CLASS TEST

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ENGINEERING MATHEMATICS

CE | ME

Date of Test : 19/06/2026

ANSWER KEY

1.	(b)	7.	(a)	13.	(c)	19.	(b)	25.	(a)
2.	(d)	8.	(d)	14.	(d)	20.	(a)	26.	(a)
3.	(a)	9.	(c)	15.	(a)	21.	(d)	27.	(a)
4.	(a)	10.	(d)	16.	(a)	22.	(a)	28.	(b)
5.	(c)	11.	(d)	17.	(b)	23.	(b)	29.	(b)
6.	(c)	12.	(a)	18.	(a)	24.	(c)	30.	(a)

DETAILED EXPLANATIONS

1. (b)

Given matrix,
$$A = \begin{bmatrix} 21 & 17 & 7 & 10 \\ 24 & 22 & 6 & 10 \\ 6 & 8 & 2 & 3 \\ 5 & 7 & 1 & 2 \end{bmatrix}$$

Operating, $R_1 \to R_1 - R_2 - R_4$, $R_2 \to R_2 - 3R_3$
and $R_3 \to R_3 - 2R_4$

$$A = \begin{bmatrix} -8 & -12 & 0 & -2 \\ 6 & -2 & 0 & 1 \\ -4 & -6 & 0 & -1 \\ 5 & 7 & 1 & 2 \end{bmatrix}$$

$$|A| = -\begin{vmatrix} -8 & -12 & -2 \\ 6 & -2 & 1 \\ -4 & -6 & -1 \end{vmatrix} = 0$$

2. (d)

The characteristic equation of the matrix A is

$$\begin{bmatrix} (2-\lambda) & 1 & 1 \\ 0 & (1-\lambda) & 0 \\ 1 & 1 & (2-\lambda) \end{bmatrix} = 0$$

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

According to Cayley-Hamilton theorem, we have

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + A^2 + A + I$$
$$= A^2 + A + I$$

$$A^2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix}$$

$$A^2 + A + I = \begin{bmatrix} 5 & 4 & 4 \\ 0 & 1 & 0 \\ 4 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

3. (a)

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(2-\lambda)(5-\lambda) = 0$$
$$\lambda = 2, 3, 5$$

4. (a)

For the equation $\frac{dy}{dx} + Py = Q$ the integrating factor is $e^{\int P dx}$,

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$
$$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$
$$P = \frac{1}{\cos^2 x} = \sec^2 x$$
$$\text{Integrating factor} = e^{\int \sec^2 x\, dx} = e^{\tan x}$$

5. (c)

$$f(u) = \sin u = \frac{x + 2y + 3z}{x^8 + y^8 + z^8}$$
$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = (-7)\frac{f(u)}{f'(u)}$$
$$= -7\frac{\sin u}{\cos u} = -7 \tan u$$

6. (c)

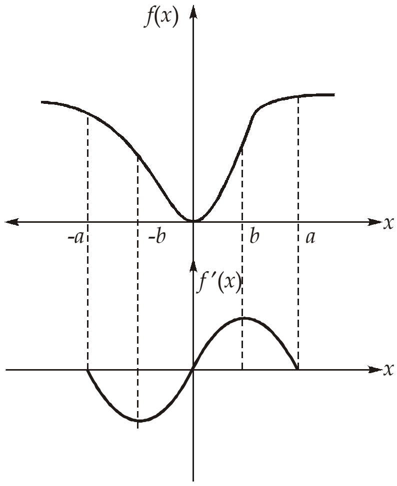
Gauss-Seidel iterative techniques are used to solve system of simultaneous linear algebraic equation.

7. (a)

For the equation $\frac{dy}{dx} + Py = Q$ the integrating factor is $e^{\int P dx}$,

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$
$$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$
$$P = \frac{1}{\cos^2 x} = \sec^2 x$$
$$\text{Integrating factor} = e^{\int \sec^2 x\, dx} = e^{\tan x}$$

8. (d)



i) From (–∞, a), $f(x)$ is constant $\Rightarrow f'(x) = 0$

ii) From (–a, –b), $f(x)$ decrases and also rate of decrement increases $\Rightarrow f'(x)$ increases with negative value.

iii) From (–b, 0), $f(x)$ decrease but rate of decrement decrease $\Rightarrow f'(x)$ decrease but remains negative.

from (2, 0), $f(x)$ increases with rate of increment increases

So

$f'(x)$ decreases but remains positive.

9. (c)

Given,
$$Z = \frac{1+i}{\sqrt{2}}$$
$$|Z| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

10. (d)

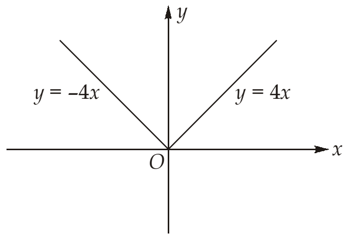
For solving the algebraic equation of the form $f(x) = 0$, the iterative scheme by Newton-Raphson methods,
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Here,
$$f(x) = x - \cos(\pi x)$$
$$f'(x) = 1 + \pi \sin(\pi x)$$

Therefore, with $x_0 = 0.5$, we get
$$f(x) = x - \cos(\pi x)$$
$$f(0.5) = 0.5$$
$$f'(x) = 1 + \pi \sin(\pi x)$$
$$f'(0.5) = 4.1416$$
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = 0.5 - \frac{0.5}{4.1416}$$
$$= 0.3793 \approx 0.38$$

11. (d)

As the graph of function $y = 4|x|$ is having sharp corner at $x = 0$.



So, it indicates that function $y = 4|x|$ is continuous but not differentiable at $x = 0$.

12. (a)

For the differential equation of the form $\frac{dy}{dx} = f(x, y)$. Solution by Euler's method is given as :

$$y_{k+1} = y_k + hf(x_k, y_k)$$

Given : Step size $h = 0.1$ and $y(0) = 1$.

And $$\frac{dy}{dx} = f(x, y) = x^2y - 1.2y. \text{ Therefore,}$$

$$y_{x=0.1} = y_1 = y_0 + h(x_0^2 y_0 - 1.2y_0)$$
$$= 1 + 0.1(0^2 \times 1 - 1.2 \times 1)$$
$$= 0.880$$

13. (c)

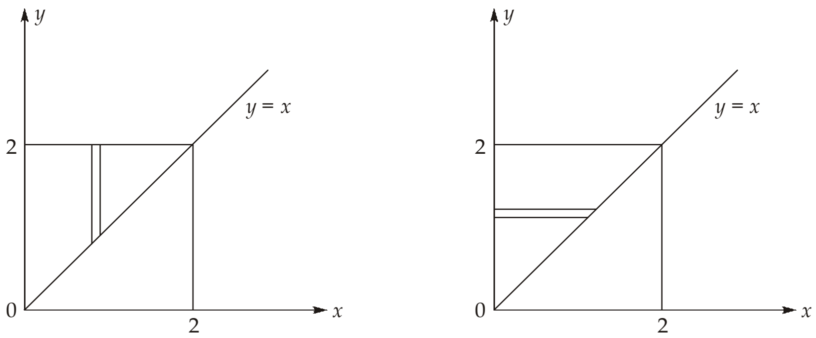
$$\text{Mean} = \frac{\text{Total marks obtained by all students}}{\text{Number of students}}$$
$$= \frac{25 \times 20 + 30 \times 20 + 35 \times 40 + 40 \times 20}{100}$$
$$= \frac{3300}{100} = 33$$

Mode is the value that occurs most frequently. Hence, frequency of 35 is more so mode is 35.

14. (d)

$$A^{-1} = \frac{(\text{adj } A)}{|A|}$$
$$|A| = -6 \times 3 = -18$$
$$|A|.(A^{-1}) = (\text{adj. } A)$$
$$\lambda \text{ of adj. } A = \frac{|A|}{\lambda_1}, \frac{|A|}{\lambda_2} = \frac{-18}{-6}, \frac{-18}{3}$$
$$= 3, -6$$

15. (a)



$x < y < 2$ $\quad$ $0 < x < y$
$0 < x < 2$ $\quad$ $0 < y < 2$

$$I = \int_0^2 \int_0^y f(x,y)\,dx\,dy$$

$$r = p = 0,\ q = y \text{ and } s = 2$$

16. (a)

The directional derivative of f in the direction of $\vec{n}$ is given by $\hat{n}\cdot\nabla f$, where $\hat{n}$ is the unit vector of n. Here,

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{\vec{i}-\vec{k}}{\sqrt{1^2+(-1)^2}} = \frac{1}{\sqrt{2}}(\vec{i}-\vec{k})$$

and $\quad f = x^2 + y^2 + z^2$

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

Therefore, the directional derivative is

$$\hat{n}\cdot\nabla f = \frac{1}{\sqrt{2}}(\vec{i}-\vec{k})\cdot(2x\hat{i}+2y\hat{j}+2z\hat{k})$$

Since, $\quad \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

and $\quad \hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0,$

We get $\quad \hat{n}\cdot\nabla f = \frac{1}{\sqrt{2}}(2x - 2z)$

$$(\hat{n}\cdot\nabla f) \text{ at } (1, 1, 1) = \frac{1}{\sqrt{2}}(2\times 1 - 2\times 1) = 0$$

17. (b)

Probability that A speaks truth $P(A) = 0.7$
Probability that B speaks truth $P(B) = 0.9$

$\therefore \quad P(\bar{A}) = 0.3, \quad P(\bar{B}) = 0.1$

Hence, the probability that they will contradict to each other

$$= P(\bar{A})P(B) + P(A)P(\bar{B})$$

$$= 0.3 \times 0.9 + 0.7 \times 0.1$$
$$= 0.27 + 0.07 = 0.34$$
$$= 34\%$$

18. (a)

$$\text{Work done} = \int_C \vec{F}.d\vec{r} = \int\left(yz\hat{i} + zx\hat{j} + xy\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$
$$= \int_C yzdx + zx\,dy + xydz$$
$$= \int_C d(xyz) = \int_{(1,1,1)}^{(3,3,2)} d(xyz)$$
$$= [xyz]_{(1,1,1)}^{(3,3,2)}$$
$$= 3 \times 3 \times 2 - 1 \times 1 \times 1$$
$$= 17$$

19. (b)

If θ be the temperature of the body at any time t, then

$$\frac{d\theta}{dt} = -k(\theta - 40)$$

where k is a constant.

Integrating, $$\int \frac{d\theta}{\theta - 40} = -k\int dt + \ln c$$ where c is a constant.

$$\ln(\theta - 40) = -kt + \ln c$$

i.e., $$\theta - 40 = ce^{-kt} \qquad \text{...(i)}$$

When $t = 0$, $\theta = 80°$ and when $t = 20$, $\theta = 60°$.

$\therefore$ $40 = c$ and $20 = ce^{-20k}$; $k = \frac{1}{20}\ln 2$

Thus, (i) becomes $$\theta - 40 = 40.e^{-\left(\frac{1}{20}\ln 2\right)t}$$

When $t = 40$ min, $$\theta = 40 + 40\,e^{-2\ln 2} = 40 + 40e^{\ln\left(\frac{1}{4}\right)}$$
$$= 40 + 40 \times \frac{1}{4}$$
$$= 50°\text{C}$$

20. (a)

Given equation can be rewritten as

$$\sin\theta\frac{d\theta}{dr} + \frac{1}{r}(1 - 2r^2)\cos\theta = -r^2 \qquad \text{...(i)}$$

Put $\cos\theta = y$

So that $$-\sin\theta\frac{d\theta}{dr} = \frac{dy}{dr}$$

Then (i) becomes

$$\frac{dy}{dr} + \left(2r - \frac{1}{r}\right)y = r^2$$

which is a Leibnitz's equation.

$$\therefore \quad \text{I.F.} = e^{\int\left(2r-\frac{1}{r}\right)dr} = e^{r^2 - \ln r} = \frac{1}{r}e^{r^2}$$

Thus, its solution is

$$y\left(\frac{1}{r}e^{r^2}\right) = \int r^2 \cdot e^{r^2} \cdot \frac{1}{r}dr + c$$

or
$$\frac{ye^{r^2}}{r} = \frac{1}{2}\int e^{r^2}\, 2r dr + c = \frac{1}{2}e^{r^2} + c$$

or
$$2e^{r^2}\cos\theta = re^{r^2} + 2cr$$

$$r(1 + 2ce^{-r^2}) = 2\cos\theta$$

21. (d)

Given:
$$P = \begin{bmatrix} x & y \\ z & w \end{bmatrix};\ Q = \begin{bmatrix} x^2+y^2 & xz+yw \\ xz+yw & z^2+w^2 \end{bmatrix}$$

$$PP^T = \begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix}^T$$

$$= \begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} x & z \\ y & w \end{bmatrix}$$

$$= \begin{bmatrix} x^2+y^2 & xz+yw \\ xz+yw & z^2+w^2 \end{bmatrix}$$

$\because \quad Q = PP^T$

and $\quad r(P) = n$

Then, $\quad r(Q) = \min\{r(P), r(P^T)\}$

$\Rightarrow \quad r(Q) = \min(n, n) \because [r(P) = r(P^T)]$

$\Rightarrow \quad r(Q) = n$

22. (a)

$$(y+x)^2\frac{dy}{dx} = a^2$$

Put $\quad y + x = z$

Then,
$$\frac{dy}{dx} + 1 = \frac{dz}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dz}{dx} - 1$$

$$\therefore \quad z^2\left(\frac{dz}{dx} - 1\right) = a^2$$

$$\Rightarrow \quad \frac{dz}{dx} - 1 = \frac{a^2}{z^2}$$

$$\Rightarrow \quad \frac{dz}{dx} = 1 + \frac{a^2}{z^2}$$

$$\Rightarrow \quad \int \frac{dz}{1+\frac{a^2}{z^2}} = \int dx$$

$$\Rightarrow \quad \int \left(\frac{z^2 dz}{z^2+a^2}\right) = \int dx$$

$$\Rightarrow \quad \int \frac{z^2+a^2-a^2}{z^2+a^2} dz = \int dx$$

$$\Rightarrow \quad \int dz - \int \frac{a^2}{z^2+a^2} dz = \int dx$$

$$\Rightarrow \quad z - a^2 \times \frac{1}{a} \tan^{-1}\left(\frac{z}{a}\right) = x + c$$

$$\Rightarrow \quad z - a\tan^{-1}\left(\frac{z}{a}\right) = x + c$$

$$\Rightarrow \quad y + x - a\tan^{-1}\left(\frac{y+x}{a}\right) = x + c$$

$$\Rightarrow \quad y - c = a\tan^{-1}\left(\frac{y+x}{a}\right)$$

$$\Rightarrow \quad \tan\left(\frac{y-c}{a}\right) = \frac{y+x}{a}$$

$$\Rightarrow \quad y + x = a\tan\left(\frac{y-c}{a}\right)$$

23. (b)

Given, DE is $(D^2+2D+2)y = 0$

It's A.E. is $\quad m^2+2m+2 = 0$

$$\therefore \quad m = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$$

It's C.F. is $\quad y = e^{-x}(C_1 \cos x + C_2 \sin x)$

Given: $y(0) = 0$

$$\therefore \quad 0 = e^{-0}(C_1 \times 1 + C_2 \times 0)$$

$$C_1 = 0$$

$$\therefore \quad y = C_2 e^{-x} \sin x$$

$$\therefore \quad y'(x) = -C_2 e^{-x} \sin x + C_2 e^{-x} \cos x$$

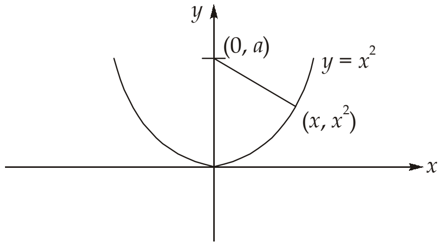
Given: $y'(0) = 1$

$$\therefore \quad 1 = -C_2 \times 0 + C_2 \times 1$$

$$\Rightarrow \quad C_2 = 1$$

Hence, $\quad y = e^{-x} \sin x$

24. (c)



Let there is a point (x, x^2) on the parabola and distance of this point from $(0, a)$ is:

$$D = \sqrt{(x-0)^2 + (x^2 - a)^2}$$

$$\Rightarrow \quad D^2 = x^2 + (x^2 - a)^2$$

When D is minimum, D^2 will also be minimum

So, $$\frac{d(D^2)}{dx} = 0$$

$$\Rightarrow \quad 2x\left[1 + 2(x^2 - a)\right] = 0$$

$$\therefore x = 0, \text{ and } 1 + 2(x^2 - a) = 0$$

$$\Rightarrow \quad x = 0 \text{ and } x^2 = \left(\frac{2a-1}{2}\right)$$

$\because$ For minimum distance, $\frac{d^2(D^2)}{dx^2}$ should be greater than zero.

$$\therefore \quad \frac{d^2(D^2)}{dx^2} = 2\left[1 + 2(x^2 - a)\right] + 2x[4x]$$

$$\frac{d^2(D^2)}{dx^2} = 2 + 4x^2 - 4a + 8x^2$$

$$\frac{d^2(D^2)}{dx^2} = 2 + 12x^2 - 4a$$

At $x = 0$, $$\frac{d^2(D^2)}{dx^2} = 2 - 4a$$

$$\because \quad a > 1$$

$$\therefore \quad \frac{d^2(D^2)}{dx^2} < 0$$

So, $x = 0$ is having maxima.

At $x^2 = \frac{2a-1}{2}$, $$\frac{d^2(D^2)}{dx^2} = 2 + 12\left(\frac{2a-1}{2}\right) - 4a$$

$$= 2 + 6(2a-1) - 4a$$

$$= 2 + 12a - 6 - 4a$$

$$= 8a-4 = 4(2a-1) > 0$$

Hence, $x = \sqrt{\frac{2a-1}{2}}$ is having minima and minimum distance is

$$D_{min} = \sqrt{x^2 + (x^2-a)^2}$$

$$= \sqrt{\frac{2a-1}{2} + \left(\frac{2a-1}{2} - a\right)^2} = \sqrt{\frac{2a-1}{2} + \frac{1}{4}}$$

$$= \sqrt{\frac{4a-1}{4}} = \frac{\sqrt{4a-1}}{2}$$

25. (a)

Given surfaces are:

Surface-1 = $\phi_1 \Rightarrow ax^2 - byz = (a+2)x$

Normal at surface-1, $\nabla\phi_1 = \frac{\partial}{\partial x}(\phi_1)\hat{i} + \frac{\partial}{\partial y}(\phi_1)\hat{j} + \frac{\partial}{\partial z}(\phi_1)\hat{k}$

$$= [2ax-(a+2)]\hat{i} - bz\hat{j} - by\hat{k}$$

$$\nabla\phi_{1\,(1,\,-1,\,2)} = (2a-a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$

$$= (a-2)\hat{i} - 2b\hat{j} + b\hat{k}$$

Surface-2 = $\phi_2 \Rightarrow 4x^2y + z^3 - 4 = 0$

Normal at surface-2, $\nabla\phi_2 = \frac{\partial}{\partial x}(\phi_2)i + \frac{\partial}{\partial y}(\phi_2)\hat{j} + \frac{\partial}{\partial z}(\phi_2)\hat{k}$

$$\Rightarrow \quad \nabla\phi_2 = 8xy\hat{i} + 4x^2\hat{j} + 3z^2\hat{k}$$

$$\nabla\phi_{2(1,\,-1,\,2)} = -8\hat{i} + 4\hat{j} + 12\hat{k}$$

If surfaces cut orthogonally to each other then their normals also cut orthogonally to each other.

$$\therefore \quad \nabla\phi_1 \cdot \nabla\phi_2 = -8(a-2) - 8b + 12b = 0$$

$$\Rightarrow \quad -8a + 16 + 4b = 0$$

$$\Rightarrow \quad -2a + b + 4 = 0 \quad \text{...(i)}$$

Point (1, –1, 2) also lies on both the surfaces

Hence, $a(1)^2 + b(2) = (a+2)$

$\Rightarrow \quad a + 2b = a + 2$

$\Rightarrow \quad 2b = 2$

$\Rightarrow \quad b = 1$

Put $b = 1$ in equation (i)

$$-2a + (1) + 4 = 0$$

$$\Rightarrow \quad a = \frac{5}{2}$$

26. (a)

$$\text{Mean } (x_1) = 0.5$$

$$\frac{1}{\lambda_1} = 0.5$$

$$\lambda_1 = \frac{1}{0.5} = 2$$

$$\text{Mean } (x_2) = 0.25$$

$$\frac{1}{\lambda_2} = 0.25$$

$$\lambda_2 = \frac{1}{0.25} = 4$$

$$y = \text{mean } (x_1, x_2)$$

$$\text{Mean } (y) = \frac{1}{\lambda_1 + \lambda_2} = \frac{1}{2+4} = \frac{1}{6}$$

27. (a)

Complete Solution CS

$$CS = CF + PI$$

Now Auxilliary equation

$$(D^2 + 4D + 6)y = 0$$

$$\Rightarrow \quad m^2 + 4m + 6 = 0$$

$$\Rightarrow \quad m = -2 \pm \sqrt{2}\, i$$

So $$C.F \rightarrow \left[c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x\right]e^{-2x} \qquad ....(1)$$

Now,

$$PI \rightarrow \frac{3^x}{D^2 + 4D + 6} = \frac{e^{x\ln 3}}{D^2 + 4D + 6}$$

$$\Rightarrow \quad P.I = \frac{e^{x\ln 3}}{(\ln 3)^2 + 4.\ln 3 + 6} = \frac{e^{x\ln 3}}{11.6} = \frac{3^x}{11.6}$$

$$\Rightarrow \quad C.S : y(x) = e^{-2x}\left[c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x\right] + \frac{3^x}{11.6}$$

28. (b)

Let, P_1, P_2, P_3, P_4 be probability of selection in 1st, 2nd, 3rd & 4th attempt respectively,

Now,

$$P_1 = \frac{1}{24};\ P_2 = \frac{1}{24}[1 + 0.5]$$

$$P_2 = \frac{1}{24} \times \frac{3}{2}$$

$$P_3 = \frac{1}{24} \times \frac{3}{2}[1 + 0.5] = \frac{1}{24} \times \left(\frac{3}{2}\right)^2$$

$$P_4 = \frac{1}{24} \times \left(\frac{3}{2}\right)^3$$

Now let A_i be selection in i^{th} attempt & $\overline{A_i}$ be unsuccessful attempt,

So,

$$P_{\text{selection}} = A_1 + \overline{A_1}A_2 + \overline{A_1}\,\overline{A_2}\,A_3 + \overline{A_1}\,\overline{A_2}\,\overline{A_3}\,A_4$$

$$= \frac{1}{24} + \frac{23}{24} \times \frac{1}{24} \times \frac{3}{2} + \frac{23}{24}\left(1 - \frac{3}{48}\right) \times \frac{1}{24} \times \left(\frac{3}{2}\right)^2 + \frac{23}{24}\left(1 - \frac{3}{48}\right)$$

$$\left(1 - \frac{9}{96}\right) . \frac{1}{24} \times \left(\frac{3}{2}\right)^3 = 0.3$$

29. (b)

To get ABC their are two ways,

i) $(AB)C$

Now, Number of multiplications in $AB = 2 \times 3 \times 4 = 24$

Now, $ABC = (AB)_{2\times4}\, C_{4\times2}$

Number of multiplication for $(AB)C = 2 \times 4 \times 2 = 16$

$\Rightarrow$ Total multiplication $= 24 + 16 = 40$

ii) $A(BC)$

Number of multiplication operations in $BC = 3 \times 4 \times 2 = 24$

Now,

$$ABC = A_{2\times3}\,(BC)_{3\times2}$$

Number of multiplication for A(BC) = 2 × 3 × 2 = 12

$\Rightarrow$ Total multiplication $= 24 + 12 = 36$

$\Rightarrow$ Minimum Number $= 36$

30. (a)

$$\vec{\nabla}\phi = \left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)\left[3x^2y - 4yz^2 + 6z^2x\right]$$

$$\Rightarrow \quad \vec{\nabla}\phi = (6xy + 6z^2)\,\hat{i} + (3x^2 - 4z^2)\,\hat{j} + (-8yz + 12\,zx)\,\hat{k}$$

Now at (1, 1, 1)

$$\vec{\nabla}\phi = 12\hat{i} - \hat{j} + 4\hat{k} \quad .....(1)$$

Also direction of line is, $\hat{A} = \dfrac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{17}}$ .....(2)

$\Rightarrow$ Directional derivative using (1) & (2)

$$\vec{\nabla}\phi.\hat{A} = \left(12\hat{i} - \hat{j} + 4\hat{k}\right)\left(\frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{17}}\right)$$

$$= \frac{24 - 2 + 12}{\sqrt{17}} = \frac{34}{\sqrt{17}} = 2\sqrt{17}$$

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