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SIGNALS AND SYSTEMS

EC + EE

Date of Test : 10/08/2026

ANSWER KEY ➤

1. (c)	7. (b)	13. (d)	19. (a)	25. (b)
2. (b)	8. (d)	14. (a)	20. (d)	26. (c)
3. (a)	9. (c)	15. (d)	21. (b)	27. (c)
4. (b)	10. (a)	16. (c)	22. (d)	28. (a)
5. (a)	11. (a)	17. (a)	23. (a)	29. (d)
6. (d)	12. (d)	18. (b)	24. (c)	30. (d)

DETAILED EXPLANATIONS

1. (c)

We know that the Laplace transform of

$$\cos(at)u(t) = \frac{s}{s^2 + a^2}$$

$$\therefore \cos(\pi t)u(t) = \frac{s}{s^2 + \pi^2}$$

now, the given function $x(t)$ can be written as,

$$\begin{aligned} &= \cos \pi t [u(t) - u(t-1)] \\ &= \cos(\pi t)u(t) - \cos \pi t u(t-1) \\ &= \cos \pi t u(t) - \cos \pi(t-1+1)u(t-1) \\ &= \cos \pi t u(t) - \cos[\pi(t-1) + \pi]u(t-1) \\ x(t) &= \cos(\pi t)u(t) + \cos[\pi(t-1)]u(t-1) \end{aligned}$$

By taking Laplace transform,

$$X(s) = \frac{s}{s^2 + \pi^2} + \frac{s e^{-s}}{s^2 + \pi^2} \quad [\because x(t-t_0) = X(s) \cdot e^{-st_0}, \text{ by shifting property}]$$

$$X(s) = \frac{s[1 + e^{-s}]}{s^2 + \pi^2}$$

2. (b)

Given, $x(t) = \frac{\sin(10\pi t)}{\pi t}$

Taking Fourier transform

$$X(j\omega) = \begin{cases} 1 & ; \quad |\omega| \leq 10\pi \\ 0 & ; \quad |\omega| > 10\pi \end{cases}$$

or

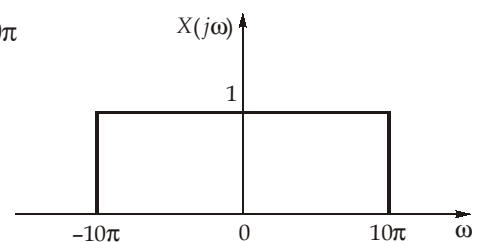
$\therefore$ The maximum frequency ' ω_m ' present in $x(t)$ is $\omega_m = 10\pi$

Hence we require,

$$\frac{2\pi}{T_s} > 2\omega_m$$

$$\frac{2\pi}{T_s} > 20\pi$$

$$\therefore T_s < \frac{1}{10}$$



3. (a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} \frac{2^{-n}}{n!} z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{(2z)^{-n}}{n!} = \sum_{n=0}^{\infty} \left(\frac{1}{2z} \right)^n$$

$$X(z) = 1 + \frac{1}{2z} + \frac{\left(\frac{1}{2z}\right)^2}{2!} + \frac{\left(\frac{1}{2z}\right)^3}{3!} + \dots$$

$$X(z) = e^{1/2z}$$

$$X(1) = e^{1/2} = \sqrt{e} = 1.648 \approx 1.65$$

4. (b)

The output of the given LTI system is,

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{+\infty} h[k] e^{j\omega(n-k)} + \sum_{k=-\infty}^{+\infty} h[k] e^{j2\omega(n-k)} \\ &= e^{j\omega n} \sum_{k=-\infty}^{+\infty} h[k] e^{-j\omega k} + e^{j2\omega n} \sum_{k=-\infty}^{+\infty} h[k] e^{-j2\omega k} \\ &= e^{j\omega n} H(e^{j\omega}) + e^{j2\omega n} H(e^{j2\omega}) \end{aligned}$$

Since the input cannot be extracted from the above expression, the sum of the complex exponential is not an eigen function.

5. (a)

6. (d)

Let us consider two signals,

$$x_1(t) = 1, \quad \forall t$$

$$x_2(t) = -1, \quad \forall t$$

Clearly $x_1(t) \neq x_2(t)$ but $(x_1(t))^2 = (x_2(t))^2$

Therefore different inputs gives the same output hence the system is non invertible.

And also it is non linear system.

7. (b)

By using FFT to compute DFT, we need $\frac{N}{2} \log_2 N$ complex multiplications.

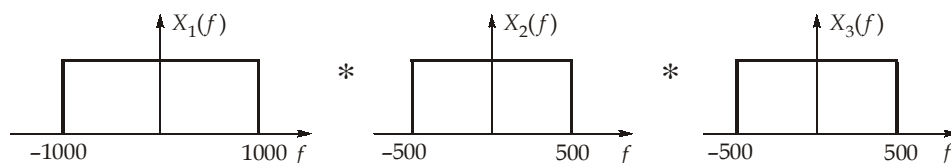
Therefore for 16-point DFT by using FFT we require $\frac{16}{2} \log_2 16 = 32$.

8. (d)

We know that,

For

$$X(f) = X_1(f) * X_2(f) * X_3(f)$$



$$\begin{aligned} \text{Sampling frequency, } f_s &= 2(1000 + 500 + 500) \\ f_s &= 4000 \text{ samples/sec} \end{aligned}$$

9. (c)

Conjugate anti-symmetric part of $x[n]$ is $\frac{x[n] - x^*[-n]}{2}$.

$$x^*[-n] = [2, 1 + j, -2 + j5]$$

$$\therefore \frac{x[n] - x^*[-n]}{2} = \frac{[-2 - j5, 1 - j, 2] - [2, (1 + j), -2 + j5]}{2} = [-2 - j2.5, -j, 2 - j2.5]$$

↑

11. (a)

Given,
$$X(e^{j\omega}) = \frac{\sin \frac{3\omega}{2}}{\sin \frac{\omega}{2}} = \frac{e^{j3\omega/2} - e^{-j3\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}} = \frac{e^{j3\omega/2} [1 - e^{-j3\omega}]}{e^{j\omega/2} [1 - e^{-j\omega}]}$$

$$= e^{j\omega} \left[\frac{1 - e^{-j3\omega}}{1 - e^{-j\omega}} \right]$$

$$X(e^{j\omega}) = \frac{e^{j\omega}}{1 - e^{-j\omega}} - \frac{e^{-j2\omega}}{1 - e^{-j\omega}}$$

by taking inverse DTFT,

$$\begin{aligned} x[n] &= u[n + 1] - u[n - 2] \\ &= \begin{cases} 1; & -1 \leq n < 2 \\ 0; & \text{otherwise} \end{cases} \end{aligned}$$

From parseval's theorem,

$$\sum_{n=-\infty}^{\infty} |nx[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \left[\frac{d}{d\omega} X(e^{j\omega}) \right] \right|^2 d\omega$$

$$\therefore \frac{1}{\pi} \int_{-\pi}^{\pi} \left| \left[\frac{d}{d\omega} X(e^{j\omega}) \right] \right|^2 d\omega = 2 \sum_{n=-\infty}^{\infty} |nx[n]|^2$$

$$= 2 \sum_{n=-1}^1 |n|^2 = 2[1 + 0 + 1] = 4$$

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \left| \left[\frac{d}{d\omega} X(e^{j\omega}) \right] \right|^2 d\omega = 4$$

12. (d)

Given,
$$\begin{aligned} X(\omega) &= \frac{1}{\omega^2 + j\omega} = \frac{1}{j\omega[1 - j\omega]} = \frac{2}{2} \times \frac{1}{j\omega} \cdot \frac{1}{(1 - j\omega)} \\ &= \frac{1}{2} \frac{2}{j\omega} \cdot \frac{1}{(1 - j\omega)} \end{aligned}$$

From the property of convolution,

$$x_1(t) * x_2(t) = X_1(\omega) X_2(\omega)$$

$$\text{sgn } t * e^t u(-t) \xrightarrow{FT} \frac{2}{j\omega} \cdot \frac{1}{1 - j\omega}$$

$$\frac{1}{2} \operatorname{sgn} t * e^t u(-t) \xrightarrow{FT} \frac{1}{j\omega} \cdot \frac{1}{1-j\omega}$$

$$\therefore x(t) = \frac{1}{2} \operatorname{sgn} t * e^t u(-t)$$

13. (d)

Given $x(t) = \sin(150\pi t)$

Time period, $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{150\pi} = \frac{1}{75} \text{ sec}$

$$3 \text{ time periods} = 3 \times T = 3 \times \frac{1}{75} = \frac{1}{25} \text{ sec}$$

$\therefore$ The signal sampled at a rate of five samples is $\frac{1}{25} \text{ sec}$

So, 1 sample in $\frac{1}{125} \text{ sec} = T_s$ [sampling interval]

$$\therefore \text{Sampling frequency} = f_s = \frac{1}{T_s} = 125 \text{ samples/sec}$$

also, Nyquist rate $= f_N = 2f_m = 2 \times 75 \quad [\because \omega_m = 150\pi \Rightarrow f_m = 75 \text{ Hz}]$
 $= 150 \text{ samples/sec}$

$$\therefore \text{The ratio, } \frac{f_s}{f_N} = \frac{125}{150} = \frac{5}{6} = 0.83$$

14. (a)

Given,

By the definition of Fourier transform,

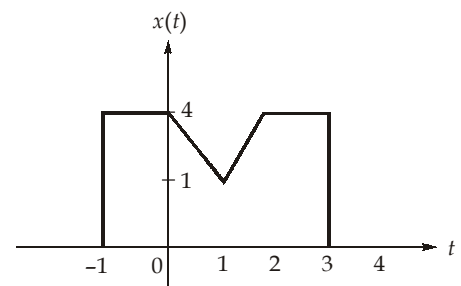
$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} d\omega$$

at $t = 0$,

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) d\omega$$

$$\therefore \int_{-\infty}^{\infty} X(j\omega) d\omega = 2\pi x(0) = 2\pi(4) = 8\pi \approx 25.13$$



15. (d)

By redrawing the given frequency response, we get,

We can write $H(\omega) = -j2 \operatorname{sgn}(\omega)$

We know that,

$$\text{For } \operatorname{sgn}(t) \xleftrightarrow{\text{FT}} \frac{2}{j\omega}$$

By duality property

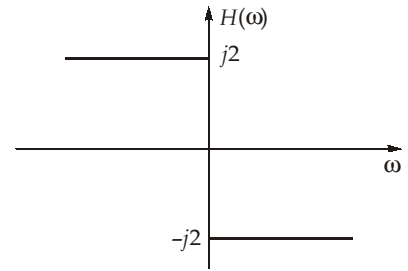
$$\frac{2}{jt} \xleftrightarrow{\text{FT}} 2\pi \operatorname{sgn}(-\omega)$$

$$\frac{2}{jt} \xleftrightarrow{\text{FT}} -2\pi \operatorname{sgn}(\omega)$$

$$\frac{2}{\pi t} \xleftrightarrow{\text{FT}} -j2 \operatorname{sgn}(\omega)$$

or

$$= 2(\pi t)^{-1}$$



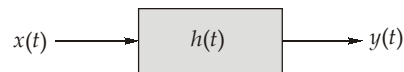
16. (c)

Given, the Causal LTI system,

$$H(j\omega) = \frac{1}{3 + j\omega}$$

and output,

$$y(t) = e^{-3t} u(t) - e^{-4t} u(t)$$



$$\text{We know that, } H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{1}{3 + j\omega} - \frac{1}{4 + j\omega} = \frac{1}{(3 + j\omega)(4 + j\omega)}$$

$$\therefore X(j\omega) = \frac{Y(j\omega)}{H(j\omega)} = \frac{1}{4 + j\omega}$$

By inverse Fourier transform of $X(j\omega)$, we have,

$$x(t) = e^{-4t} u(t)$$

17. (a)

Given,

$$\begin{aligned} X(z) &= \frac{10 - 8z^{-1}}{2 - 5z^{-1} + 2z^{-2}} \\ &= \frac{2}{(2 - z^{-1})} + \frac{4}{(1 - 2z^{-1})} \end{aligned}$$

$$X(z) = \frac{2z}{2z - 1} + \frac{4z}{z - 2}$$

$$X(z) = \frac{z}{\left(z - \frac{1}{2}\right)} + \frac{4z}{(z - 2)}$$

Since, ROC includes unit circle,

∴ ROC of $X(z)$ is $\frac{1}{2} < |z| < 2$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 4(2^n)u[-n-1]$$

∴ $x(1) = \frac{1}{2} = 0.5$

18. (b)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

$$g[n] = x[n-2]_{\text{mod } N}$$

$$G[k] = e^{-j\frac{2\pi}{N}(2)k} X[k]$$

$$G[1] = e^{-j\frac{2\pi}{4}(2)1} X[1] = e^{-j\pi} X[1]$$

$$G[1] = -X[1] = -7$$

19. (a)

We know that, from the definition of DTFT,

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

where, $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

by putting $n = -1$,

∴ $x[-1] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega} d\omega$

∴ $\int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega} d\omega = 2\pi x[-1]$
 $= 2\pi[-1] = -2\pi = -6.28$

20. (d)

Given pole-zero plot can be written as,

$$X(z) = \frac{(z-1)(z+1)}{z^2} = \frac{z^2-1}{z^2} = 1 - z^{-2}$$

put $z = e^{j\omega}$,

$$|X(e^{j\omega})| = |1 - \cos 2\omega + j \sin 2\omega|$$

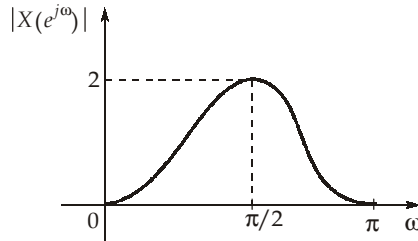
$$|X(e^{j\omega})| = |\sqrt{2-2\cos 2\omega}|$$

put, $\omega = 0 \Rightarrow |X(e^{j\omega})| = 0$

$$\omega = \pi/2 \Rightarrow |X(e^{j\pi/2})| = 2$$

$$\omega = \pi \Rightarrow |X(e^{j\pi})| = 0$$

It is a bandpass filter response,



If poles are moved towards $\pm \frac{\pi}{2}$

Suppose we take poles are at $z = \pm j0.5$

$$X(z) = \frac{(z+1)(z-1)}{(z+j0.5)(z-j0.5)}$$

still response will remain same as BPF.

Because of zeros at the same position.

21. (b)

Given signals, $x(t) = \sin \omega_0 t$

$$h(t) = \text{sgnt}$$

from the multiplication property of Fourier transform,

$$x(t)h(t) = \frac{1}{2\pi} [X(\omega) * H(\omega)]$$

Fourier transform of $x(t)$ is $X(\omega)$,

$$X(\omega) = \frac{\pi}{j} [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

Fourier transform of $h(t)$ is $H(\omega)$,

$$H(\omega) = \frac{2}{j\omega}$$

$\therefore$

$$\begin{aligned} x(t)h(t) &\xrightarrow{\text{FT}} \frac{1}{2\pi} \left[\frac{\pi}{j} (\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) * \frac{2}{j\omega} \right] \\ &\xrightarrow{\text{FT}} \frac{1}{2\pi} \left[\left[\frac{\pi}{j} \times \frac{2}{j(\omega - \omega_0)} \right] - \left[\frac{\pi}{j} \times \frac{2}{j(\omega + \omega_0)} \right] \right] \\ &\quad (\because X(\omega) * \delta(\omega - \omega_0) = X(\omega - \omega_0)) \end{aligned}$$

$$\xrightarrow{\text{FT}} \left[\frac{-1}{\omega - \omega_0} + \frac{1}{\omega + \omega_0} \right] = \frac{-\omega - \omega_0 + \omega - \omega_0}{\omega^2 - \omega_0^2}$$

$\therefore$

$$x(t)h(t) \xrightarrow{\text{FT}} \frac{-2\omega_0}{\omega^2 - \omega_0^2}$$

22. (d)

Given signal, Let $x(t) = \frac{1}{\pi(1+t^2)}$

We know that,

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2 + \omega^2}$$

Put $a = 1$

$$e^{-|t|} \xleftrightarrow{FT} \frac{2}{1 + \omega^2}$$

By using duality property,

$$\frac{2}{1+t^2} \xleftrightarrow{FT} 2\pi e^{-|\omega|}$$

$$\frac{2}{1+t^2} \xleftrightarrow{FT} 2\pi e^{-|\omega|}$$

$$\frac{1}{\pi(1+t^2)} \xleftrightarrow{FT} e^{-|\omega|}$$

23. (a)

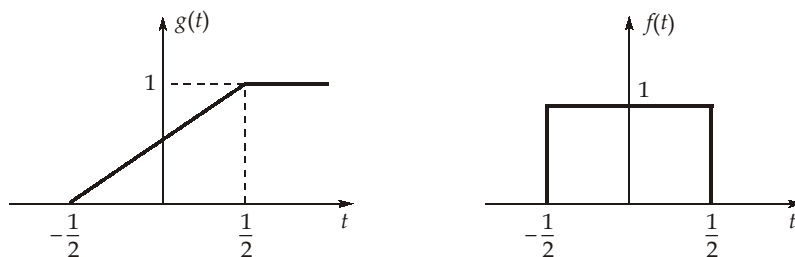
By the definition of Fourier series,

We can write $C_{N_0/2}$ for N_0 is even,

$$\begin{aligned} C_{N_0/2} &= \frac{1}{N_0} \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{N_0}{2}\right)\left(\frac{2\pi}{N_0}\right)n} \\ &= \frac{1}{N_0} \sum_{n=0}^{N-1} x[n] e^{-j\pi n} = \frac{1}{N_0} \sum_{n=0}^{N-1} (-1)^n x[n] = \text{real} \end{aligned}$$

24. (c)

Given,



We can write,

$$g(t) = \int_{-\infty}^t f(t) dt = \begin{cases} 0; & t < -\frac{1}{2} \\ t + \frac{1}{2}; & -\frac{1}{2} < t < \frac{1}{2} \\ 1; & t > \frac{1}{2} \end{cases}$$

$\therefore$

$$g(t) \xleftrightarrow{FT} G(\omega)$$

$$g(t) = \int_{-\infty}^t f(t) dt \xrightarrow{FT} \frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$$

($\because$ from time integration property)

$$\therefore G(\omega) = \frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$$

$$\text{but, } F(\omega) = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j\omega t} dt = \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= -\frac{1}{j\omega} \left[e^{-j\frac{\omega}{2}} - e^{+j\frac{\omega}{2}} \right] = \frac{2}{\omega} \sin\left(\frac{\omega}{2}\right)$$

$$F(\omega) = Sa\left(\frac{\omega}{2}\right)$$

$$\therefore G(\omega) = \frac{Sa\left(\frac{\omega}{2}\right)}{j\omega} + \pi \delta(\omega) \quad [\because F(0) = 1]$$

25. (b)

Given system is,

$$\frac{dy(t)}{dt} + 4y(t) = 2x(t)$$

$$(s + 4) Y(s) = 2 X(s)$$

$$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s + 4}$$

$$\frac{e^{j4t} - e^{-j4t}}{j} = 2 \sin 4t$$

$$2 \sin 4t \longrightarrow \boxed{\frac{2}{s+4}} \longrightarrow y(t)$$

$$y(t) = 2 \left[\frac{2}{j4 + 4} \right] \sin \left[4t - \tan^{-1} \left(\frac{4}{4} \right) \right]$$

$$y(t) = \frac{1}{\sqrt{2}} \sin[4t - 45^\circ]$$

$$\therefore A = \frac{1}{\sqrt{2}} = 0.707$$

26. (c)

$$x_1(t) \xrightarrow{\text{L.T.}} \frac{1}{s+2}$$

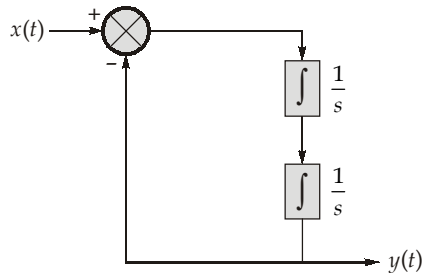
$$x_2(t) \xrightarrow{\text{L.T.}} \frac{1}{s+3}$$

$$x_1(t-2) \xrightarrow{\text{L.T.}} \frac{e^{-2s}}{s+2}$$

$$x_2(t+3) \xrightarrow{\text{L.T.}} \frac{e^{3s}}{s+3}$$

$$x_2(-t+3) \xrightarrow{\text{L.T.}} \frac{e^{-3s}}{3-s}$$

27. (c)



$$H(s) = \frac{\frac{1}{s^2}}{1 + \frac{1}{s^2}} = \frac{1}{s^2 + 1}$$

$$h(t) = \sin t \, u(t)$$

28. (a)

The difference equation is,

$$y[n] + 0.5 y[n-1] = x[n]$$

Characteristic equation,

$$\lambda + 0.5 = 0, \quad \text{root, } \lambda = -0.5$$

$$y_H[n] = C_1(-0.5)^n$$

$$y_p[n] = K$$

$$K + 0.5 K = 1$$

$$\Rightarrow K = \frac{2}{3}$$

So, $y_p[n] = \frac{2}{3}u[n]$

Complete response,

$$\begin{aligned} y[n] &= y_h[n] + y_p[n] \\ &= C_1(-0.5)^n + \frac{2}{3} \end{aligned}$$

Using given initial conditions,

$$y[-1] = C_1(-0.5)^{-1} + \frac{2}{3} = 0$$

$$\Rightarrow C_1 = \frac{1}{3}$$

and, $y[n] = \left[\frac{1}{3} \left(\frac{-1}{2} \right)^n + \frac{2}{3} \right] u[n]$

Alternative Solution:

$$x[n] = 0.5y[n-1] + y[n]$$

Taking z-transform,

$$X(z) = Y(z) + 0.5[z^{-1}Y(z) + 0]$$

$$\Rightarrow \text{Put, } X(z) = \frac{z}{z-1} \quad [\because x[n] = u(n)]$$

$$Y(z) = \frac{z}{(z-1)(z+0.5)}$$

by partial fraction method,

$$\frac{A}{(z+0.5)} + \frac{B}{(z-1)} = \frac{z}{(z-1)(z+0.5)}$$

$$\text{Solving, } A = \frac{1}{3},$$

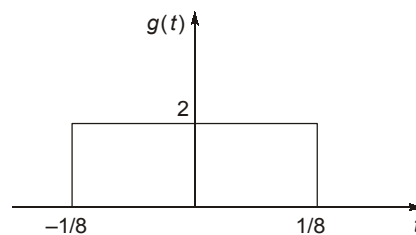
$$B = \frac{2}{3}$$

$$\text{Hence, } y[n] = \left[\frac{1}{3}[-0.5]^n + \frac{2}{3} \right] u[n]$$

29. (d)

$$\begin{aligned} g(t) &= \text{rect}(4t) * 4\delta(-2t) \\ &= 4 \text{rect}(4t) * \delta(-2t) \quad (\because \delta(-t) = \delta(t)) \\ &= 2 \text{rect}(4t) \quad \left(\because \delta(at) = \frac{1}{|a|} \delta(t) \right) \end{aligned}$$

thus $g(t)$ is given as



now,

$$\text{rect}(t) \xrightarrow{F.T} \text{sinc}(f)$$

$$2\text{rect}(t) \xrightarrow{F.T} 2\text{sinc}(f)$$

$$2\text{rect}(4t) \xrightarrow{F.T} 2 \cdot \frac{1}{4} \text{sinc}\left(\frac{f}{4}\right) \quad (\text{scaling property})$$

$$\therefore 2\text{rect}(4t) \xrightarrow{F.T} \frac{1}{2} \text{sinc}\left(\frac{f}{4}\right)$$

30. (d)

Given, $x(t) = 2 + \cos(50\pi t)$

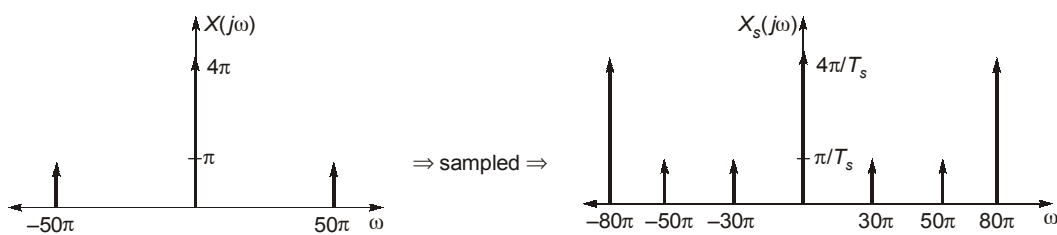
Frequency of signal $\omega_{\text{sig}} = 50\pi$
 $T_s = 0.025 \text{ sec}$

$\therefore$ sampling frequency $\omega_s = \frac{2\pi}{T_s} = 80\pi \text{ rad/sec}$

then, $X(j\omega) = 4\pi\delta(\omega) + \pi[\delta(\omega + 50\pi) + \delta(\omega - 50\pi)]$

Let the sampled signal be represented as $X_s(j\omega)$, where $X_s(j\omega)$ is given as

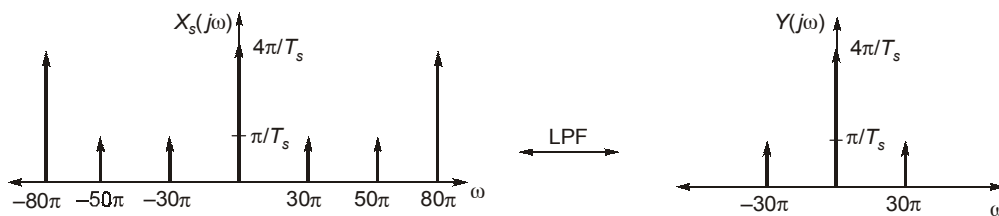
$$X_s(j\omega) = \frac{1}{T_s} \sum_{m=-\infty}^{\infty} X(j(\omega - m\omega_s))$$



$$X_s(j\omega) = 40 \sum_{m=-\infty}^{\infty} [4\pi\delta(\omega - 80\pi m) + \pi\delta(\omega - 50\pi - 80\pi m) - \pi\delta(\omega + 50\pi - 80\pi m)]$$

now, the sampled input $X_s(j\omega)$ is passed through a low passed filter having cut-off frequency at $\omega = 40\pi$.

Therefore the output $Y(j\omega)$ will contain only the components which are less than $\omega = 40\pi$.



Now by putting $T_s = 0.025$, we will get

