CLASS TEST

Sl. : 02DLSKCE_DEFGH_300826

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STRENGTH OF MATERIALS

CIVIL ENGINEERING

Date of Test : 30/08/2026

ANSWER KEY

1.	(d)	7.	(a)	13.	(c)	19.	(c)	25.	(b)
2.	(b)	8.	(d)	14.	(b)	20.	(d)	26.	(b)
3.	(d)	9.	(a)	15.	(b)	21.	(b)	27.	(d)
4.	(c)	10.	(d)	16.	(d)	22.	(a)	28.	(d)
5.	(c)	11.	(a)	17.	(c)	23.	(b)	29.	(b)
6.	(b)	12.	(c)	18.	(a)	24.	(c)	30.	(c)

DETAILED EXPLANATIONS

1. (d)

For the given stress condition,

$\sigma_x = +50\ \text{N/mm}^2$, $\sigma_y = 0$, $\tau_{xy} = \pm 20\ \text{N/mm}^2$

Now, principal stresses,

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

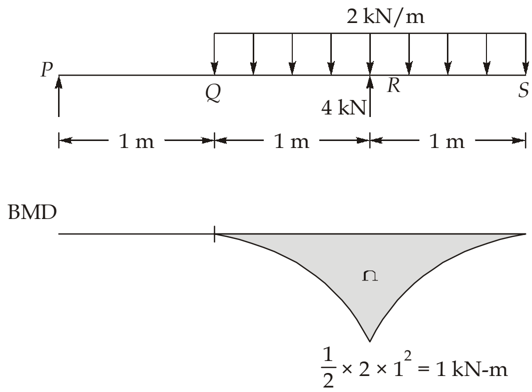
$$= \frac{50+0}{2} \pm \sqrt{\left(\frac{50-0}{2}\right)^2 + 20^2}$$

$$= 57, -7\ \text{N/mm}^2$$

So, $\sigma_1 = 57\ \text{N/mm}^2$ (T)

$\sigma_2 = 7\ \text{N/mm}^2$ (C)

2. (b)



3. (d)

As we know, $\frac{dV}{dx} = w$

$\Rightarrow$ $w = \frac{d}{dx}(5x^2) = 10x$

For midspan, $x = 1$ m

So, load intensity, $w = 10$ N/m

4. (c)

$$\text{Deflection due to load} = \frac{wl^4}{8EI} = \frac{10 \times (3000)^4}{8 \times 5 \times 10^{11}} = 202.5\ \text{mm}$$

Since gap is only 3 mm.

$$\therefore \quad 202.5 - 3 = \frac{Rl^3}{3EI}$$

$$\Rightarrow \quad \frac{(202.5-3)\times 3\times 5\times 10^{11}}{(3000)^3} = R$$

$$\Rightarrow \quad R = 11.083 \text{ kN} \simeq 11.08 \text{ kN}$$

5. (c)

$$\Delta = \frac{PL}{(b_2-b_1)tE}\ln\left(\frac{b_2}{b_1}\right)$$

$$= \frac{PL}{atE}\ln\left(\frac{c}{q}\right)$$

On comparison, $a = b_2 - b_1 = 40 - 20 = 20$ mm

$c = b_2 = 40$ mm

$\Rightarrow \quad a + c = 20 + 40 = 60$ mm

6. (b)

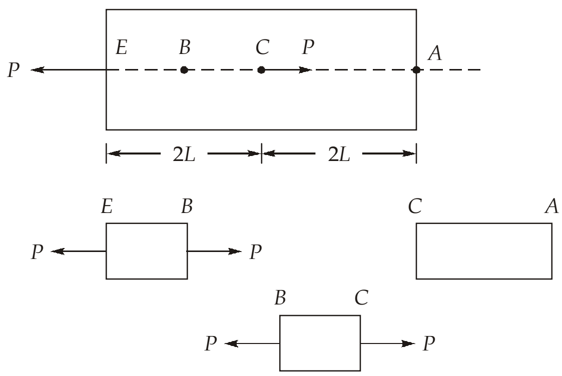
$$\sigma_x = 50 \text{ unit}, \sigma_y = 40 \text{ unit}, \tau_{xy} = 20 \text{ unit}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \quad \sigma_2 = \frac{50+40}{2} - \sqrt{\left(\frac{50-40}{2}\right)^2 + 20^2}$$

$$= 45 - 20.615 = 24.384 \text{ unit}$$

7. (a)



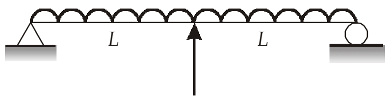
Displacement of point A, $x_A = \delta_A + \delta_C$

$$= \frac{Pl}{AE} + \frac{Pl}{AE} = \frac{2Pl}{AE}$$

Displacement of point B, $x_B = \delta_B = \dfrac{Pl}{AE}$

$\therefore$ Required ratio $= \dfrac{x_A}{x_B} = \dfrac{\frac{2Pl}{AE}}{\frac{Pl}{AE}} = 2$

8. (d)



$$P = F_s = \frac{4wL}{3}$$

$$\left[\frac{5}{384}\frac{(2w)(2L)^4}{EI}\right] - \left[\frac{\left(\frac{4wL}{3}\right)(2L)^3}{48EI}\right] = \delta_{\text{spring}}$$

$$\Rightarrow \quad \delta_{\text{spring}} = \frac{5wL^4}{12EI} - \frac{2wL^4}{9EI} = \frac{7wL^4}{36EI}$$

$$P = F_s = k\delta_{\text{spring}}$$

$$\Rightarrow \quad k = \frac{4wL/3}{\frac{7}{36}\frac{wL^4}{EI}}$$

$$\Rightarrow \quad k = \frac{48}{7}\frac{EI}{L^3}$$

9. (a)

$$\text{Radius of Mohr's circle} = \frac{\phi_{\max}}{2}$$

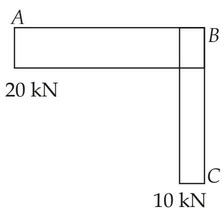
$$\Rightarrow \quad \frac{\phi_{\max}}{2} = \sqrt{\left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$\Rightarrow \quad \frac{1000\mu}{2} = \sqrt{\left(\frac{1000 - 200}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

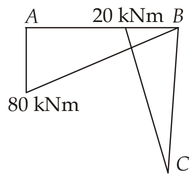
$$\Rightarrow \quad 500^2 = 400^2 + \left(\frac{\phi_{xy}}{2}\right)^2$$

$$\therefore \quad \phi_{xy} = 600\ \mu$$

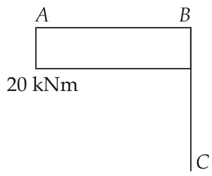
10. (d)



SFD



BMD



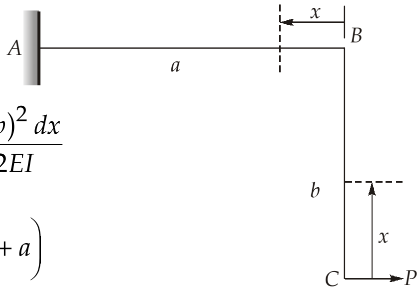
Torque diagram

11. (a)

Moment,

$$M_x = Px \qquad 0 < x < b$$

$$= Pb \qquad 0 < x < a$$



$$\text{Strain energy} = \int \frac{M_x^2 dx}{2EI} = \int_0^b \frac{(Px)^2\, dx}{2EI} + \int_0^a \frac{(Pb)^2\, dx}{2EI}$$

$$= \frac{P^2b^3}{6EI} + \frac{P^2b^2a}{2EI} = \frac{P^2b^2}{2EI}\left(\frac{b}{3} + a\right)$$

12. (c)

$$\text{Deflection due to load} = \frac{wl^4}{8EI} = \frac{10 \times (3000)^4}{8 \times 5 \times 10^{11}} = 202.5 \text{ mm}$$

Since gap is only 3 mm.

$$\therefore \qquad 202.5 - 3 = \frac{Rl^3}{3EI}$$

$$\Rightarrow \qquad \frac{(202.5 - 3) \times 3 \times 5 \times 10^{11}}{(3000)^3} = R$$

$$\Rightarrow \qquad R = 11.083 \text{ kN} \simeq 11.08 \text{ kN}$$

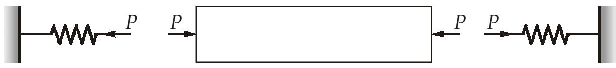
13. (c)

Change in length in dx strip is,

$$\Delta L_{\text{strip}} = dx\alpha T_x = \frac{Tx^4 \alpha dx}{L^4}$$

Total change in length of bar is,

$$\Delta L_{\text{bar}} = \int_0^L \Delta L_{\text{strip}} = \int_0^L \frac{Tx^4}{L^4}\alpha dx = \frac{T}{L^4}\alpha\left[\frac{x^5}{5}\right]_0^L = \frac{T}{L^4}\alpha\frac{L^5}{5} = \frac{T\alpha L}{5}$$



Now, total deformation is 0.

$$\Rightarrow \qquad \left[\frac{T\alpha L}{5} + \left(\frac{-PL}{AE}\right)\right] + \left[\frac{-P}{k_{\text{spring}}}\right] + \left[\frac{-P}{k_{\text{spring}}}\right] = 0$$

$$\Rightarrow \qquad \frac{T\alpha L}{5} = \frac{PL}{AE} + \frac{2P}{k_{\text{spring}}}$$

$$\Rightarrow \qquad \frac{T\alpha L}{5} = \frac{PL}{AE}\left[1 + \frac{2AE}{Lk_{\text{spring}}}\right]$$

$$\Rightarrow \qquad \frac{P}{A} = \frac{E\alpha T}{5\left(1 + \frac{2AE}{Lk_{\text{spring}}}\right)} \qquad \left[\because k_{\text{bar}} = \frac{AE}{L}\right]$$

$$\therefore \qquad \sigma_{\text{Thermal}} = \frac{E\alpha T}{5\left(1 + 2\frac{k_{\text{bar}}}{k_{\text{spring}}}\right)}$$

14. (b)

As it is given that, $\varepsilon = \frac{\sigma}{E} = \frac{y}{R} = 3.0 \times 10^{-5}$

So, $\frac{1}{R} = \frac{3.0 \times 10^{-5}}{30} \text{mm}^{-1} = 10^{-6} \text{mm}^{-1}$

Also, in pure bending, $\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} = \text{constant}$

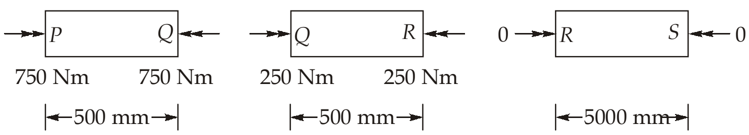
For σ_{max}, y_{max} has to be used

So, $\sigma_{max} = \frac{E}{R} y_{max} = \frac{200 \times 10^3}{R} \text{MPa} \times y_{max}$

$\Rightarrow$ $\sigma_{max} = 200 \times 10^3 \times 10^{-6} \times 50 \text{ MPa}$

$\Rightarrow$ $\sigma_{max} = 10 \text{ MPa}$

15. (b)



Angle of twist $\phi = \frac{Tl}{GJ}$

$$\phi_{PS} = \phi_{PQ} + \phi_{QR} + \phi_{RS}$$

$$= \frac{750 \times 10^3 \times 500}{80 \times 10^3 \times \frac{\pi}{32} \times 50^4} + \frac{250 \times 10^3 \times 500}{80 \times 10^3 \times \frac{\pi}{32} \times 50^4} + 0$$

$$= 10 \times 10^{-4} \times \frac{32}{\pi} \text{ rad} = 0.58°$$

16. (d)

For a thin walled tube, angle of twist

$$\theta = \frac{TL}{4A_m^2 G} \oint \frac{dS}{t} = \frac{TL}{4A_m^2 Gt} \oint dS$$

The integral represents the length around the centreline boundary which is $2\pi r_m$

$$\therefore \quad \theta = \frac{TL \times 2\pi r_m}{4\left(\pi r_m^2\right)^2 Gt} = \frac{TL}{2\pi r_m^3 Gt}$$

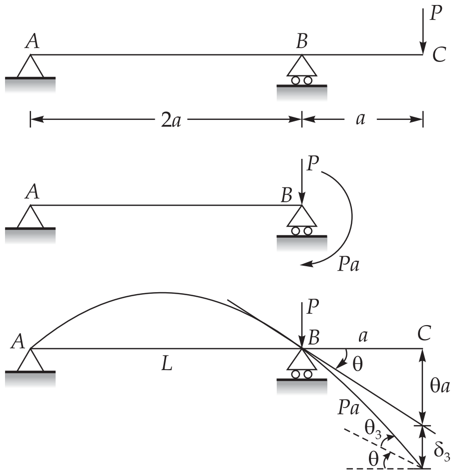
Alternative solution:

$$\frac{\tau}{r_m} = \frac{G\theta}{L} \text{ (From torsion equation)}$$

But, $\tau = \frac{T}{2A_m t}$

$$\theta = \frac{\tau L}{G r_m} = \frac{T}{2A_m t G r_m} L = \frac{TL}{2\left(\pi r_m^2\right) t G r_m} = \frac{TL}{2\pi r_m^3 Gt}$$

17. (c)



$$\theta = \frac{(Pa)L}{3EI}$$

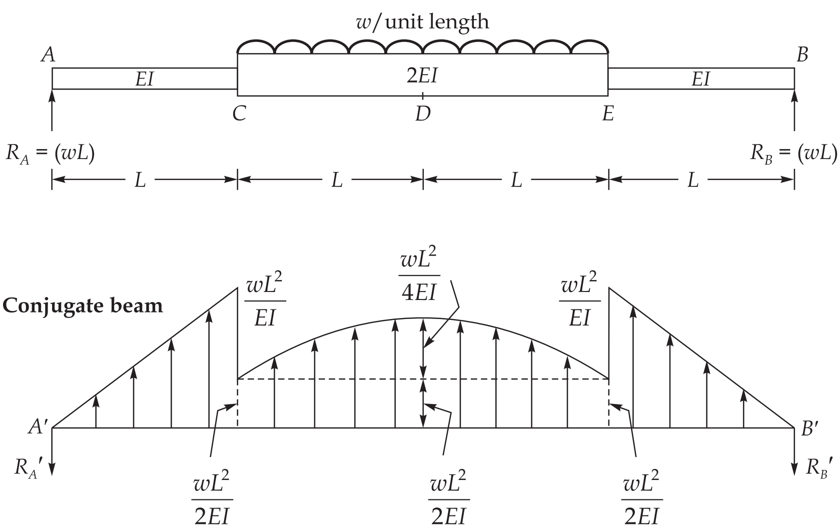
$$\delta_3 = \frac{Pa^3}{3EI}$$

Deflection at free end, $\delta_c = \theta a + \delta_3 = \dfrac{(Pa)(2a)a}{3EI} + \dfrac{Pa^3}{3EI}$

$$= \frac{2Pa^3}{3EI} + \frac{Pa^3}{3EI} = \frac{Pa^3}{EI}$$

18. (a)

Using conjugate beam method.



The reaction at A' and B' are equal due to symmetry.

$$R_A' = \frac{1}{2}\times L\times\frac{wL^2}{EI} + \left(\frac{wL^2}{2EI}\right)\times L + \frac{2}{3}\times\frac{wL^2}{4EI}\times L$$

$$= \frac{7wL^3}{6EI}$$

From conjugate beam 'Theorem-1':

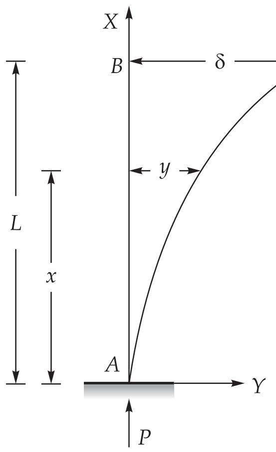
The slope at any point in a real beam will be equal to shear force at the corresponding point in conjugate beam.

$$\therefore \quad \theta_A = \text{SF at } A' \text{ in conjugate beam}$$

$$= -R_A' = -\frac{7wL^3}{6EI}$$

$$= \frac{7wL^3}{6EI}(\text{CW})$$

19. (c)



Given differential equation is:

$$EI\frac{d^2y}{dx^2} = P(\delta - y)$$

$$\Rightarrow \quad EI\frac{d^2y}{dx^2} = P\delta - Py$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{P}{EI}\times y = \frac{P\delta}{EI}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \alpha^2 y = \frac{P\delta}{EI} \qquad \left(\text{where } \alpha^2 = \frac{P}{EI}\right)$$

The solution of the above differential equation

$$y = A\sin\alpha x + B\cos\alpha x + \frac{P\delta}{EI\alpha^2}$$

$$= A\sin\alpha x + B\cos\alpha x + \delta$$

Applying boundary condition.

At $x = 0$; $\quad y = 0$

$$\therefore \quad B = -\delta$$

Also, $$\frac{dy}{dx} = A\alpha\cos\alpha x - B\alpha\sin\alpha x$$

At $x = 0$, $$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 0 = A\alpha - 0$$

$$\Rightarrow \quad A = 0$$

$\therefore \quad y = -\delta\cos\alpha x + \delta = \delta(1-\cos\alpha x)$

Also, at $x = L$, $\quad y = \delta$

$\therefore \quad \delta = \delta(1-\cos\alpha L)$

$\Rightarrow \quad \cos\alpha L = 0$

$\Rightarrow \quad \cos\alpha L = \cos\frac{\pi}{2}$ or $\cos\frac{3\pi}{2}$ or $\cos\frac{5\pi}{2}$ or , ...

$\Rightarrow \quad \alpha L = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ or $\frac{5\pi}{2}$ or , ...

For second critical load, $\quad \alpha L = \frac{3\pi}{2}$

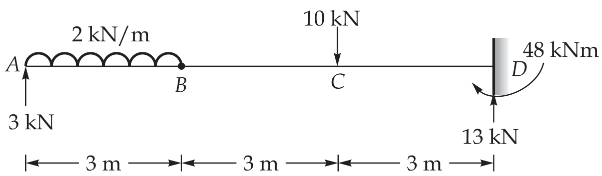
$\Rightarrow \quad \sqrt{\frac{P}{EI}}\times L = \frac{3\pi}{2}$

Squaring both sides, $\quad \frac{P}{EI}\times L^2 = \frac{9\pi^2}{4}$

$\Rightarrow \quad P = \frac{9\pi^2 EI}{4L^2}$

20. (d)

The loading diagram for the given SFD is



$\therefore \quad$ Total downward load $= 10 + 2 \times 3 = 16$ kN

21. (b)

$\because$ The rigid connecting plate remains horizontal

$\therefore$ The strain in all three bars remains same

$\therefore \quad \epsilon_{Cu} = \epsilon_{Zn} = \epsilon_{Al}$

$\Rightarrow \quad \frac{\sigma_{Cu}}{E_{Cu}} = \frac{\sigma_{Zn}}{E_{Zn}} = \frac{\sigma_{Al}}{E_{Al}}$

$\Rightarrow \quad \sigma_{Zn} = \frac{E_{Zn}}{E_{Cu}}\times\sigma_{Cu} = 0.769\sigma_{Cu}$ ...(i)

$\sigma_{Al} = \frac{E_{Al}}{E_{Cu}}\times\sigma_{Cu} = 0.615\sigma_{Cu}$ ...(ii)

Now, $\quad 100\times 10^3 = P_{Cu} + P_{Zn} + A_{Al}$

$\Rightarrow \quad 100\times 10^3 = \sigma_{Cu}A_{cu} + \sigma_{Zn}A_{Zn} + \sigma_{Al}A_{Al}$

$\Rightarrow \quad 100\times 10^3 = \sigma_{Cu}\left[A_{Cu} + 0.769A_{Zn} + 0.615A_{Al}\right]$

$$\Rightarrow \qquad \sigma_{Cu} = \frac{100\times10^3}{[600+0.769\times700+0.615\times800]}$$

$$\sigma_{Cu} = \frac{100\times10^3}{1630.3} = 61.34 \text{ N/mm}^2$$

22. (a)

From figure,

Twisting torque, $T = 1 \times 0.5 + 1 \times 0.3 = 0.8$ kNm

From torsion equation:

$$\frac{T}{I_P} = \frac{G\theta}{L}$$

$$\therefore \qquad \theta = \frac{TL}{GI_P} = \frac{0.8\times10^6\times3000}{28\times10^3\times\frac{\pi}{32}\times\left(150^4-120^4\right)}$$

$$= 2.92\times10^{-3} \text{ rad} = \left(\frac{2.92\times10^{-3}}{\pi}\times180\right)^\circ$$

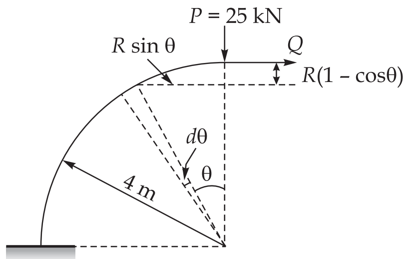
$$= 0.167^\circ \simeq 0.17^\circ$$

23. (b)

Apply a horizontal load (Q) at tip.

$$M_x = PR\sin\theta + QR(1-\cos\theta)$$

$$\therefore \qquad \frac{\partial M_x}{\partial Q} = R(1-\cos\theta)$$



$$\therefore \qquad \Delta_H = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$

$$= \left.\frac{1}{EI}\int M_x\frac{dM_x}{dQ}dx\right|_{Q=0}$$

$$= \left.\frac{1}{EI}\int\left\{\left(PR\sin\theta+QR(1-\cos\theta)\right)R(1-\cos\theta)dx\right\}\right|_{Q=0}$$

$$= \frac{1}{EI}\int_0^{\pi/2}PR\sin\theta R(1-\cos\theta)Rd\theta$$

$$\therefore \qquad \Delta_H = \frac{PR^3}{EI}\int_0^{\pi/2}\sin\theta(1-\cos\theta)d\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}\left(\sin\theta - \frac{\sin 2\theta}{2}\right)d\theta$$

$$= \frac{PR^3}{EI}\left(-\cos\theta + \frac{\cos 2\theta}{4}\right)_0^{\pi/2}$$

$$= \frac{PR^3}{EI}\left(-0 - \frac{1}{4} - \left(-1 + \frac{1}{4}\right)\right)$$

$$= \frac{PR^3}{2EI}$$

Given $P = 25$ kN, $R = 4$ m, $E = 200 \times 10^3$ N/mm^2

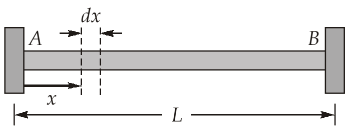
Dia. of cross-section, $d = 150$ mm

$$\therefore \quad \Delta_H = \frac{25\times10^3\times(4000)^3}{2\times200\times10^3\times\frac{\pi}{64}\times(150)^4}$$

$= 160.963$ mm $\simeq 160$ mm (nearest multiple of 10)

24. **(c)**

Consider an element of length 'dx' at a distance x from A,



Increase in length of element dx due to temperature increase, ΔT is given by,

$$d\delta = (dx)\left(\frac{\alpha}{3}\right)(\Delta T)$$

$$\Rightarrow \quad d\delta = dx\left(\frac{\alpha}{3}\right)\frac{\Delta T_1 x^2}{3L^2}$$

$\therefore$ Total elongation of bar,
$$\delta = \int_0^L \frac{\alpha}{3}\frac{\Delta T_1 x^2}{3L^2}dx$$

$$\Rightarrow \quad \delta = \frac{\alpha\Delta T_1}{9L^2}\times\frac{x^3}{3}\Big|_0^L$$

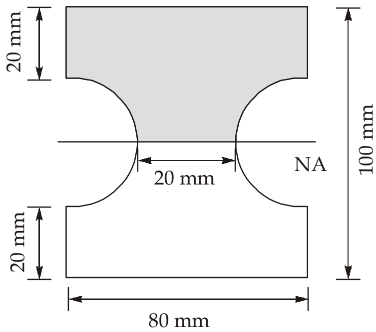
$$\Rightarrow \quad \delta = \frac{\alpha\Delta T_1}{9L^2}\times\frac{L^3}{3}$$

$$\Rightarrow \quad \delta = \frac{\alpha\Delta T_1 L}{27}$$

$$\therefore \quad \text{Compressive stress} = (2E)\times\frac{\delta}{L} = 2E\times\frac{\alpha\Delta T_1}{27} = \frac{2}{27}E\alpha\Delta T_1$$

25. (b)

$$I_{NA} = \left[\frac{80(100)^3}{12}\right] - \left[\frac{\pi}{64}(60)^4\right] = 6.03 \times 10^6\,\text{mm}^4$$



$$Q = \int y dA = y_c A$$

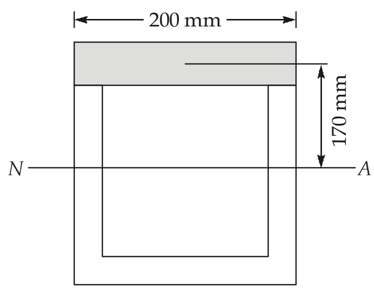
$$= [(80 \times 50 \times 25)] - \left[\frac{\pi}{2}(30)^2 \times \frac{4(30)}{3\pi}\right] = 82000\,\text{mm}^3$$

Now, $$\tau = \frac{VQ}{Ib} = \frac{(20 \times 10^3)(82000)}{(6.03 \times 10^6) \times (20)}$$

$\Rightarrow$ $$\tau = 13.599\text{ MPa} \simeq 13.6\text{ MPa}$$

26. (b)

$$I_{NA} = \frac{200(360)^3}{12} - \frac{160(320)^3}{12} = 340693333.3\text{ mm}^4$$



Shear flow, $$q = \frac{VQ}{I} = \frac{(3.2 \times 10^3)(200 \times 20 \times 170)}{(340693333.3)}$$

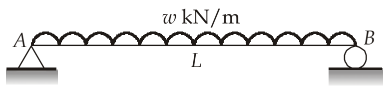
$\Rightarrow$ $$q = 6.387\text{ N/mm}$$

$\therefore$ Shear flow × spacing = 250 × 2 N

$\Rightarrow$ $$\text{Spacing} = \frac{500}{6.387}$$

$\Rightarrow$ Spacing = 78.3 mm c/c

27. (d)

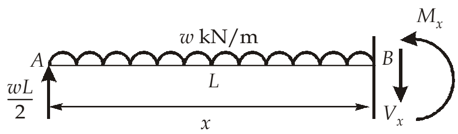


$$y = \frac{1}{EI}\left[2x^3 - \frac{x^4}{6} - 36x\right]$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{EI}\left[6x^2 - \frac{1}{6}4x^3 - 36\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left[12x - \frac{12x^2}{6}\right]$$

$$\therefore \quad M_x = EI\frac{d^2y}{dx^2} = 12x - 2x^2 \qquad \text{...(1)}$$



$$\left(\frac{wL}{2}x\right) + \left(-wx \times \frac{x}{2}\right) + (-M_x) = 0$$

$$\Rightarrow \quad M_x = \frac{wL}{2}x - \frac{wx^2}{2} \qquad \text{...(2)}$$

By comparing (1) and (2),

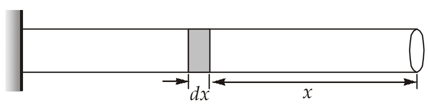
$$\frac{w}{2} = 2$$

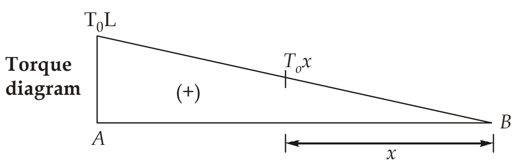
$$\Rightarrow \quad w = 4 \text{ kN/m}$$

$$\frac{wL}{2} = 12$$

$$\Rightarrow \quad L = 6 \text{ m}$$

28. (d)





$$U_{\text{Total}} = \int^{A} dU_{\text{strip}} = \int_0^L \frac{T_x^2\,dx}{2GJ}$$

$$\Rightarrow \qquad U_{\text{Total}} = \int_0^L \frac{T_0^2 x^2}{2GJ} dx$$

$$\Rightarrow \qquad U_{\text{total}} = \frac{T_0^2}{2GJ} \cdot \frac{L^3}{3} = \frac{T_0^2 L^3}{6GJ} = 0.167 \frac{T_0^2 L^3}{GJ}$$

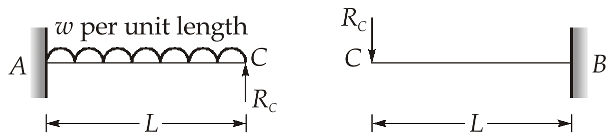
29. **(b)**

After perfect plastic yielding, if the load is further increased, the steel begins to strain harden. During strain hardening, the material appears to regain some of its strength and offers more resistance, thus requiring increased tensile load for further deformation. This is so because the material undergoes changes in its atomic and crystalline structure in the strain hardening region.

30. **(c)**

We know that at internal hinge deflection will be same at just left and right of hinge.

So,



$$(\delta_c)_{\text{left}} \downarrow = \frac{wL^4}{8EI} - \frac{R_c L^3}{3EI} \qquad \ldots\text{(i)}$$

$$(\delta_c)_{\text{right}} \downarrow = \frac{R_c L^3}{3EI} \qquad \ldots\text{(ii)}$$

So from eq. (i) and (ii)

$$\frac{wL^4}{8EI} - \frac{R_c L^3}{3EI} = \frac{R_c L^3}{3EI}$$

$$\Rightarrow \qquad \frac{2R_c L^3}{3EI} = \frac{wL^4}{8EI}$$

$$\Rightarrow \qquad R_c = \frac{3}{16} wL$$

■■■■