S.No. : 01 CH_A_280220

Engineering Mathematics

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CLASS TEST 2020-2021

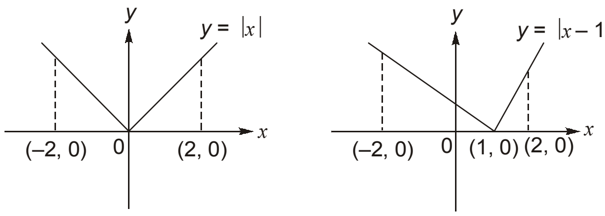
Date of Test : 28/02/2020

ANSWER KEY ➤ Engineering Mathematics

1.	(c)	7.	(a)	13.	(d)	19.	(d)	25.	(c)
2.	(d)	8.	(d)	14.	(b)	20.	(b)	26.	(b)
3.	(b)	9.	(d)	15.	(b)	21.	(c)	27.	(b)
4.	(d)	10.	(b)	16.	(d)	22.	(c)	28.	(c)
5.	(b)	11.	(c)	17.	(b)	23.	(a)	29.	(d)
6.	(c)	12.	(c)	18.	(a)	24.	(d)	30.	(c)

DETAILED EXPLANATIONS

1. (c)



$$\int_{-2}^{2}(|x|\,dx)+\int_{-2}^{2}(|x-1|\,dx) = \text{Area under the curves}$$

$$= 2\times\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 3\times 3+\frac{1}{2}\times 1\times 1 = 4+\frac{9}{2}+\frac{1}{2} = 9 \text{ unit}^2$$

2. (d)

Here,

$$n = 10000$$
$$p = 0.02$$
$$q = 1-0.02 = 0.98$$

$$\text{Standard deviation} = \sqrt{npq} = \sqrt{10000\times 0.02\times 0.98} = \sqrt{196} = 14$$

3. (b)

$$\text{div}\,\phi = \nabla\cdot\phi = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot(\phi)$$

$$= \frac{\partial}{\partial x}(3xz)+\frac{\partial}{\partial y}(2xy)-\frac{\partial}{\partial z}yz^2 = 3z+2x-2yz$$

$$= 3\times 2+2\times 2-2\times 1\times 2 = 6$$

4. (d)

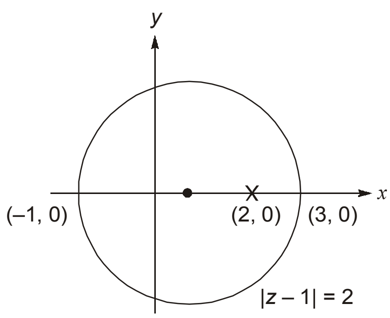
$$\frac{1}{2\pi i}\oint_C\frac{f(z)}{z-a}dz = f(a)$$

$$= 2\times x|_{z=2+i0}$$

$$= 2x|_{x+iy=2}$$

$$= 2\times 2$$

$$= 4$$



5. (b)

If A is orthogonal, then

$$AA^T = I$$

$$\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}\begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$x^2+y^2+z^2 = 1$$

6. (c)

$$f(0) = \lim_{x \to 0} \left\{ \frac{\log(1+x)^{(1+x)} - x}{x^2} \right\} \qquad \left[\frac{0}{0} \text{ form}\right]$$

Applying L'Hospital Rule,

$$f(0) = \lim_{x \to 0} \frac{\log(1+x) + 1 - 1}{2x} = \frac{1}{2} \lim_{x \to 0} \frac{\log(1+x)}{x} = \frac{1}{2}$$

7. (a)

$$\begin{bmatrix} 3 \\ b \\ c \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + k_2 \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} + k_3 \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}$$

$$= k_1 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + 2k_2 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} - k_3 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ b \\ c \end{bmatrix} = (k_1 + 2k_2 - k_3) \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$k_1 + 2k_2 - k_3 = 3$$

$$3(k_1 + 2k_2 - k_3) = 3 \times 3 = 9$$

$$2(k_1 + 2k_2 - k_3) = 2 \times 3 = 6$$

8. (d)

Let x is the thickness of ice at time t

$$V = \frac{4\pi}{3}(x+10)^3$$

Where x is thickness of ice

$$\frac{dV}{dt} = \frac{4\pi}{3} \times 3 \times (x+10)^2 . \frac{dx}{dt}$$

$$\left.\frac{dx}{dt}\right|_{x=5} = \left.\frac{50}{4\pi(x+10)^2}\right|_{x=5}$$

$$= \frac{50}{4\pi(5+10)^2} = \frac{1}{18\pi} \text{ cm/min}$$

9. (d)

$$y = \log_e (x + e)$$

$$x + e = e^y$$

$\Rightarrow$ $$x = e^y - e$$

For $x = 0$

$$y = \log_e (0 + e) = 1$$

For $y = 0$

$$0 = \log_e (x + e)$$

$$x + e = 1$$

$$x = 1 - e$$

$$\text{Area} = \left|\int_{y_1}^{y_2} x \cdot dy\right| = \left|\int_0^1 (e^y - e)dy\right| = \left|e^y - ey\Big|_0^1\right|$$

$$= |e - 1 - e(1-0)|$$

$$= |-1| = 1$$

10. (b)

Given that the partial differential equation is parabolic.

$\therefore$ $B^2 - 4AC = 0$ Here $A = 3$

$\therefore$ $B^2 - 4(3)(3) = 0$ $C = 3$

$$B^2 - 36 = 0$$

$$B^2 = 36$$

11. (c)

$$[A : B]$$

$$\begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 1 & 1 & -3 & : & -1 \\ 5 & -3 & 9 & : & 21 \end{bmatrix}$$

$R_2 \to 4R_2 - R_1$; $R_3 \to 4R_3 - 5R_1$

or $\begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 0 & 6 & -18 & : & -12 \\ 0 & -2 & 6 & : & 44 \end{bmatrix}$

$R_3 \to 3R_3 + R_2$

$$\begin{bmatrix} 4 & -2 & 6 & : & 8 \\ 0 & 6 & -18 & : & 12 \\ 0 & 0 & 0 & : & 120 \end{bmatrix}$$

From here

$$\rho(A) \neq \rho(B) = \rho(A : B) < 3$$

$\Rightarrow$ System is inconsistent with no solution.

12. (c)

Here unit digit of the number is 3.

$$3^1 = \underline{3}$$
$$3^2 = \underline{9}$$
$$3^3 = 2\underline{7}$$
$$3^4 = 8\underline{1}$$
$$3^5 = 34\underline{3}$$

From here, cyclicity of 3 is 4. So, the probability of having 3 in the unit place = $\frac{1}{4}$

13. (d)

Let $f(x + iy) = u(x, y) + iv(x, y)$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

Since $f(z)$ is analytic function

So $$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = 2y = \frac{\partial v}{\partial y}$$

$$u = \int \frac{\partial u}{\partial x} \cdot dx + \int \frac{\partial u}{\partial y} \cdot dy$$

$$= 2xy + Q(x, y)$$

$$f(1 + i) = 2$$

$$u(1, 1) = 2$$

$$v(1, 1) = 0$$

$$2 \times 1 \times 1 + Q(1, 1) = 2$$

$$y(x, y) = 0$$

$$\Rightarrow \quad u(x, y) = 2xy$$

$$\frac{\partial u}{\partial y} = 2x = -\frac{\partial v}{\partial x}$$

$$v = \int \frac{\partial v}{\partial x} \cdot dx + \int \frac{\partial v}{\partial y} \cdot dy = \int -2x \cdot dx + \int 2y dy = -x^2 + y^2 + c$$

$$u(1, 1) = 1 - 1 + c = 0$$

$$\Rightarrow \quad c = 0$$

$$v(x, y) = y^2 - x^2$$

14. (b)

The maximum variation is in direction of grad T.

$$T = x^2 + 4xy + y^2$$

$$\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} = (2x + 4y)\hat{i} + (4x + 2y)\hat{j}$$

$$\nabla T|_{(2,2)} = (4 + 8)\hat{i} + (8 + 4)\hat{j}$$

$$= 12\hat{i} + 12\hat{j}$$

The direction in which rate is slowest is perpendicular the direction in which variation is maximum.

$$\nabla T|_{\min} = 12\hat{i} - 12\hat{j} \text{ or } \hat{i} - \hat{j}$$

15. (b)

By Trapezoidal rule, the area under the curve or $\int_{x_1}^{x_2} y dx$ is

$$= \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$$

where, $y = f(x)$

Here, $h = 1$

$$\Rightarrow \quad \text{Area} = \frac{1}{2}\left(10 + 2\left(50 + 70 + 80\right) + 100\right) = 255 \text{ sq. unit}$$

16. (d)

Eigen value of A

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -2 \\ -3 & 2-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)(2 - \lambda) - 6 = 0$$

$$\lambda^2 - 3\lambda + 2 - 6 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = -1, 4$$

Eigen values of A^4 are $(-1)^4$ and $(4)^4$

$$= 1 \text{ and } 256$$

17. (b)

$$f(x) = \frac{\log_e(1+ax) - \log_e(1-bx)}{x}$$

For function to be continuous

$$f(0) = \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{\log_e(1+ax) - \log(1-bx)}{x} = \lim_{x \to 0} \frac{\log_e(1+ax) \times a}{ax} + \frac{\log(1-bx) \times b}{-bx}$$

$$= a + b$$

18. (a)

$$A = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$$

$$A + I = \begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

$$[A+I]^2 = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2\alpha \\ 0 & 1 \end{bmatrix}$$

$$[A+I]^3 = \begin{bmatrix} 1 & 2\alpha \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3\alpha \\ 0 & 1 \end{bmatrix}$$

$$[A+I]^{50} = \begin{bmatrix} 1 & 50\alpha \\ 0 & 1 \end{bmatrix}$$

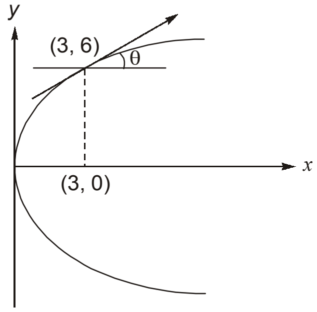
$$50A = \begin{bmatrix} 0 & 50\alpha \\ 0 & 0 \end{bmatrix}$$

$$[A+I]^{50} - 50A = \begin{bmatrix} 1 & 50\alpha \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 50\alpha \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a + b + c + d = 1 + 0 + 0 + 1 = 2$$

19. (d)



Direction of velocity at point (3, 6) is in the direction of tangent at that point.

$\Rightarrow$ Slope of tangent = Slope of velocity

$$\tan\theta = \left.\frac{dy}{dx}\right|_{(3,6)} = \frac{12}{2y} = \frac{12}{2\times 6} = 1$$

$$\Rightarrow \quad \theta = 45^\circ$$

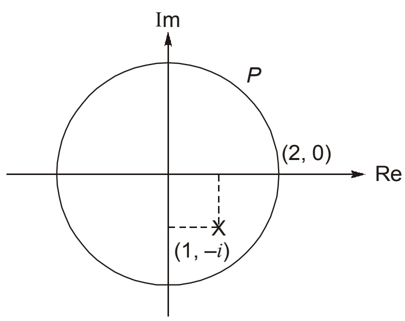
$$v = v_x\hat{i} + v_y\hat{j}$$

$$|v| = \sqrt{v_x^2 + v_y^2} = 10 \text{ m/s}$$

$$v_x = 10\cos 45^\circ = 5\sqrt{2} \text{ m/s}$$

$$v_y = 10\sin 45^\circ = 5\sqrt{2} \text{ m/s}$$

20. (b)



Since (1, $-i$) is inside the circle.

$$\Rightarrow \quad f(\alpha) = \left(\frac{1}{2\pi i}\left.(3z^2 + 7z + 1)\right|_{z=\alpha} \times 2\pi i\right)$$

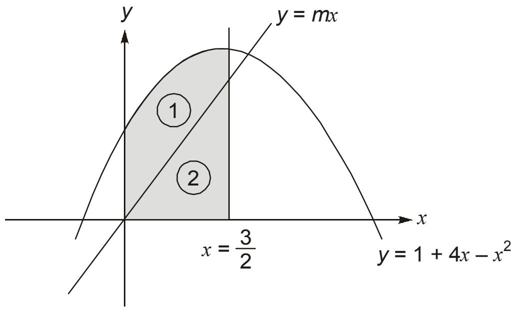
$$f(\alpha) = 3\alpha^2 + 7\alpha + 1$$

$$f'(\alpha) = 6\alpha + 7 = 6(1 - i) + 7 = 13 - 6i$$

$$|f'(\alpha)| = \sqrt{(13)^2 + (6)^2}$$

$$= \sqrt{205} = 14.32$$

21. (c)



$$\text{Area of (1)} = \text{Area of (2)} = \int_0^{3/2} mx\,dx = \frac{1}{2}[\text{Area of (1)} + \text{Area of (2)}]$$

$$\frac{1}{2}\times\int_0^{3/2}\left(1+4x-x^2\right)dx = \frac{1}{2}\left[x+\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^{3/2}$$

$$\int_0^{3/2} mxdx = \frac{1}{2}\left[\left(\frac{3}{2}-0\right)+2\left(\frac{9}{4}-0\right)-\left(\frac{27}{8\times 3}-0\right)\right]$$

$$\left[m\frac{x^2}{2}\right]_0^{3/2} = \frac{1}{2}\left[\frac{3}{2}+\frac{9}{2}-\frac{9}{8}\right]$$

$$m\times\frac{9}{4\times 2} = \frac{1}{2}\times\frac{39}{8}$$

$$m = \frac{13}{6} = 2.17$$

22. (c)

$$N = 5$$

$$h = \frac{b-a}{N} = \frac{5-0}{5} = \frac{5}{5} = 1$$

Therefore

$$\int_0^5 f(x)dx = \frac{1}{2}[0+2.236+2(1+1.414+1.732+2)] = \frac{1}{2}(14.528) = 7.264$$

23. (a)

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\,[(3-\lambda)^2-1] = 0$$

$$(1-\lambda)\,[\lambda^2+9-6\lambda-1] = 0$$

$$(1-\lambda)\,(\lambda^2-6\lambda+8) = 0$$

$$(1-\lambda)\,(\lambda^2-6\lambda+8) = 0$$

$$(1-\lambda)\,(\lambda-2)\,(\lambda-4) = 0$$

$$\lambda = 1, 2, 4$$

$$[A-\lambda_1 I]\,[x] = 0$$

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 3-1 & -1 \\ 0 & -1 & 3-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_2 - x_3 = 0$$

$$-x_2 + 2x_3 = 0$$

$x_1 = k_1, \quad x_2 = 2x_3, \quad 2x_2 = x_3$

$$\text{eigen vector} = \begin{bmatrix} k_1 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda_2 = 2$

$$[A - \lambda_2 I][x] = 0$$

$$\begin{bmatrix} 1-2 & 0 & 0 \\ 0 & 3-2 & -1 \\ 0 & -1 & 3-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = 0$$

$$x_2 - x_3 = 0 \qquad x_2 = x_3 \qquad \begin{bmatrix} 0 \\ k \\ k \end{bmatrix}$$

$$-x_2 + x_3 = 0 \qquad x_2 = x_3$$

$\lambda_3 = 4$

$$[A - \lambda_3 I][X] = [0]$$

$$\begin{bmatrix} 1-4 & 0 & 0 \\ 0 & 3-4 & -1 \\ 0 & -1 & 3-4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = 0$$

$$-x_2 - x_3 = 0 \qquad x_2 = -x_3 \qquad \begin{bmatrix} 0 \\ k \\ -k \end{bmatrix}$$

$$-x_2 - x_3 = 0 \qquad x_2 = -x_3$$

24. (d)

$$P(X = r) = \frac{e^{-m} m^r}{r!}$$

$$P(X = r) = P(X = 0) + P(X = 1) = e^{-m} + \frac{e^{-m} m}{1!}$$

Here is given that $m = 5$

$\therefore \quad P(X = 5) = e^{-5} + e^{-5} \cdot 5$

$$= \frac{6}{e^5}$$

25. (c)

$$9x^2 + 36x + 16y^2 + 96y + 36 = 0$$
$$9(x^2 + 4x) + 16(y^2 + 6y) + 36 = 0$$
$$9(x^2 + 4x + 4) - 36 + 16(y^2 + 6y + 9) - 144 + 36 = 0$$
$$9(x+2)^2 + 16(y+3)^2 = 144$$
$$\frac{(x+2)^2}{16} + \frac{(y+3)^2}{9} = 1$$

The given curve is ellipse.

The area of ellipse $= \pi ab$ [a = semi major axis, b = semi minor axis]

$$= \pi \times \sqrt{16} \times \sqrt{9} = 12\pi = 37.69 \text{ sq. unit}$$

26. (b)

By trapezoidal rule,

$$\text{Area} = \int_{7.40}^{7.90} f(x)\,dx$$
$$= \frac{h}{2}\left[y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)\right]$$
$$= \frac{0.10}{2}\left[1.93 + 3.10 + 2(1.92 + 2.05 + 2.30 + 2.60)\right] = 1.1385 \approx 1.14$$

27. (b)

$$f(x) = a_0 + a_1x^2 + a_2x^4 \ldots a_nx^{2n}$$
$$\frac{df(x)}{dx} = a_1 \cdot 2x + a_2 4x^3 \ldots a_n \cdot 2nx^{2n-1}$$

For maxima or maxima

$$\frac{df(x)}{dx} = 0$$
$$a_1 \cdot 2x + a_2 \cdot 4x^3 \ldots a_n \cdot 2nx^{2n-1} = 0$$
$$x\left(a_1 \cdot 2 + a_2 4x^2 \ldots a_n \cdot 2nx^{2n-2}\right) = 0$$
$$x = 0, \quad 2a_1 + 4a_2x^2 \ldots\ldots 2n a_n x^{2n-2} \neq 0$$

For $x = 0$

$$f''(x) = 2a_1 > 0$$

Minima at $x = 0$ and exactly one minima.

28. (c)

$$|A| = \begin{vmatrix} {}^xC_0 & {}^xC_1 & {}^xC_2 \\ {}^yC_0 & {}^yC_1 & {}^yC_2 \\ {}^zC_0 & {}^zC_1 & {}^zC_2 \end{vmatrix} = \begin{vmatrix} 1 & \frac{x}{1} & \frac{x(x-1)}{2} \\ 1 & \frac{y}{1} & \frac{y(y-1)}{2} \\ 1 & z & \frac{z(z-1)}{2} \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & x & x^2-x \\ 1 & y & y^2-y \\ 1 & z & z^2-z \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \frac{1}{2}\begin{vmatrix} 1 & x & x \\ 1 & y & y \\ 1 & z & z \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$R_3 \to R_3 - R_2,\ R_2 \to R_2 - R_1$

$$= \frac{1}{2}\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-y & z^2-y^2 \end{vmatrix}$$

$$= \frac{1}{2}[(y-x)(z^2-y^2)-(y^2-x^2)(z-y)]$$

$$= \frac{1}{2}[(y-x)(z-y)(z+y)-(y-x)(y+x)(z-y)]$$

$$= \frac{1}{2}\ [(y-x)(z-y)(z+y-y-x)]$$

$$= \frac{1}{2}\ (y-x)(z-y)(z-x)$$

$$|A| = \frac{1}{2}\ (x-y)(y-z)(z-x)$$

$$|A| = \frac{1}{2}\ (x-y)(y-z)(z-x) = 0$$

Then, either $x = y$

or $y = z$

or $z = x$

29. (d)

Let $d \rightarrow$ defective

$y \rightarrow$ supplied by y

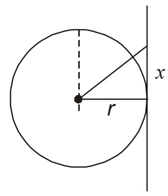
$$P\left(\frac{y}{d}\right) = \frac{P(y \cap d)}{P(d)}$$

$$P(y \cap d) = 0.3 \times 0.02 = 0.006$$

$$P(d) = 0.6 \times 0.01 + 0.3 \times 0.02 + 0.1 \times 0.03 = 0.015$$

$$P\left(\frac{y}{d}\right) = \frac{0.006}{0.015} = 0.4$$

30. (c)



$$\tan\theta = \frac{x}{r}$$

$$x = r\tan\theta$$

$$\frac{dx}{dt} = r\sec^2\theta\,\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = r\frac{d\theta}{dt}\sec^2\theta = v\sec^2\theta$$

when 1/8 of circle is covered

$$\theta = \frac{1}{8}\times 2\pi = \frac{\pi}{4}$$

$$\frac{dx}{dt} = 20\times\sec^2\left(\frac{\pi}{4}\right) = 20\times 2 = 40 \text{ km/h}$$

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