d)	17.	(a)	23.	(a)	29.	(d)
6.	(a)	12.	(a)	18.	(d)	24.	(c)	30.	(d)

1. **(a)**

$$\ln y = \sin^{-1}x, \qquad \ln z = -\cos^{-1}x$$

$$\ln y - \ln z = \sin^{-1}x + \cos^{-1}x$$

$$\ln\left(\frac{y}{z}\right) = \frac{\pi}{2}$$

$$y = ze^{\pi/2}$$

$$\frac{dy}{dz} = e^{\pi/2}$$

$$\frac{d^2y}{dz^2} = 0$$

2. **(d)**

For function to be differentiable it should be continuous $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$

$$f(0^-) = \lim_{x\to 0^-}\frac{\sin(3p-1)x}{3x}\times\frac{(3p-1)}{(3p-1)} = \lim_{x\to 0^-}\frac{\sin(3p-1)x}{(3p-1)x}\times\frac{(3p-1)}{3} = \frac{(3p-1)}{3}$$

$$f(0^+) = \lim_{x\to 0^+}\frac{\tan(3p+1)x}{2x}\times\frac{(3p+1)}{(3p+1)}$$

$$= \lim_{x\to 0^+}\frac{\tan(3p+1)x}{(3p+1)x}\times\frac{3p+1}{2} = \frac{3p+1}{2}$$

For function to be continuous,

$$\frac{3p-1}{3} = \frac{3p+1}{2}$$

By solving, we get, $p = -\dfrac{5}{3}$

3. **(a)**

$$\frac{dy}{dx} = e^{ax}\times e^{by}$$

$$\frac{dy}{e^{by}} = e^{ax}\times dx$$

$$\frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + c$$

$y(0) = 0$

$$\Rightarrow \qquad c = -\left[\frac{1}{b}+\frac{1}{a}\right] = -\left[\frac{a+b}{ab}\right]$$

4. **(a)**

$$\int f(x)dx = \frac{h}{3}\big[(y_0+y_n)+4(y_1+y_3...)+2(y_2+y_4...)\big]$$

Here $h = \dfrac{1}{2}$. So, $\displaystyle\int_0^2 f(x)dx = \frac{1}{2\times 3}\big[(0+4)+4(0.25+2.25)+2(1)\big]$

$$= \frac{1}{6}\big[4+10+2\big] = 2.667$$

5. **(c)**

Case-I: White ball is transferred from urn A to urn B

Probability of drawing white ball from urn $B = \dfrac{2}{2+4} \times \dfrac{6}{13} = \dfrac{2}{13}$

Case-II: Black ball is transferred from A to B

Probability of drawing white ball from urn $B = \dfrac{4}{2+4} \times \dfrac{5}{13} = \dfrac{10}{39}$

Required probability $= \dfrac{2}{13} + \dfrac{10}{39} = \dfrac{16}{39}$

6. **(a)**

$$\nabla \cdot \vec{F} = 0 \qquad \text{[For solenoidal vector]}$$

$$\frac{\partial(y^2 - z^2 + 3yz - 2x)}{\partial x} + \frac{\partial(3xz + 2xy)}{\partial y} + \frac{\partial(2xy - axz + 2z)}{\partial z} = 0$$

$$-2 + 2x - ax + 2 = 0$$

From here, $\qquad a = 2$

7. **(a)**

$$(2y - 3x)dx + x\,dy = 0$$

$$\frac{dy}{dx} + \frac{2}{x}y = 3$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$$

$$y \cdot x^2 = 3\int x^2 dx = x^3 + c$$

For $x = 0$, $y = 0$

$\Rightarrow \qquad\qquad 0 = 0 + c$

$\Rightarrow \qquad\qquad c = 0$

For $x = 2$, $\qquad y \times 2^2 = 2^3$

$$y = 2$$

8. **(c)**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

$$f(u) = e^u = \frac{x^4 + y^4}{x + y}$$

From here $n = 3$.

$\Rightarrow \qquad x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3\dfrac{e^u}{e^u} = 3$

9. **(b)**

The roots of auxiliary equation are $2, \pm 2i$

$$a = -(2 + 2i - 2i) = -2$$
$$b = 2 \times (2i) + 2 \times (-2i) + 2i \times (-2i) = 4$$
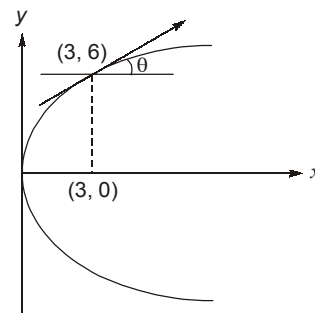$$c = -(2 \times 2i \times (-2i)) = -8$$
$$a + b + c = -2 + 4 - 8 = -6$$

10. **(a)**

Let the numbers appeared on dices are X_1 and X_2.

$$\text{Expectation of the sum} = E[X_1 + X_2] = E[X_1] + E[X_2]$$
$$= 2E[X]$$

$$E[X] = \sum_{i=1}^{6} x_i p(x_i) = \frac{1}{6}(1 + 2 + 3 + \dots + 6)$$

$$= \frac{1}{6} \times \frac{6 \times 7}{2} = \frac{7}{2}$$

So, $\text{Expected sum} = 2E[X] = 2 \times \dfrac{7}{2} = 7$

11. **(d)**

Direction of velocity at point (3, 6) is in the direction of tangent at that point.

$\Rightarrow$ Slope of tangent = Slope of velocity

$$\tan\theta = \left.\frac{dy}{dx}\right|_{(3,6)} = \frac{12}{2y} = \frac{12}{2 \times 6} = 1$$

$\Rightarrow$ $\theta = 45°$

$$v = v_x \hat{i} + v_y \hat{j}$$

$$|v| = \sqrt{v_x^2 + v_y^2} = 10 \text{ m/s}$$

$$v_x = 10\cos 45° = 5\sqrt{2} \text{ m/s}$$

$$v_y = 10\sin 45° = 5\sqrt{2} \text{ m/s}$$

12. **(a)**

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 kx\,dx + \int_2^4 2k\,dx + \int_4^6 (-kx + 6k)\,dx = 1$$

$$\frac{kx^2}{2}\Big|_0^2 + 2kx\Big|_2^4 + \left(\frac{-kx^2}{2} + 6kx\right)\Big|_4^6 = 1$$

$$\frac{k}{2}(2^2 - 0) + 2k(4 - 2) - \frac{k}{2}(6^2 - 4^2) + 6k(6 - 4) = 1$$

$$2k + 4k - 10k + 12k = 1$$

$$8k = 1 \quad \Rightarrow k = \frac{1}{8}$$

$$\text{Mean} = \int_{-\infty}^{\infty} xf(x)dx = \int_0^2 \frac{1}{8}x^2 dx + \int_2^4 \frac{1}{4}x dx + \int_4^6 \left(-\frac{1}{8}x^2 + \frac{3}{4}x\right)dx$$

$$= \frac{1}{8}\frac{x^3}{3}\Big|_0^2 + \frac{1}{4}\frac{x^2}{2}\Big|_2^4 - \frac{1}{8}\frac{x^3}{3}\Big|_4^6 + \frac{3}{4}\frac{x^2}{2}\Big|_4^6$$

$$= \frac{1}{3} + \frac{3}{2} - \frac{19}{3} + \frac{15}{2} = 3$$

13.   (b)

If one of the eigen values is zero, then

$$\begin{vmatrix} x + 1 & \omega & \omega^2 \\ \omega & x + \omega^2 & 1 \\ \omega^2 & 1 & x + \omega \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3 : \begin{vmatrix} x + 1 + \omega + \omega^2 & x + 1 + \omega + \omega^2 & x + 1 + \omega + \omega^2 \\ \omega & x + \omega^2 & 1 \\ \omega^2 & 1 & x + \omega \end{vmatrix} = 0$$

Since ω is cube root of unity,

$$1 + \omega + \omega^2 = 0$$

$$x\begin{vmatrix} 1 & 1 & 1 \\ \omega & x + \omega^2 & 1 \\ \omega^2 & 1 & x + \omega \end{vmatrix} = 0$$

$$x[(x + \omega^2)(x + \omega) + \omega^2 + \omega - \omega^2(x + \omega^2) - 1 - \omega(x + \omega)] = 0$$

$$x[(x^2 + (\omega + \omega^2)x + \omega^3 + \omega^2 + \omega - x\omega^2 - \omega^4 - 1 - \omega x - \omega^2] = 0$$
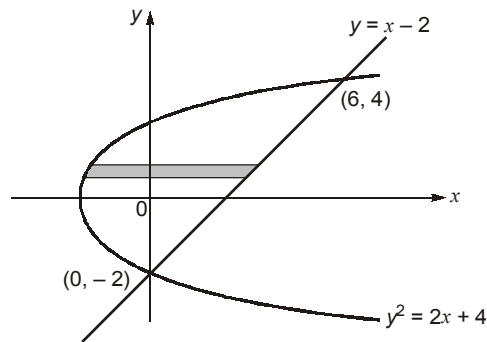
$$x[x^2 + \omega^3 + \omega - \omega^4 - 1] = 0 \qquad\qquad (\because \omega^4 = \omega \text{ and } \omega^3 = 1)$$
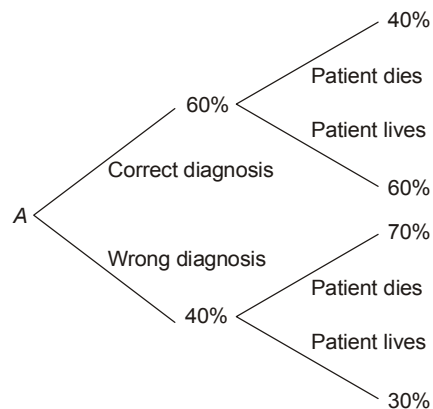
$$x^3 = 0$$

$$x = 0$$

14. (a)

The point of intersection of line and parabolic are $(0, -2)$ and $(6, 4)$.

$$\text{Area} = \int_{-2}^{4} \int_{\left(\frac{y^2-4}{2}\right)}^{y+2} dx\,dy = \int_{-2}^{4} x\Big|_{\frac{y^2-4}{2}}^{y+2} dy$$

$$= \int_{-2}^{4}\left(y + 2 - \frac{y^2}{2} + 2\right) dy = \left(\frac{y^2}{2} + 4y - \frac{y^3}{6}\right)\Bigg|_{-2}^{4} = 18$$

15. (c)

Probability that patient dies who had diseases X is

$$= \frac{40}{100} \times \frac{60}{100} + \frac{70}{100} \times \frac{40}{100} = \frac{52}{100}$$

Probability that he dies of correct diagnosis

$$= \frac{60 \times 40}{100 \times 100} = \frac{24}{100}$$

$$\text{Required probability} = \frac{24/100}{52/100} = \frac{6}{13}$$

16. (c)

$$\begin{vmatrix} 1-\lambda & 2 \\ p & 5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(5-\lambda) - 2p = 0$$

$$\lambda^2 - 6\lambda + 5 - 2p = 0$$

Let the roots are λ_1 and λ_2.

From the characteristic equation,

$$\lambda_1 + \lambda_2 = 6$$

$$\lambda_1 \lambda_2 = 5 - 2p \geq 0 \qquad \text{[For roots to be positive]}$$

$$p \leq \frac{5}{2} \qquad \qquad \text{... (i)}$$

For roots to be real,

$$6^2 - 4(5 - 2p) \geq 0$$

$$36 - 20 + 8p \geq 0$$

$$p \geq -2 \qquad \qquad \text{... (ii)}$$

From equations (i) and (ii),

$$-2 \leq p \leq \frac{5}{2}$$

17.   (a)

$$h = 0.2$$

$$\int_{4.0}^{5.2} f(x)dx = \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2.....y_5)]$$

$$= \frac{0.2}{2}\left[\begin{array}{c}(2.3863 + 2.6484) + 2(2.4351 + 2.4816 \\ + 2.5261 + 2.5686 + 2.6094)\end{array}\right]$$

$$= 3.0276$$

18.   (d)

Probability of showing even number $= \dfrac{2}{1+2} = \dfrac{2}{3}$

Probability of showing odd number $= \dfrac{1}{1+2} = \dfrac{1}{3}$

For sum to be odd = (Even + Even + odd)/(Even + Odd + Even)/(Odd + Even + Even)/(odd + odd + odd)

$$\text{Required probability} = \left(\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}\right) \times 3 + \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}\right) = \frac{12+1}{27} = \frac{13}{27}$$

19.   (b)

For non-trivial solution to exist,

$$\begin{vmatrix} 1 & -2 & 1 \\ k & -1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$-1 - 8 - k + 2 + 2 + 2k = 0$$

$$k = 5$$

For $k = 5$,

$$x - 2y + z = 0$$

$$5x - y + 2z = 0$$

$$\frac{x}{-4+1} = \frac{y}{5-2} = \frac{z}{-1+10}$$

$$\frac{x}{-3} = \frac{y}{3} = \frac{z}{9}$$

$$x : y : z = -1 : 1 : 3$$

20. **(c)**

There are 3 ways of selecting 1 girl and 2 boys.

	Group 1	Group 2	Group 3	Probability
Case I	Girl	Boy	Boy	$\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{18}{64}$
Case II	Boy	Girl	Boy	$\frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{6}{64}$
Case III	Boy	Boy	Girl	$\frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{2}{64}$

Total probability $= \frac{18}{64} + \frac{6}{64} + \frac{2}{64} = \frac{26}{64} = 0.40625 \simeq 0.41$

21. **(d)**

$$\oint_C \vec{F} \cdot \vec{dr} = \oint_C (yz\,dx + zx\,dy + xy\,dz)$$

$$= \oint_C (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\oint_C \vec{F} \cdot \vec{dr} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds$$

where S is the surface bounded by the circle C

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = (x-x)\hat{i} - (y-y)\hat{j} + (z-z)\hat{k} = 0$$

So, $$\oint_C \vec{F} \cdot \vec{dr} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS = 0$$

22. **(d)**

$$f(x) = 2x^3 - 3x^2 - 12x + 5$$

$$f'(x) = 6x^2 - 6x - 12$$

For minima/maxima, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

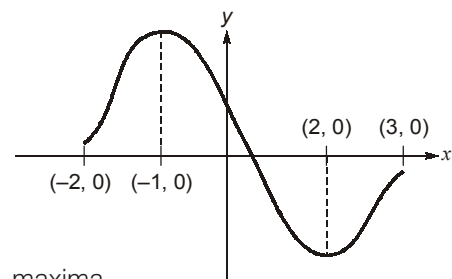$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

$$x = -1, 2$$

$$f''(x) = 12x - 6$$

$$f''(-1) = -12 - 6 = -18 < 0 \implies \text{maxima}$$

$$f''(2) = 24 - 6 = 18 > 0 \implies \text{minima}$$

The function has maxima at $x = -1$ and minima at $x = 2$.

The function is decreasing between -1 and 2.

23. **(a)**

$\sin x \cos y\, dx + \cos x \sin y\, dy = 0$

Divide by $\cos x \cos y$, we get,

$\tan x\, dx + \tan y\, dy = 0$

Integrating the equation,

$\log \sec x + \log \sec y = C_1$

$\log \dfrac{1}{\cos x \cos y} = C_1$

$\cos x \cos y = C$

Since it passes through $\left(0, \dfrac{\pi}{3}\right)$

$\cos(0) \cos\left(\dfrac{\pi}{3}\right) = C$

$\dfrac{1}{2} = C$

$\Rightarrow$ The equation of curve is,

$\cos x \cos y = \dfrac{1}{2}$

24. **(c)**

$\dfrac{\partial M}{\partial y} = 3xy^2 + 1$

$\dfrac{\partial N}{\partial x} = 4xy^2 + 2$

$\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

So, the given equation is not exact.

$\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M} = \dfrac{4xy^2 + 2 - 3xy^2 - 1}{y(xy^2 + 1)} = \dfrac{1}{y}$

$IF = e^{\int \frac{1}{y}dy} = e^{\log y} = y$

The given equation can be made exact by multiplying with integrating factor, i.e. y for this problem.

25. **(c)**

If the two curves intersects, then at point of intersection,

$3x^3 + 2x^2 + 8x - 5 = 2x^3 + 3x + 2$

$x^3 + 2x^2 + 5x - 7 = 0$

$f(x) = x^3 + 2x^2 + 5x - 7$

$f(0) = 0 + 0 + 0 - 7 = -7 < 0$

$f(1) = 1 + 2 + 5 - 7 = 1 > 0$

$\Rightarrow$ One root lies between 0 and 1. Let us assume 1 as initial value.

$f'(x) = 3x^2 + 4x + 5$

$x_1 = 1 - \dfrac{f(x)}{f'(x)}\Big|_{x=1} = 1 - \dfrac{1^3 + 2 \times 1^2 + 5 \times 1 - 7}{3 \times 1^2 + 4 \times 1 + 5} = 0.9167$

$x_2 = x_1 - \dfrac{f(x)}{f'(x)}\Big|_{x=0.9167} = 0.9136$

26. (b)

Number of ways of throwing 6 is five $\Rightarrow$ (1 + 5), (2 + 4), (3 + 3), (4 + 2), (5 + 1)

Number of ways of throwing 7 is six $\Rightarrow$ (1 + 6), (2 + 5), (3 + 4), (4 + 3), (5 + 2), (6 + 1)

Probability of throwing 6, $p_1 = \dfrac{5}{36}$

Probability of failing to throw 6, $p_2 = 1 - \dfrac{5}{36} = \dfrac{31}{36}$

Probability of throwing 7, $q_1 = \dfrac{6}{36}$

Probability of failing to throw 7, $q_2 = 1 - \dfrac{6}{36} = \dfrac{30}{36}$

$$\begin{aligned} \text{Probability of } B \text{ winning} &= p_2 q_1 + p_2 q_2\, p_2 q_1 + p_2 q_2 p_2 q_2 p_2 q_1 + \dots \\ &= p_2 q_1 [1 + p_2 q_2 + (p_2 q_2)^2 + (p_2 q_2)^3 + \dots] \\ &= \frac{p_2 q_1}{(1 - p_2 q_2)} = \frac{\dfrac{31}{36} \times \dfrac{6}{36}}{1 - \dfrac{31}{36} \times \dfrac{30}{36}} = \frac{31 \times 6}{366} = \frac{31}{61} \end{aligned}$$

27. (b)

The characteristic equation of matrix A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

or $\qquad A^2 - 4A - 5I = 0$

Now the given polynomial can be written as,

$$A^5 - 4A^4 - 7A^3 + 11A^2 - 2A + kI = (A^3 - 2A + 3I)(A^2 - 4A - 5I) + (k + 15)I$$

Since, $\qquad\qquad A^2 - 4A - 5I = 0$

For the given polynomial to be zero,

$$k + 15 = 0$$
$$k = -15$$

28. (b)

z varies from 0 to $\dfrac{x^2 + y^2}{4}$; y varies from 0 to $\sqrt{16 - x^2}$; x varies from 0 to 4.

$$\begin{aligned} \text{Volume} &= \iiint dx\,dy\,dz = \int_0^4 \int_0^{\sqrt{16-x^2}} \int_0^{\frac{x^2+y^2}{4}} dz\,dy\,dx \\ &= \frac{1}{4}\int_0^4 \int_0^{\sqrt{16-x^2}} (x^2 + y^2)\,dy\,dx = \frac{1}{4}\int_0^4 \left(x^2 y + \frac{y^3}{3} \right)\Bigg|_0^{\sqrt{16-x^2}} dx \\ &= \frac{1}{4}\int_0^4 \left(x^2 \sqrt{16 - x^2} + \frac{\left(\sqrt{16 - x^2}\right)^3}{3} \right) dx \end{aligned}$$

Let, $\qquad\qquad x = 4\sin\theta \qquad\qquad x \to 0 \text{ to } 4$

$$dx = 4\cos\theta\,d\theta \qquad \theta \to 0 \text{ to } \frac{\pi}{2}$$

$$\text{Volume} = \frac{1}{4}\left[4^4\int_0^{\pi/2}\sin^2\theta\cos^2\theta\,d\theta + \frac{4^4}{3}\int_0^{\pi/2}\cos^4\theta\,d\theta\right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{\left\lfloor\frac{3}{2}\times\left\lfloor\frac{3}{2}\right.\right.}{2\left\lfloor\frac{6}{2}\right.} + \frac{4^4}{3}\times\frac{\left\lfloor\frac{5}{2}\times\left\lfloor\frac{3}{2}\right.\right.}{2\left\lfloor\frac{6}{2}\right.}\right]$$

$$= \frac{1}{4}\left[4^4 \times \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2!}\pi + \frac{4^4}{3}\times\frac{3}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2!}\times\pi\right]$$

$$= \frac{1}{4}\left[16\pi + 16\pi\right] = 8\pi\ \text{unit}^3$$

29. (d)

$$IF = e^{\int f'(x)dx} = e^{f(x)}$$

Solution of differential equation,

$$y \times IF = \int IF\cdot f(x)\cdot f'(x)dx$$

$$y \times e^{f(x)} = \int e^{f(x)}\cdot f(x)\cdot f'(x)dx$$

Let $$f(x) = t$$

$$f'(x)\,dx = dt$$

$$y \times e^t = \int e^t \cdot t\,dt$$

$$y.e^t = t.e^t - e^t + c$$

$$y = t - 1 + ce^{-t}$$

$$\log(y + 1 - t) = -t + c'$$

$$\log[y + 1 - f(x)] + f(x) = c'$$

30. (d)

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t - 5)\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$$

$$\vec{v}\Big|_{t=1} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

Component of velocity in direction $\hat{i} - 3\hat{j} + 2\hat{k}$ will be,

$$\frac{\vec{v}\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{(4\hat{i} - 2\hat{j} + 3\hat{k})\cdot(\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}} = \frac{4 + 6 + 6}{\sqrt{14}} = \frac{16}{\sqrt{14}}$$

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