CLASS TEST

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ELECTROMAGNETICS THEORY

ELECTRICAL ENGINEERING

Date of Test : 28/04/2026

ANSWER KEY

1.	(a)	7.	(a)	13.	(c)	19	(d)	25.	(d)
2.	(b)	8.	(a)	14.	(c)	20	(a)	26.	(b)
3.	(d)	9.	(c)	15.	(d)	21	(a)	27.	(a)
4.	(a)	10.	(b)	16.	(b)	22.	(b)	28.	(d)
5.	(c)	11.	(b)	17.	(c)	23.	(a)	29.	(b)
6.	(a)	12.	(b)	18.	(b)	24.	(b)	30.	(a)

DETAILED EXPLANATIONS

1. (a)

In cylindrical coordinate system:

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ 0 & \rho\sin 2\phi & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\hat{a}_\rho(0) - \rho\hat{a}_\phi(0) + \hat{a}_z\frac{\partial}{\partial \rho}(\rho \sin 2\phi)\right] = \frac{1}{\rho}(\sin 2\phi)\hat{a}_z$$

at point $P(4, \pi/6, 0)$,

$$\nabla \times \vec{A} = \frac{1}{4}\sin\left(2\times\frac{\pi}{6}\right)\hat{a}_z = \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\hat{a}_z = \frac{\sqrt{3}}{8}\hat{a}_z$$

2. (b)

The given vector $\vec{A}$ is in spherical coordinates

$$\vec{A} = 2r\cos\theta\cdot\cos\phi\hat{a}_r + r^{1/2}\hat{a}_\phi$$

$$\text{div.}\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot A_r) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial \theta}(\sin\theta\cdot A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(2r^3\cos\theta\cos\phi\right) + 0 + \frac{1}{r\sin\theta}(0)$$

$$= 6\cos\theta\cos\phi$$

At point $\left(1, \frac{\pi}{4}, \frac{\pi}{3}\right)$,

$$\nabla\cdot\vec{A} = 6\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) = 2.121$$

3. (d)

The incremental work is given by:

$$dW = -Q\cdot\bar{E}\cdot dl$$

Now dl in the direction of $\hat{a}_\phi$;

$$d\bar{l} = r\,d\phi\,\hat{a}_\phi = 6\times10^{-6}\hat{a}_\phi$$

Thus $\vec{E}\cdot d\bar{l} = -200\times6\times10^{-6} = -1200\times10^{-6}$

$$dW = -40\times10^{-6}\times(-1200\times10^{-6})$$

$$= 48 \text{ nJ}$$

4. (a)

$$C = 4\pi\varepsilon a \qquad \text{where } \varepsilon = \varepsilon_0,$$

$$a = 18 \text{ cm}$$

$$C = 4\pi \times \frac{10^{-9}}{36\pi} \times 18 \times 10^{-2}$$
$$= 2 \times 10^{-11} = 20 \text{ pF}$$

5. (c)

Given, $\vec{H} = 4y\,\hat{a}_x + (z^2 - x^2)\hat{a}_y + 3y\,\hat{a}_z$ A/m

So,
$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4y & z^2 - x^2 & 3y \end{vmatrix} = \hat{a}_x(3 - 2z) + \hat{a}_z(-2x - 4)$$

At origin (0, 0, 0), $\nabla \times H = \hat{a}_x - \hat{a}_z$

$$|\nabla \times H| = \sqrt{3^2 + 4^2} = 5$$

6. (a)

At the centre of the loop, assuming loop is horizontal,

$$\vec{H} = \frac{I}{2r}\hat{a}_z$$

Magnetic flux density, $\vec{B} = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1 \times 10^{-3}}{2 \times 2}\hat{a}_z$

$$\vec{B} = 0.314\,\hat{a}_z \text{ (n Wb/m}^2)$$

7. (a)

8. (a)

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t} \text{ is continuity equation}$$

9. (c)

Given, $\phi = 4x^2 + y^2 + cz^2$

In source free region,

$$\nabla \cdot \vec{D} = 0$$

Also $D = \in E$

So, $\in (\nabla \cdot \vec{E}) = 0$

or, $\nabla \cdot \vec{E} = 0$

Also $\vec{E} = -\nabla V = -\nabla \phi$

$$\nabla V = \frac{\partial \phi}{\partial x}\hat{a}_x + \frac{\partial \phi}{\partial_y}\hat{a}_y + \frac{\partial \phi}{\partial z}\hat{a}_z$$

$$-\vec{E} = 8x\,\hat{a}_x + 2y\,\hat{a}_y + 2cz\,\hat{a}_z$$

Again, $\nabla \cdot \vec{E} = 0$

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$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z} = 8 + 2 + 2c = 0$$

$$c = -5$$

10. (b)

When spheres are brought in contact total charge gets redistributed

$$Q = \frac{Q_1+Q_2}{2}=\frac{4.5-1.5}{2} = 1.5 \text{ nC}$$

After separation of 50 cm between spheres

$$\text{Force, } |F| = \frac{Q_1 \times Q_2}{4\pi \epsilon_0 \times r^2}$$

$$= \frac{(1.5\times10^{-9})\times(1.5\times10^{-9})}{4\pi\epsilon_0 \times(50\times10^{-2})^2}$$

$$= \frac{2.25\times10^{-18}}{4\pi\times8.854\times10^{-12}\times(50\times10^{-2})^2} = 80.89 \text{ nN}$$

11. (b)

To the surface, $r = 4$; $\hat{a}_r$ is perpendicular

Hence, $\vec{ds} = r^2 \sin\theta d\theta d\phi \cdot \hat{a}_r$

Now, $\vec{J}\cdot\vec{ds} = \left(\frac{4}{r^2}\right)\cos\theta\times r^2\sin\theta\, d\theta\, d\phi$

$$= 4 \sin\theta \cdot \cos\theta \cdot d\theta d\phi$$

$$= 2 \sin 2\theta\ d\theta\ d\phi$$

$$\text{Total current} = \oint \vec{J}\cdot\vec{ds}$$

$$I = \int_{\theta=0^\circ}^{30^\circ}\int_{\phi=0}^{2\pi} 2\sin2\theta\, d\theta\, d\phi$$

$$I = 2\left[-\frac{\cos2\theta}{2}\right]_{0^\circ}^{30^\circ}\times[2\pi-0]$$

$$= [-\cos 60^\circ + \cos 0^\circ]\times 2\pi$$

$$= \left[-\frac{1}{2}+1\right]\times2\pi = 3.14 \text{ Amp}$$

12. (b)

$$\text{Magnetic flux} = \Phi = \int \vec{B}.\vec{ds}$$

$$= \int_0^{2.0}\int_{0.5}^{2.5}\left(\frac{2.0}{r}\right)\hat{a}_\phi . dr\, dz\, \hat{a}_\phi$$

$$= 2 \times 2 \times [\ln 2.5 - \ln 0.5]$$

$$\Phi = 4 \times \ln\left(\frac{2.5}{0.5}\right) = 6.4377 \text{ Wb}$$

13. (c)

The turns per unit length $= \frac{400}{0.5} = 800$

So, the axial field is,

$$B = \mu_0 H = \mu_0\, 800\, I \text{ (Wb/m}^2\text{)}$$

$$\frac{L}{l} = \frac{N\Phi}{I} = N\left(\frac{B}{I}\right)A$$

$$\frac{L}{l} = \frac{400(800I \times (4\pi \times 10^{-7}))}{I} \times \pi(0.02)^2$$

Inductance, $L = 505.32\ \mu\text{H}$

14. (c)

The potential is given by:

$$V_{AB} = -\int_B^A \bar{E} \cdot d\bar{l}$$

Now, we know that $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$ for infinite line charge

$$\vec{E} = \frac{10^{-9}}{2\pi\left(\frac{10^{-9}}{36\pi}\right)\rho}\hat{a}_\rho = \frac{18}{\rho}\hat{a}_\rho \text{ V/m}$$

$$d\bar{l} = d\rho \cdot \hat{a}_\rho + \rho d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

$$\bar{E} \cdot d\bar{l} = \frac{18}{\rho}d\rho$$

$$V_{AB} = -\int_4^2 \frac{18}{\rho}d\rho = \left[-18\ln\rho\right]_4^2 = -18\left[\ln 2 - \ln 4\right]$$

$$= 18 \ln 2 = 12.48 \text{ volts}$$

15. (d)

With in the conductor,

$$\vec{J} = \nabla \times \vec{H}$$

$$= -\frac{\partial}{\partial z}\left(\frac{Ir}{2\pi a^2}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{Ir^2}{2\pi a^2}\right)\hat{a}_z$$

$$\vec{J} = \frac{1}{r}\frac{I}{2\pi a^2} \cdot 2r\, \hat{a}_z$$

$$\vec{J} = \frac{I}{\pi a^2}\hat{a}_z$$

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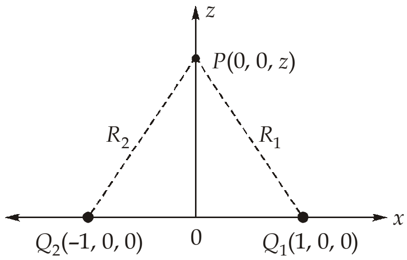
Outside the conductor, $\vec{J} = \nabla \times \vec{H}$

$$= \frac{-\partial}{\partial z}\left(\frac{I}{2\pi r}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{I}{2\pi}\right)\hat{a}_z$$

$$\vec{J} = 0$$

16. (b)

Consider the diagram shown below:



Distance of Q_1 and Q_2 from point P are:

$$R_1 = R_2 = \sqrt{z^2+1}$$

Since, $Q_1 = Q_2$

and $R_1 = R_2$

Now, potential at P is twice that of single charge

$$V = 2\times\frac{Q}{4\pi\varepsilon_0 R_1}$$

$$= 2\times\frac{8\times10^{-9}}{4\pi\times\left(\frac{10^{-9}}{36\pi}\right)\sqrt{z^2+1}} = \frac{144}{\sqrt{z^2+1}} \text{ Volts}$$

$$\frac{dV}{dz} = \frac{d}{dz}\left(\frac{144}{(z^2+1)^{1/2}}\right)$$

$$= 144\times\left[-\frac{1}{2}(z^2+1)^{-3/2}\right]\times 2z$$

$$\frac{dV}{dz} = \frac{-144z}{(z^2+1)^{3/2}};$$

$$\left|\frac{dV}{dz}\right| = \frac{144z}{(z^2+1)^{3/2}} \text{ V/m}$$

17. (c)

$$E = \frac{\Delta V}{\Delta Z} = \frac{250-100}{5\times10^{-3}} = 3\times10^4 \text{ V/m}$$

$$\vec{E} = -\nabla V = -3\times10^4\,\hat{a}_z \text{ V/m}$$

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E} = \left(\frac{10^{-9}}{36\pi}\right) \times 2.4 \times (-3 \times 10^4)\hat{a}_z$$

$$\vec{D} = -6.366 \times 10^{-7} \hat{a}_z \text{ C/m}^2$$

Since $\vec{D}$ is constant between the disks, and $D_n = \rho_s$ at a conductor surface.

$\therefore$ $\rho_s = \pm 6.366 \times 10^{-7}$ C/m^2

Positive sign on the upper plate and negative sign on the lower plate.

18. (b)

Magnitude of electrical flux density,

$$|\vec{D}| = \frac{Q}{4\pi |R|^2}$$

$$R = \sqrt{1^2 + (3)^2 + (-4)^2} = 5.099 \text{ m}$$

$$|\vec{D}| = \frac{40 \times 10^{-9}}{4\pi (5.099)^2} = 122.43 \text{ pC/m}^2$$

19 (d)

$$\vec{R}_{21} = (\vec{R}_1 - \vec{R}_2)$$
$$= (1, -2, 3) - (2, -1, 0)$$
$$= (-1, -1, 3)$$
$$= -\hat{i} - \hat{j} + 3\hat{k}$$

$$F_{21} = \frac{Q_1 Q_2}{4\pi \epsilon_0 |\vec{R}_{21}|^2} \hat{R}_{21}$$

$$= \frac{25 \times 20 \times 10^{-12} \times 9 \times 10^9}{\left(\sqrt{1+1+9}\right)^2} \left(\frac{-\hat{i} - \hat{j} + 3\hat{k}}{\sqrt{1+1+9}}\right)$$

$$= 0.123(-\hat{i} - \hat{j} + 3\hat{k}) \text{N}$$

20 (a)

Vector from the line to the point P

$$\vec{r} = -2u_x + 3u_y$$

$$\vec{E} = \frac{\lambda}{2\pi \epsilon_0 r} \hat{r}$$

$$\vec{E} = \frac{0.1 \times 10^{-6}}{2\pi \epsilon_0 \sqrt{4+9}} \left(\frac{-2\hat{u}_x + 3\hat{u}_y}{\sqrt{13}}\right)$$

$$= -276.92\hat{u}_x + 415.38\hat{u}_y$$

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21 **(a)**

Electric field for free space is

$$E = \left(\frac{5z^3}{\varepsilon_o}\right)\hat{a}_z \text{ V/m}$$

$$\vec{D} = 5z^3\hat{a}_z \qquad \{\because D = \varepsilon_o E\}$$

Convert to spherical system

$$Z = r\cos\theta$$

$$\hat{a}_z = \cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta$$

$$\vec{D} = 5r^3\cos^3\theta\left[\cos\theta\,\hat{a}_r - \sin\theta\,\hat{a}_\theta\right]$$

By Gauss's divergence theorem,

$$\oint \vec{D}\cdot\vec{ds} = \iiint(\vec{\nabla}\cdot\vec{D})dV$$

$$\nabla\cdot\vec{D} = 15r^2\cos^2\theta$$

$$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$Q = \int_{r=0}^{3}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} 15r^2\cos^2\theta r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$Q = 15\int_{r=0}^{3} r^4 dr\int_0^{\pi}\cos^2\theta\sin\theta\,d\theta\int_0^{2\pi} d\phi$$

$$= -15\left[\frac{r^5}{5}\right]_0^3 \times \left[\frac{\cos^3\theta}{3}\right]_0^{\pi} \times 2\pi$$

$$= -729 \times \left[-\frac{1}{3} - \frac{1}{3}\right] \times 2\pi$$

$$= -729 \times \frac{-2}{3} \times 2\pi$$

$$= 972\pi$$

22. **(b)**

$$V = x^2yz - By^3z$$

$\nabla^2 V = 0$ is Laplace's equation.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V}{\partial x} = 2xyz$$

$$\frac{\partial^2 V}{\partial x^2} = 2yz$$

$$\frac{\partial V}{\partial y} = x^2z - 3y^2Bz$$

$$\frac{\partial^2 V}{\partial y^2} = -6yBz$$

$$\frac{\partial V}{\partial z} = x^2y + By^3$$

$$\frac{\partial^2 V}{\partial z^2} = 0$$

$$2yz - 6yBz = 0$$

$$yz - 3yzB = 0$$

$$B = \frac{1}{3}$$

23. (a)

Within the conductor, $\vec{J} = \nabla \times \vec{H}$

$$= \frac{-\partial}{\partial z}\left(\frac{Ir}{2\pi a^2}\right)\hat{a}_r + \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(\frac{Ir^2}{2\pi a^2}\right)\hat{a}_z$$

$$\vec{J} = \frac{1}{r}\cdot\frac{I}{2\pi a^2}(2r)\hat{a}_z = \frac{I}{\pi a^2}\hat{a}_z$$

Outside the conductor,

$$\vec{J} = \vec{\nabla} \times \vec{H}$$

$$= -\frac{\partial}{\partial z}\left(\frac{I}{2\pi r}\right)\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{I}{2\pi}\right)\hat{a}_z$$

$$J = 0$$

$\therefore$ (a) option is correct.

24. (b)

$$\vec{E} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$d\vec{l} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

Potential difference is given by,

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$= -\int (2x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= -\int_1^2 2x\,dx\,\hat{a}_x - \int_2^0 y\,dy\,\hat{a}_y - \int_3^0 z\,dz\,\hat{a}_z$$

$$= -2\left[\frac{x^2}{2}\right]_1^2 - \left[\frac{y^2}{2}\right]_2^0 - \left[\frac{z^2}{2}\right]_3^0$$

$$= -2\left[\frac{3}{2}\right] - [-2] - \left[\frac{-9}{2}\right]$$

$$V = -\frac{1}{2}[6 - 4 - 9] = 3.5 \text{ V}$$

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25. (d)

We know, $$\frac{L}{l} = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times10^{-7}}{2\pi}\ln\left(\frac{3}{1}\right) \quad [\because \mu = \mu_0 \text{ as } \mu_r = 1]$$

$$\frac{L}{l} = 0.21972\ \mu\text{H/m}$$

∴ Inductance for 1 km, $$L = 0.21972 \times 1000 = 219.72\ \mu\text{H}$$

26. (b)

Poisson's equation is given by,

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(4y^3+8x^2) = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial^2}{\partial y^2}(4y^3)+\frac{\partial^2}{\partial x^2}(8x^2) = \frac{-\rho}{\varepsilon_0} \quad \text{At point } P(4, 0)$$

$$\rho = -16\varepsilon_0$$

27. (a)

$$E = \frac{\Delta V}{\Delta z} = \frac{250-100}{5\times10^{-3}} = 3\times10^4\ \text{V/m}$$

$$\vec{E} = -\nabla\vec{V} = -3\times10^4\ \text{V/m}$$

$$D = \epsilon_0\epsilon_r\,\vec{E} = \frac{10^{-9}}{36\pi}\times2.4\times(-3\times10^4)\hat{a}_z$$

$$= -6.37\times10^{-7}\hat{a}_z\ \text{C/m}^2$$

As D is constant between the disks and $D_n = \rho_s$ at a conductor surface

$$\rho_s = \pm 6.37\times10^{-7}\ \text{C/m}^2$$

Positive sign on upper plate and negative sign on lower plate.

28. (d)

Using boundary condition at dielectric - dielectric interface for oblique incidence, we have

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2}$$

$$\frac{\tan 60°}{\tan\alpha_2} = \frac{3}{\sqrt{3}}$$

$$\tan\alpha_2 = \frac{\sqrt{3}}{3}\tan 60° = 1$$

∴ $$\alpha_2 = 45°$$

29. (b)

$$\rho_v = \nabla \cdot \vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho^2 z \cos\phi\right)\Big|_{(1,2\pi,2)}$$

$$= \frac{1}{\rho}2\rho \cdot z\cos\phi\Big|_{(1,2\pi,2)}$$

$$= \frac{1}{1}\times 2\times 1\times 2\times \cos 2\pi$$

$$= 4\ \text{C/m}^2$$

30. (a)

Given, $N = 300$ turns, $I = 5$ A

and $l = 50\ \text{cm} = 0.5\ \text{m}$

$\because$ $H \cdot l = NI$

$$H = \frac{NI}{l}$$

Also, $B = \mu_0 H$

$$B = 4\pi\times 10^{-7}\times\left(\frac{NI}{l}\right)$$

$$= 4\pi\times 10^{-7}\times\left(\frac{300\times 5}{0.5}\right) = 3.77\times 10^{-3}T = 3.77\ \text{mT}$$

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