S.No. : 01 IG_CE_C+D_6320

Engineering Mechanics

MADE EASY

India's Best Institute for IES, GATE & PSUs

Delhi | Noida | Bhopal | Hyderabad | Jaipur | Lucknow | Indore | Pune | Bhubaneswar | Kolkata | Patna

Web: www.madeeasy.in | **E-mail:** info@madeeasy.in | **Ph:** 011-45124612

CLASS TEST 2020-2021

CIVIL ENGINEERING

Date of Test : 06/03/2020

ANSWER KEY ➤ Engineering Mechanics

1.	(d)	7.	(a)	13.	(d)	19.	(c)	25.	(d)
2.	(a)	8.	(b)	14.	(b)	20.	(b)	26.	(d)
3.	(b)	9.	(a)	15.	(c)	21.	(c)	27.	(d)
4.	(b)	10.	(a)	16.	(b)	22.	(a)	28.	(b)
5.	(b)	11.	(a)	17.	(c)	23.	(a)	29.	(a)
6.	(b)	12.	(a)	18.	(b)	24.	(a)	30.	(a)

DETAILED EXPLANATIONS

1. (d)

Given: Mass of elevator = 500 kg

Mass of operator = 100 kg

Upward acceleration = $3\ m/s^2$

Total tension in the cable of the elevator = $(m_1 + m_2)(g + a)$

$= (500 + 100)(10 + 3) = 600 \times 13$

Total tension in the cable of the elevator = 7800 N = 7.8 kN

2. (a)

Given:Velocity of first particle, $u_1 = 10$ m/s

Angle of projection for first particle, $\alpha_1 = 60°$

Angle of projection for second particle, $\alpha_2 = 30°$

Velocity of second particle, u_2 = ?

Given, Time of flight is same.

$$t_1 = t_2$$

$$\frac{2u_1 \sin\alpha_1}{g} = \left(\frac{2u_2 \sin\alpha_2}{g}\right)$$

$$u_2 = \frac{10\times\sin 60°}{(\sin 30°)} = \frac{10\times\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 10\times\sqrt{3}$$

$$u_2 = 17.32\ m/s$$

3. (b)

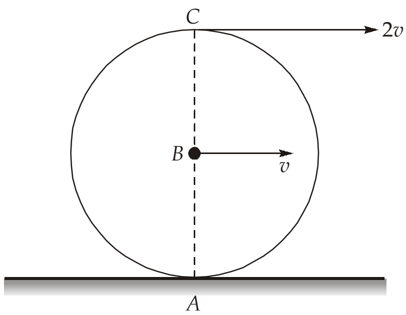
Given: Velocity, v = 54 kmph = $(54)\times\frac{5}{18}$ = 15 m/s

Diameter, d = 1 m

Radius, r = 0.5 m

(i) Velocity of the top of the wheel relative to the person sitting in the carriage:

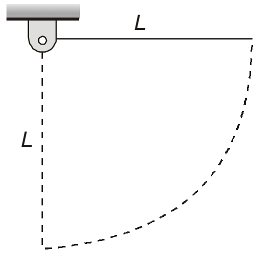
We know that the velocity of the top of the wheel (C) = $2v = 2 \times 15 = 30$ m/s



Velocity of the person sitting in the carriage, v = 15 m/s

Velocity of the top of the wheel relative to the person sitting in the carriage = 30 – 15 = 15 m/s

4. **(b)**



Applying conservation of energy,

$$mgL = \frac{mgL}{2} + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad I\omega^2 = mgL$$

$$\Rightarrow \quad \frac{mL^2}{3}\omega^2 = mgL \quad \text{[The moment of inertia about the end of the rod is } \frac{mL^2}{3}\text{]}$$

$$\therefore \quad \omega = \sqrt{\frac{3g}{L}}$$

5. **(b)**

Using conservation of energy,

$$mgh = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2\times0.04\times9.81\times4.9}{400}}$$

$$\therefore \quad x = 0.098 \text{ m} = 9.8 \text{ cm}$$

6. **(b)**

$$\text{Work done, } dW = F\cdot dx = (10 + 0.5 \ln x)dx$$

Thus,

$$\int_0^W dW = \int_2^4 (10 + 0.5\ln x)dx$$

$$W = 10(4 - 2) + 0.5\int_2^4 \ln x\, dx$$

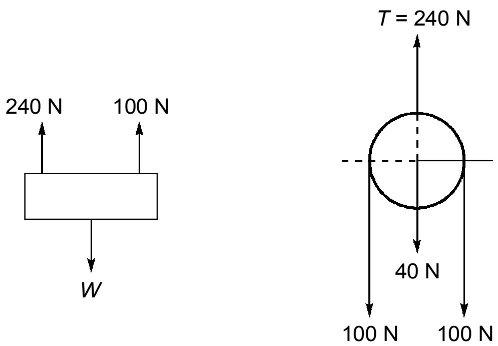
$$W = 20 + 0.5\,(x\ln x - x)_2^4$$

$$W = 20 + 0.5\,(4\ln 4 - 4 - 2\ln 2 + 2)$$

$$W = 21.079 \text{ J}$$

7. **(a)**

The FBD of the weight W is

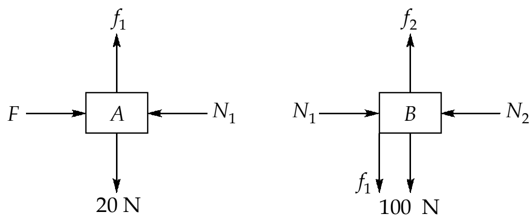


So, $240 + 100 = W$ (240 N includes weight of pulley and tension carried by rope)

$\therefore$ $W = 340$ N

8. (b)

The *FBD* of blocks *A* and *B* are,



So, $f_1 = 20$ N

f_2(friction on *B* due to wall) $= 100 + f_1 = 120$ N

9. (a)

Given: $\vec{F} = 10\hat{i} + 5\hat{j} + \hat{k}$ (N)

$x = \sqrt{106}$ m

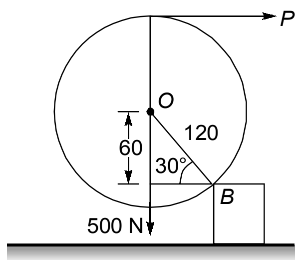
Now, $\vec{A} \times \vec{B} = (3\hat{i} + 4\hat{j}) \times (3\hat{j} + \hat{k}) = 4\hat{i} - 3\hat{j} + 9\hat{k}$

Now, $W = \left[(10\hat{i} + 5\hat{j} + \hat{k}) \times \sqrt{106}\right] \cdot \dfrac{(4\hat{i} - 3\hat{j} + 9\hat{k})}{\sqrt{4^2 + 3^2 + 9^2}}$

or $W = 40 - 15 + 9$

$\therefore$ $W = 34$ Nm

10. (a)



Taking moment about *B*,

$P \times (60 + 120) = 500 \times 120 \cos 30°$

$\therefore$ $P = 288.68$ N

11. (a)

Given: Mass of first ball $= m$ kg

Mass of second ball $= 2m$ kg

Initial velocity of first mass $= u$ m/s

Initial velocity of second mass $= \dfrac{u}{7}$ m/s

Coefficient of restitution, $e = 0.75$

Let, Velocity of the first ball after impact $= v_1$ m/s

Velocity of the second ball after impact $= v_2$ m/s

By momentum conservation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$mu + 2m \times \frac{u}{7} = mv_1 + 2mv_2$$

$$\frac{9u}{7} = (v_1 + 2v_2) \quad \text{...(i)}$$

We also know that, $(v_2 - v_1) = e(u_1 - u_2)$

$$(v_2 - v_1) = 0.75\left(u - \frac{u}{7}\right)$$

$$v_2 - v_1 = \frac{9u}{14} \quad \text{...(ii)}$$

Multiplying equation (ii) by 2,

$$2v_2 - 2v_1 = \frac{9u}{7} \quad \text{...(iii)}$$

$$v_1 + 2v_2 = \frac{9u}{7} \quad \text{from eq.(i)}$$

By equation (iii) -(i), $v_1 = 0$ m/s

12. (a)

Given:Speed, $u = 25$ m/s; diameter, $d = 50$ cm $= 0.5$ m; Radius, $r = 0.25$ m

We know that, $v^2 = u^2 + 2as$

$$0 = (25)^2 + 2 \times a \times (25)$$

$$a = \frac{-625}{50} = -12.5 \text{ m/s}^2$$

[Here minus sign represents retardation]

Angular retardation of the wheel,

$$\alpha = \frac{a}{r} = \frac{-12.5}{0.25} = -50 \text{ rad/s}^2$$

[Minus sign indicates retardation]

$$\alpha = 50 \text{ rad/s}^2 \text{ (Retardation)}$$

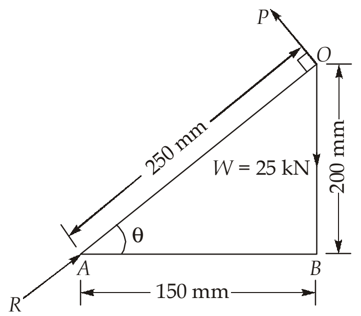
13. (d)

Given, Diameter of wheel = 500 mm, weight of wheel = 25 kN

For the least pull, force P must be normal to the line AO.

$$AB = \sqrt{(OA)^2 - (OB)^2} = \sqrt{(250)^2 - (200)^2}$$

$$AB = 150 \text{ mm}$$



Taking moment about A,

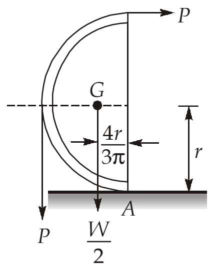
$$P \times 250 = W \times 150$$

$$P = \frac{25 \times 150}{250}$$

$$P = 15 \text{ kN}$$

14. (b)

Taking one halve of cylinder. Centre of gravity of a semicircle is at a distance of $\frac{4r}{3\pi}$ from centre.



Taking moment about A,

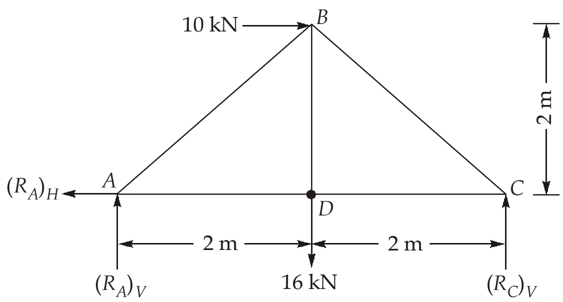
$$P \times 2r = P \times r + \left(\frac{W}{2}\right) \times \left(\frac{4r}{3\pi}\right)$$

$$P \times r = W\left(\frac{2r}{3\pi}\right)$$

$$P = \frac{2W}{3\pi}$$

15. (c)

Let, R_A and R_C are the support reactions at A and C respectively. Support C is a roller support so there will be only vertical reaction at C.



Now, $\Sigma F_x = 0$

$(R_A)_H = 10 \text{ kN} (\leftarrow)$

Now, Taking moment about A,

$(R_C)_V \times 4 = (16 \times 2) + (10 \times 2) = 52$

$\Rightarrow (R_C)_V = 13 \text{ kN}$

Now, $\Sigma F_y = 0$

$\Rightarrow (R_A)_V + (R_C)_V = 16$

$\Rightarrow (R_A)_V = 16 - 13 = 3 \text{ kN}$

$$\text{Reaction at } A, R_A = \sqrt{(R_A)_H^2 + (R_A)_V^2} = \sqrt{10^2 + 3^2} = 10.44 \text{ kN}$$

16. (b)

Assume, Initial angular velocity = ω_0

Angular acceleration = α

Condition I:

Angular velocity after 4 sec = ω

$$\omega = \omega_0 + (\alpha t)$$

$$\omega = \omega_0 + 4\alpha \quad \text{...(i)}$$

We know that, $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$100 = (4\omega_0) + \frac{1}{2}\times\alpha\times 4^2$$

$$100 = 4\omega_0 + 8\alpha \quad \text{...(ii)}$$

Condition II:

$$\theta = \omega \times t \quad \{\because \alpha = 0 \text{ in } 2^{nd} \text{ case}\}$$

$$80 = (\omega_0 + 4\alpha) \times 4$$

$$20 = \omega_0 + 4\alpha \quad \text{...(iii)}$$

Multiply equation (iii) by (2), $40 = 2\omega_0 + 8\alpha$ ...(iv)

By equation (ii) – equation (iv),

$$(100 - 40) = (4\omega_0 + 8\alpha) - (2\omega_0 + 8\alpha)$$

$$60 = 2\omega_0$$

Initial angular velocity, ω_0 = 30 rad/s

17. (c)

Given: Mass, m = 80000 kg,

Resistance = 2% of (80000 × 10)N

$$= \frac{2\times 80000\times 10}{100} = 16000 \text{ N} = 16 \text{ kN}$$

Available force = Tractive force – Resistance

= (26 – 16) = 10 kN

$$\text{Acceleration of train} = \frac{\text{Available force}}{\text{mass}} = \frac{10\times 10^3}{80\times 10^3} = \frac{1}{8} \text{ m/s}^2$$

Final velocity of the train, v = 10 m/s

$$\therefore \quad v = u + at$$

$$10 = 0 + \left(\frac{1}{8}\times t\right)$$

$$t = 80 \text{ s}$$

18. (b)

Given: $a = \frac{5}{v+3}$, where 'v' is velocity and 's' is distance.

We know that,

$$v\frac{dv}{ds} = a$$

$$\frac{vdv}{ds} = \left(\frac{5}{v+3}\right)$$

$$v(v+3)dv = 5ds$$

Integrating on both sides,

$$\left(\frac{v^3}{3}+\frac{3v^2}{2}\right) = 5s + c_1$$

$\therefore$ at, $t = 0$, $s = 0$ and $v = 0$

$$\therefore \quad 0 + 0 = 0 + c_1$$

$$\therefore \quad c_1 = 0$$

Now, $$\frac{v^3}{3}+\frac{3v^2}{2} = 5s$$

at, $v = 30$ m/s

$$\frac{(30)^3}{3}+\frac{3(30)^2}{2} = 5s$$

$$\frac{(30)^3}{3}+\frac{3\times30^2}{2} = 5s$$

$$9000 + 1350 = 5s$$

$$s = \frac{10350}{5}$$

$$s = 2070 \text{ m}$$

19. **(c)**

Given: $m_A = 15$ kg, $m_B = 10$ kg

For mass B, $m_Bg - T = m_Ba$

$$10g - T = 10a \quad \text{...(i)}$$

For mass A, $T = m_Aa$

$$T = 15a \quad \text{...(ii)}$$

Addition equation (i) and (ii)

$$(10g - T) + (T) = (15 + 10)a$$

$$a = \frac{10g}{25} = \frac{10\times10}{25} = 4 \text{ m/s}^2$$

Acceleration, $a = 4 \text{ m/s}^2$

20. **(b)**

Given: Angle of inclination, $\alpha = 30°$; Deceleration, $a = 1$ m/s^2; Weight of block, $W = 5$ kN

Coefficient of friction, $\mu = 0.3$

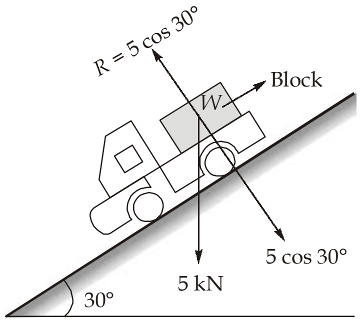
$\because$ As truck is decelerated, the load will tend to slip forward (i.e. downward)

Force due to deceleration, $F_1 = m \cdot a$

$$= \left(\frac{5\times10^3}{10}\right)\times1 = 500 \text{ N}$$

Component of the load along the plane,

$$F_2 = W\sin\theta = (5 \text{ kN}) \sin 30° = 2.5 \text{ kN}$$



∴ Total force that will cause slipping:

$$F_{net} = F_1 + F_2 = 0.5 + 2.5 = 3.0 \text{ kN}$$

$$\text{Force of friction} = \mu W\cos\theta = 0.3 \times 5 \times \cos 30°$$

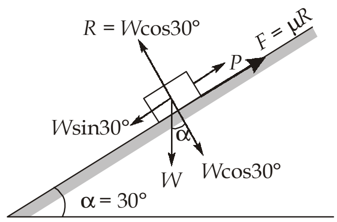
$$= 1.5 \times 0.866 \text{ kN} = 1.30 \text{ kN}$$

$$\text{Factor of safety} = \frac{\text{Force of friction}}{\text{Force causing slipping}} = \frac{1.30}{3.00} = 0.433$$

21. (c)

Given: Weight of body, $W = 1000$ N

Angle of plane of inclination, $\alpha = 30°$, Angle of friction, $\phi = 15°$



For minimum value of P, the body will be at the point of sliding downwards. In this condition, friction force will act in upward direction parallel to the plane.

Let, F and R are friction force and normal reactions respectively.

In equilibrium condition, $W\sin 30° = P + \mu R$

$$P = W\sin 30° - \mu W\cos 30°$$

[∵ $\mu = \tan\phi = \tan 15°$]

$$= W\left[0.5 - 0.268 \times \frac{\sqrt{3}}{2}\right] = W[0.5 - 0.268 \times 0.866]$$

$$= 1000 \times 0.268$$

Minimum force required for equilibrium, $P = 268$ N

Alternate:

$$P_{min} = W\frac{\sin(\alpha - \phi)}{\cos\phi}$$

$$= 1000 \times \frac{\sin(30-15)°}{\cos 15°} = 1000 \times \tan 15°$$

$$P_{min} = 268 \text{ N}$$

22. (a)

Given: P = 250 N; BF_1 = 25 mm; F_1A = 325 mm; CD = 360 mm; DF_2 = 40 mm

Leverage of the upper lever, $AB = \frac{AF_1}{BF_1} = \frac{325}{25} = 13$

Leverage of the lower lever, $CF_2 = \frac{CF_2}{DF_2} = \frac{360+40}{40} = 10$

Total leverage of the compound lever = 13 × 10 = 130

We know that,Total leverage $= \frac{W}{P} = \frac{W}{250}$

$$130 = \frac{W}{250}$$

$$W = 130 \times 250 = 32500 \text{ N} = 32.5 \text{ kN}$$

23. (a)

MOI of triangle about base AB,

$$I_1 = \frac{1}{12}\times(2r)\times(2r)^3 = \left(\frac{16}{12}\right)r^4$$

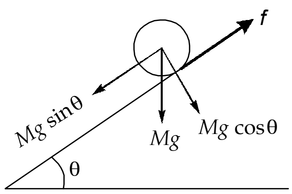
MOI of semi-circle about diameter, $I_2 = \left(\frac{1}{2}\right)\times\left(\frac{\pi}{64}\right)\times(2r)^4 = \left(\frac{\pi}{8}\right)r^4$

MOI of smaller circle about diameter, $I_3 = \left(\frac{\pi}{64}\right)r^4$

MOI of whole section about AB axis, $I = I_1 + I_2 - I_3$

$$= \left(\frac{4}{3}+\frac{\pi}{8}-\frac{\pi}{64}\right)r^4 = \left(\frac{4}{3}+\frac{7\pi}{64}\right)r^4 = \left(\frac{4}{3}+\frac{22}{64}\right)r^4 = 1.677\, r^4$$

24. (a)



Let the friction force be 'f'

So, $\quad Mg\sin\theta - f = Ma$

$$f = Mg\sin\theta - Ma$$

and $\quad fR = I\alpha = I\frac{a}{R}$

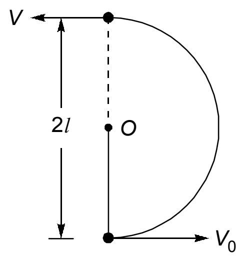
$$f = \frac{Ia}{R^2}$$

$$\frac{Ia}{R^2} = Mg\sin\theta - Ma$$

$$\therefore \quad a = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)}$$

25. (d)

Let the bob is given horizontal speed V_0 at the bottom.



By energy conservation,

So, $$\frac{1}{2}mV_0^2 = \frac{1}{2}mV^2 + mg\times(2l)$$

or, $$mV^2 = mV_0^2 - 4\,mgl \quad ...(1)$$

Also, at the top most point, force balance is,

$$mg + T = \frac{mV^2}{l}$$

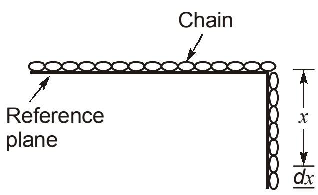
$$mV^2 = mgl + Tl$$

or $$mV_0^2 = 5\,mgl + Tl \quad \text{(using (1))}$$

For minimum V_0, T should be zero

$$\therefore \quad V_0 = \sqrt{5gl}$$

26. (d)



The potential energy of $\frac{l}{3}$ of the chain that overhangs is

$$u_1 = \int_0^{l/3} -\frac{mgx}{l}dx = \frac{-mgl}{18}$$

The potential energy of the full chain when it completely slips off the table is

$$u_2 = \int_0^{l} -\frac{mgx}{l}dx = \frac{-mgl}{2}$$

$$\text{The loss in } PE = \frac{-mgl}{18} - \left(\frac{-mgl}{2}\right) = \frac{4mgl}{9}$$

This should be equal to gain in kinetic energy, but the initial kE is zero. Hence this is the kE when the chain completely falls off the table.

27. (d)

Using conservation of angular momentum,

$$MR^2\omega = \left(MR^2 \times \frac{8\omega}{9}\right) + \left(\frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9}\right) + \left(\frac{M}{8} \times x^2 \times \frac{8\omega}{9}\right)$$

$$\therefore \quad x = \frac{4R}{5}$$

28. (b)

Now, $\Sigma F_x = 0$

$R_{B2} = P$

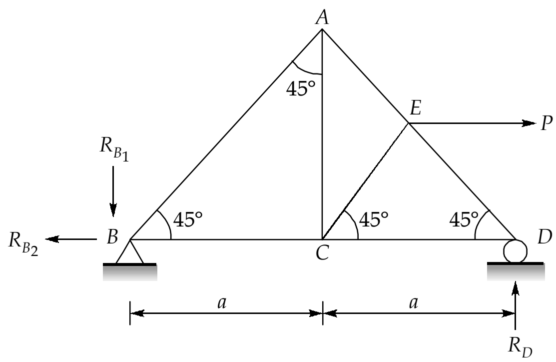
and, $\Sigma F_y = 0$

$R_D = R_{B1}$

Also, $\Sigma M_B = 0$

$$R_D \times 2a = P \times \frac{a}{2}$$

$$R_D = R_{B1} = \frac{P}{4}$$



Analysis of joint B,

So, $F_{AB}\sin 45° = \dfrac{P}{4}$

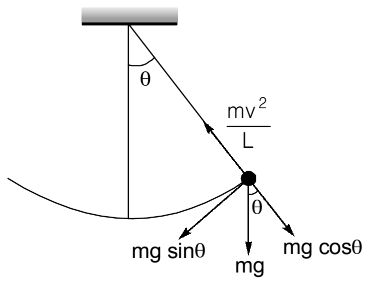
$$\Rightarrow \quad F_{AB} = \frac{\sqrt{2}P}{4}$$

Also, $P = F_{BC} + F_{AB}\cos 45°$

$$\Rightarrow \quad F_{BC} = P - F_{AB}\cos 45° = P - \frac{\sqrt{2}P}{4} \times \frac{1}{\sqrt{2}} = \frac{3P}{4}$$

Hence, $F_{BC} = 0.75\,P$

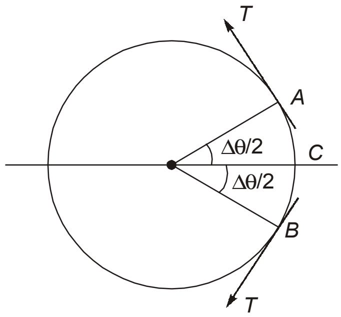
29. (a)



$$\Rightarrow \quad T - mg\cos\theta = \frac{mv^2}{L}$$

$$\Rightarrow \quad T = m\left(g\cos\theta + \frac{v^2}{L}\right)$$

30. (a)



Consider a small part ACB of the ring that subtends an angle $\Delta\theta$ at the centre. Let T be tension in the ring.

Δm be the mass of small element.

$$\Rightarrow \quad 2T\sin\frac{\Delta\theta}{2} = \Delta m\left(\frac{v^2}{r}\right) \quad \text{...(1)}$$

Length of arc ACB is $R\Delta\theta$.

Now, $$\Delta m = \frac{M}{2\pi R}\times R\Delta\theta = \frac{M\Delta\theta}{2\pi}$$

$$\Rightarrow \quad 2T\sin\frac{\Delta\theta}{2} = \frac{M\Delta\theta}{2\pi}\times\frac{v^2}{R}$$

$$\Rightarrow \quad T = M\times\frac{v^2}{2\pi R}\times\frac{\Delta\theta/2}{\sin(\Delta\theta/2)} = \frac{Mv^2}{2\pi R} \qquad \left[\because \text{for small angle, } \sin\frac{\theta}{2} = \frac{\theta}{2}\right]$$

■■■■