CLASS TEST

S.No.:03DLSK_CDEFGH_100926


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# ENGINEERING MATHEMATICS

## EC & EE

Date of Test : 10/09/2026

## ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (b) | 7. | (a) | 13. | (a) | 19. | (c) | 25. | (b) |
| 2. | (d) | 8. | (a) | 14. | (a) | 20. | (d) | 26. | (b) |
| 3. | (d) | 9. | (c) | 15. | (d) | 21. | (c) | 27. | (c) |
| 4. | (b) | 10. | (c) | 16. | (b) | 22. | (d) | 28. | (c) |
| 5. | (a) | 11. | (c) | 17. | (b) | 23. | (b) | 29. | (d) |
| 6. | (a) | 12. | (a) | 18. | (c) | 24. | (c) | 30. | (a) |

## DETAILED EXPLANATIONS

**1.** **(b)**

$$f(x) = \frac{1}{15-0} = \frac{1}{15}$$

$$P\{5 < x < 9\} = \int_5^9 \frac{1}{15}dx = \frac{4}{15}$$

**2.** **(d)**

Given
$$f(x) = 2x^2 - 5x - 6$$
$$f'(x) = 4x - 5$$
$$f''(x) = 4$$

For minima/maxima,
$$f'(x) = 0$$
$$4x - 5 = 0$$
$$x = \frac{5}{4}$$
$$f''(x) = 4 > 0 \Rightarrow \text{Minima}$$

**3.** **(d)**

$$y = \frac{2}{3}\frac{\ln x}{x}$$

$$\frac{dy}{dx} = \frac{2}{3x}\cdot\frac{1}{x} + \frac{2}{3}\ln x\left(\frac{-1}{x^2}\right) = \frac{2}{3x^2}(1-\ln x)$$

For maxima,
$$\frac{dy}{dx} = 0$$
$$\ln x = 1 \quad \Rightarrow \quad x = e \text{ is a stationary point}$$
$$\frac{d^2y}{dx^2} = \frac{-2}{3x^3}(3-2\ln x)$$

At $x = e$
$$\left(\frac{d^2y}{dx^2}\right)_{x=e} = \frac{-2}{3e^3} < 0$$

Hence maxima at $x = e$

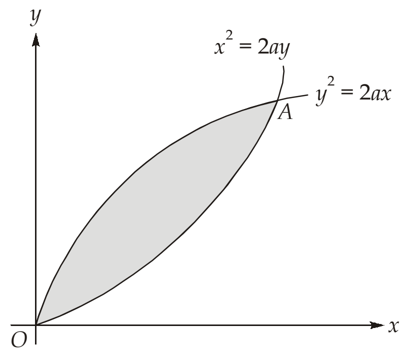
**4.** **(b)**

$$y^2 = 2ax$$
$$x = \frac{y^2}{2a} \qquad \ldots(i)$$
$$x^2 = 2ay$$

Using equation (i)

$$\frac{y^4}{4a^2} = 2ay$$
$$y^4 - 8a^3y = 0$$
$$y = 0, y = 2a$$

The parabolas intersect at 0 (0, 0) and $A$ $(2a, 2a)$




$$\text{Required area} = \int_0^{2a}\int_{x^2/2a}^{\sqrt{2ax}} dy\,dx$$
$$= \int_0^{2a}\left(\sqrt{2ax} - \frac{x^2}{2a}\right)dx$$
$$= \left|\sqrt{2a}\,\frac{2}{3}x^{3/2} - \frac{1}{2a}\cdot\frac{x^3}{3}\right|_0^{2a}$$
$$= \sqrt{2a}\,\frac{2}{3}(2a)^{3/2} - \frac{1}{2a}\,\frac{(2a)^3}{3} = \frac{8a^2}{3} - \frac{4a^2}{3} = \frac{4a^2}{3}$$

**5. (a)**

$$\vec{a}\times(\vec{b}\times\vec{c}) = (a\cdot c)b - (a\cdot b)c$$

**6. (a)**

$$P(F) = 0.3$$
$$P(G) = 0.4$$
$$P(F\cap G) = 0.2$$

Now, $\quad P(F\cup G) = P(F) + P(G) - P(F\cap G)$

$$= 0.3 + 0.4 - 0.2 = 0.5$$

$\therefore \quad P(F\cup G) = 0.5$

**7. (a)**

The auxilary equation is

$$2\lambda^2 + 8 = 0$$
$$\lambda = \pm 2\,i$$

Therefore the complimentary function,

$$y(t) = c_1\cos 2t + c_2\sin 2t$$
$$y'(t) = -2c_1\sin 2t + 2c_2\cos 2t$$

But we have,

$$y(0) = 0,$$
$$y'(0) = 10$$

Therefore, we get

$$c_1 = 0 \text{ and } c_2 = 5$$

Therefore the solution is

$$y(t) = 5 \sin 2t$$

So, $$y(1) = 5 \sin 2$$

**8. (a)**

The valid equality for an orthogonal matrix $Q$ would be

$$Q^T = Q^{-1}$$

**9. (c)**

Given functions are

$$f(x\ ,\ y) = x^3 - 3xy^2$$

$$g(x, y) = 3x^2\, y - y^2$$

So, $$\frac{\partial f}{\partial x} = 3x^2 - 3y^2$$

and $$\frac{\partial g}{\partial y} = 3x^2 - 2y$$

$\therefore$ $$\frac{\partial f}{\partial y} = -6\, xy$$

$$\frac{\partial g}{\partial x} = 6\, xy$$

$\therefore$ $$\frac{\partial f}{\partial y} = -\frac{\partial g}{\partial x}$$

**10. (c)**

Given function, $$f(x) = (K^2 - 4)x^2 + 6x^3 + 8x^4$$

$$f'(x) = 32x^3 + 18x^2 + 2(K^2 - 4)x$$

$$f''(x) = 96x^2 + 36x + 2(K^2 - 4)$$

For maxima, $$f''(0) = 2(K^2 - 4) < 0$$

$\Rightarrow$ $$K^2 - 4 < 0$$

$\Rightarrow$ $$K^2 < 4$$

$\Rightarrow$ $$-2 < K < 2$$

**11. (c)**

The given equation will be consistent, if

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ 0 & \lambda - 3 & 3 - \lambda \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$$C_3 \rightarrow C_3 + C_2$$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 5\lambda+1 \\ 0 & \lambda-3 & 0 \\ 2 & 3\lambda+1 & 6\lambda-2 \end{vmatrix} = 0$$

$$(\lambda-3)\begin{vmatrix} \lambda-1 & 5\lambda+1 \\ 2 & 2(3\lambda-1) \end{vmatrix} = 0$$

$$2(\lambda - 3)[(\lambda - 1)(3\lambda - 1)-(5\lambda + 1] = 0$$

$$6\lambda(\lambda - 3)^2 = 0$$

$$\lambda = 0 \text{ or } 3$$

The largest value $\lambda = 3$

**12.** **(a)**

Parabola given : $x^2 = 4y$ ...(i)

Straight line is $x - 2y + 4 = 0$

$$y = \frac{x+4}{2}, \text{ put in (i)}$$

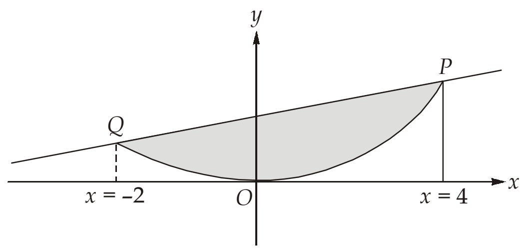
$\Rightarrow$ $x^2 = 2(x + 4)$

$\Rightarrow$ $x^2 - 2x - 8 = 0$

$\Rightarrow$ $x^2 - 4x + 2x - 8 = 0$

$\Rightarrow$ $x(x - 4) + 2(x - 4) = 0$

$\Rightarrow$ $x = 4, -2$




Required area $= POQ$

$$= \int_{-2}^{4} y\,dx \text{ from straight line} - \int_{-2}^{4} y\,dx \text{ from parabola}$$

$$= \int_{-2}^{4}\left(\frac{x+4}{2}\right)dx - \int_{-2}^{4}\frac{x^2}{4}dx = \frac{1}{2}\left|\frac{x^2}{2}+4x\right|_{-2}^{4} - \frac{1}{4}\left|\frac{x^3}{3}\right|_{-2}^{4}$$

$$= \frac{1}{2}\{8+16-(-6)\} - \frac{1}{12}(64+8) = \frac{1}{2}\times 30 - \frac{1}{12}\times 72 = 15 - 6 = 9 \text{ unit}^2$$

**13.** **(a)**

For $f(x)$ to be probability density function $= \int_{-\infty}^{\infty} f(x)dx = 1$

$$\frac{1}{A}\int_{3}^{6}(3x+5)\,dx = 1 \Rightarrow \frac{1}{A}\left|\frac{3x^2}{2}+5x\right|_{3}^{6} = 1$$

$$A = \left(\frac{3}{2}\cdot 6^2 - \frac{3}{2}\cdot 3^2\right) + 5(6-3) = \frac{3}{2}\cdot 27 + 15 = 55.5$$

**14. (a)**

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 2 & 6 \\ 2 & 10-\lambda & 2 \\ 6 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 14\lambda^2 + 288 = 0$$

From here

$$p+q+r = 14$$
$$pq+qr+rp = 0$$
$$pqr = -288$$
$$pq+qr+rp-pqr = 0-(-288) = 288$$

**15. (d)**

$$f_x(X) = \begin{cases} 1 & 0<x<1 \\ 0 & \text{Otherwise} \end{cases}$$

$$E(X^4) = \int_{-\infty}^{\infty} x^4 \cdot f_x(X)\,dx$$

$$= \int_0^1 x^4 = \left.\frac{x^5}{5}\right|_0^1 = \frac{1}{5} = 0.2$$

**16. (b)**

Since the probability of occurrence is very small, this follows Poisson distribution.

$$\text{mean} = m = np$$
$$= 1500 \times 0.002$$
$$= 3$$

Probability that more than 2 will get a hanging problem

$$= 1 - P(0) - P(1) - P(2)$$

$$= 1 - \left[e^{-m} + \frac{e^{-m}\cdot m^1}{1!} + \frac{e^{-m}\cdot m^2}{2!}\right]$$

$$= 1 - \left[e^{-3} + \frac{e^{-3}\cdot 3}{1} + \frac{e^{-3}\cdot 3^2}{2}\right] = 1 - \left[\frac{1}{e^3} + \frac{3}{e^3} + \frac{9/2}{e^3}\right] = 1 - \frac{17}{2e^3}$$

**17. (b)**

Rearranging the equation,

$$\frac{dy}{dx} - \frac{y}{(2x+1)} = e^{4x}(2x+1)$$

The equation is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$IF = e^{\int P(x)dx} = e^{\int \frac{-1}{2x+1}dx}$$

$$= e^{-\frac{\ln(2x+1)}{2}} = \frac{1}{\sqrt{2x+1}}$$

**18. (c)**

$$\text{Output produced by } A = 40\%$$
$$\therefore \quad P(A) = 0.4$$
$$\text{Output produced by } B = 60\%$$
$$\therefore \quad P(B) = 0.6$$

Let, $P\left(\frac{D}{A}\right)$ = probability that item produced by $A$ is defective

$$\therefore \quad P\left(\frac{D}{A}\right) = \frac{9}{1000} = 0.009$$

similarly, $P\left(\frac{D}{B}\right) = \frac{1}{250} = 0.004$

$P\left(\frac{A}{D}\right)$ = Probability that product is produced by $A$ given that it is defective.

$$P\left(\frac{A}{D}\right) = \frac{P(A) \times P\left(\frac{D}{A}\right)}{P(A) \times P\left(\frac{D}{A}\right) + P(B) \times P\left(\frac{D}{B}\right)}$$

$$= \frac{0.4 \times 0.009}{0.4 \times 0.009 + 0.6 \times 0.004}$$

$$P\left(\frac{A}{D}\right) = \frac{0.0036}{0.0036 + 0.0024} = \frac{0.0036}{0.006} = 0.6$$

$$\therefore \quad P\left(\frac{A}{D}\right) = 0.6$$

**19. (c)**

$$\text{Volume of solid} = \int_a^b \pi y^2 dx$$

Given $y = \frac{1}{2\sqrt{x}}$

$$\text{Volume of the solid} = \int_3^4 \frac{\pi}{4x} \cdot dx = \frac{\pi}{4}(\ln x)_3^4 = \frac{\pi}{4} \ln\left(\frac{4}{3}\right)$$

**20. (d)**

$$\frac{d^2y}{dx^2} + y = \cos x$$

$$(D^2 + 1)y = \cos x$$

$$PI = \frac{\cos x}{D^2 + 1}$$

Putting $D^2 = -1$

$$PI = \frac{\cos x}{-1+1} \quad [\text{Makes denominator zero}]$$

$\therefore$ Differentiating numerator and denominator

$$PI = x\cdot \frac{\cos x}{2D}$$

$$= \frac{1}{2}x\int \cos x\,dx = \frac{1}{2}x\sin x$$

**21. (c)**

$$\int \vec{F}\cdot\vec{dl} = \int_1^2 2x\,dx = \left[x^2\right]_1^2 = 3$$

**22. (d)**

Angle between the vectors is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = -\frac{3}{4}+\frac{1}{4} = -\frac{1}{2}$$

$$\Rightarrow \qquad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \qquad \theta = 120°$$

**23. (b)**

Probability of defective item, $p = \frac{1}{10}$,

Probability of non defective item,

$$q = \frac{9}{10}$$

$$n = 10,\ \ r = 2$$

$$p(r) = {}^nC_r\, p^r q^{n-r}$$

$$p(2) = {}^{10}C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^{10-2} = \frac{10\times 9}{1\times 2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^8$$

$$= 0.1937$$

**24. (c)**

Let, $\quad r = y - z$ , $s = z - x$, $t = x - y$

$$u = u(r, s, t)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\cdot\frac{dr}{du}+\frac{\partial u}{\partial s}\cdot\frac{ds}{dx}+\frac{\partial u}{\partial t}\cdot\frac{dt}{dx}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z}(0)+\frac{\partial u}{\partial z}(-1)+\frac{\partial u}{\partial t}(1)$$

$$= \frac{\partial u}{\partial t}-\frac{\partial u}{\partial s} \qquad \text{...(i)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}\cdot\frac{dr}{dy}+\frac{\partial u}{\partial s}\cdot\frac{ds}{dy}+\frac{\partial u}{\partial t}\cdot\frac{dt}{dy}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r}(1) + \frac{\partial u}{\partial z}(0) + \frac{\partial u}{\partial t}(-1)$$

$$= \frac{\partial y}{\partial r} - \frac{\partial u}{\partial t} \quad \text{...(ii)}$$

$$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial r}\cdot\frac{dr}{dz} + \frac{\partial u}{\partial s}\cdot\frac{ds}{\partial z} + \frac{\partial u}{\partial t}\cdot\frac{dt}{dz}$$

$$\frac{\partial y}{\partial z} = \frac{\partial u}{\partial r}(-1) + \frac{\partial u}{\partial s}(1) + \frac{\partial u}{\partial t}(0)$$

$$= \frac{\partial u}{\partial s} - \frac{\partial u}{\partial r} \quad \text{...(iii)}$$

By adding equation (i), (ii) and (iii),

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

**25. (b)**

Given, $f(x) = x^3 - 9x^2 + 24x + 10$

$$f'(x) = 3x^2 - 18x + 24 = 0$$

$$x^2 - 6x + 8 = 0$$

$$(x - 4)(x - 2) = 0$$

$$x = 2, 4$$

$$f''(x) = 6x - 18 = 6(x - 3)$$

$$f''(2) = -6 \Rightarrow x = 2 \text{ is local maxima}$$

$$f''(4) = 6 \Rightarrow x = 2 \text{ is local minima}$$

Global maximum value of $f(x) = \max[f(1), f(2), f(6)] = \max[26, 30, 46] = 46$

**26. (b)**

Given, $$I = \int_0^{\pi/2} \sin^4 x \cos^6 x\, dx$$

$$= \frac{(4-1)(4-3)\cdot(6-1)(6-3)(6-5)}{(4+6)(4+6-2)(4+6-4)(4+6-6)(4+6-8)} \times \frac{\pi}{2}$$

$$= \frac{3\cdot 1\cdot 5\cdot 3\cdot 1}{10\cdot 8\cdot 6\cdot 4\cdot 2} \times \frac{\pi}{2} = \frac{45\pi}{7860} = \frac{3\pi}{512}$$

**27. (c)**

$$s = x^2 + y^2 + z^2 = 4$$

$$x^2 + y^2 + z^2 - 4 = 0$$

grad (s) $$\nabla(s)\big|_{(1,1,-1)} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

Maximum value of directional derivative $= |\nabla(s)|$

$$= \sqrt{2^2 + 2^2 + 2^2} = 2\sqrt{3}$$

**28. (c)**

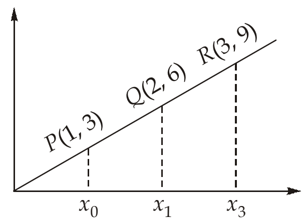
$$I = \oint_c (yzdx + zxdy + xydz)$$

$$\oint_c \vec{F} \cdot dr = \iint_R (\nabla \times \vec{F}) \cdot \hat{n}\, ds$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = \hat{i}(x-x) - \hat{j}(y-y) + \hat{k}(z-z) = 0$$

$$I = 0$$

**29. (d)**




| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $y$ | 3 | 6 | 9 |

$$h = \frac{b-a}{n} = \frac{3-1}{2} = 1$$

Value of integral using trapezoidal rule

$$I = \frac{h}{2}\left[(y_0 + y_2) + 2y_1\right] = \frac{1}{2}\left[(3+9) + 2 \times 6\right]$$

$$I = 12$$

Value of integral using Simpson's 1/3 method

$$I = \frac{h}{3} \times \left[(y_0 + y_2) + 4y_1\right] = \frac{1}{3} \times \left[(3+9) + 4 \times 6\right] = \frac{1}{3}[36] = 12$$

Difference = 12 – 12 = 0

**30. (a)**

In the equation $Mdx + Ndy = 0$

**Theorem-1:**

If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N}$ be a function of $x$ only (say $f(x)$), then $e^{\int f(x)dx}$ is an integrating factor.

**Theorem-2:**

If $\dfrac{\dfrac{\partial N}{\partial x} - \dfrac{\partial M}{\partial y}}{M}$ be a function of $y$ only (say $f(y)$), then $e^{\int f(y)dy}$ is an integrating factor.

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