CLASS TEST

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ENGINEERING MATHEMATICS

EC | EE

Date of Test : 02/02/2026

ANSWER KEY

1.	(c)	7.	(c)	13.	(c)	19.	(a)	25.	(c)
2.	(b)	8.	(b)	14.	(d)	20.	(b)	26.	(a)
3.	(c)	9.	(d)	15.	(a)	21.	(d)	27.	(b)
4.	(b)	10.	(d)	16.	(b)	22.	(b)	28.	(d)
5.	(b)	11.	(d)	17.	(b)	23.	(d)	29.	(d)
6.	(c)	12.	(c)	18.	(b)	24.	(a)	30.	(c)

DETAILED EXPLANATIONS

1. (c)

$$\text{Volume of solid} = \int_a^b \pi y^2 dx$$

Given $$y = \frac{1}{2\sqrt{x}}$$

$$\text{Volume of the solid} = \int_3^4 \frac{\pi}{4x}\cdot dx = \frac{\pi}{4}(\ln x)_3^4 = \frac{\pi}{4}\ln\left(\frac{4}{3}\right)$$

2. (b)

$$P(W \cup L) = P(W) + P(L) - P(W \cap L)$$

$$P(W \cup L) = 0$$ [If winning and losing are mutually exclusive]

$$P(W \cup L) = 0.45 + 0.25 = 0.70$$

$$P(W' \cup L') = 1 - P(W \cup L) = 1 - 0.70 = 0.3$$

3. (c)

We know, $$AA^{-1} = I,$$

$$A \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I$$

$$\frac{A}{6} \times \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = I$$

$$\frac{A}{6} = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}^{-1}$$

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

4. (b)

The given equation will be consistent, if

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ \lambda - 1 & 4\lambda - 2 & \lambda + 3 \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{vmatrix} \lambda - 1 & 3\lambda + 1 & 2\lambda \\ 0 & \lambda - 3 & 3 - \lambda \\ 2 & 3\lambda + 1 & 3(\lambda - 1) \end{vmatrix} = 0$$

$C_3 \rightarrow C_3 + C_2$

$$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 5\lambda+1 \\ 0 & \lambda-3 & 0 \\ 2 & 3\lambda+1 & 6\lambda-2 \end{vmatrix} = 0$$

$$(\lambda-3)\begin{vmatrix} \lambda-1 & 5\lambda+1 \\ 2 & 2(3\lambda-1) \end{vmatrix} = 0$$

$$2(\lambda - 3)[(\lambda - 1)(3\lambda - 1)-(5\lambda + 1] = 0$$

$$6\lambda(\lambda - 3)^2 = 0$$

$$\lambda = 0 \text{ or } 3$$

The largest value $\lambda = 3$

5. (b)

Here exhaustive no of cases = ${}^{40}C_4$.

If t_3 = 25, then tickets t_1 and t_2 must come out of 24 tickets numbered 1 to 24. This can be done in ${}^{24}C_2$ ways.

Then t_4 must come out of the 15 tickets (numbered 26 to 40) which can be done in ${}^{15}C_1$ ways.

$\therefore$ favorable number of cases = ${}^{24}C_2 \times {}^{15}C_1$

Hence the probability of t_3 being 25

$$= \frac{{}^{24}C_2 \times {}^{15}C_1}{{}^{40}C_4}$$

6. (c)

$$AA^{-1} = I$$

$$\therefore \quad \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad 2x = 1$$

$$\therefore \quad x = \frac{1}{2}$$

7. (c)

$$f(z) = \frac{z-1}{z+1} = 1 - \frac{2}{z+1}$$

$$\Rightarrow \quad f(0) = -1, f(1) = 0$$

$$\Rightarrow \quad f'(z) = \frac{2}{(z+1)^2} \qquad f'(0) = 2;$$

$$f''(z) = -\frac{4}{(z+1)^3} \qquad f''(0) = -4$$

$$f'''(z) = \frac{12}{(z+1)^4} \qquad f'''(0) = 12 \quad \text{and so on}$$

Taylor series : $f(z) = f(z_0) + (z - z_0) f'(z_0) + \frac{(z - z_0)^2}{2!} f''(z_0) + \frac{(z - z_0)^3}{3!} f'''(z_0) + \ldots$

$$f(z) = -1 + z(2) + \frac{z^2}{2}(-4) + \frac{z^3}{6}(12) + \ldots = -1 + 2z - 2z^2 + 2z^3 \ldots$$

$$f(z) = -1 + 2(z - z^2 + z^3 \ldots)$$

8. (b)

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x - 2) = 0$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^3 - 8) = 0$$

Also $f(2) = 0$

Thus $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

$\therefore$ f is continuous at $x = 2$

and $Lf'(2) = 1$ and $Rf'(2) = 12$

$\therefore$ f is not differentiable at $x = 2$.

9. (d)

$D = -96$ for the given matrix

$$|A| = \begin{vmatrix} 4 & 12 & 0 \\ 8 & 24 & 16 \\ -4 & 0 & 8 \end{vmatrix} = 2^3 \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$

(Taking 2 common from each row)

$\therefore$ $\text{Det}(A) = (2)^3 \times D$

$= 8 \times (-96) = -768$

10. (d)

$2x - 8 = 2h$ (say)

$\Rightarrow$ $2x = 8 + 2h$

$\Rightarrow$ $x = 4 + h$

$\therefore$ $\lim_{h \to 0} \frac{(8 + 2h)^{1/3} - 2}{2h}$

Above form is $\left(\frac{0}{0}\right)$ by putting the value $h = 0$

Applying L' Hospital rule $\lim_{h \to 0} \frac{\frac{1}{3}(8 + 2h)^{\left(\frac{1}{3} - 1\right)} \times 2}{2}$

$$= \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$$

11. (d)

$$A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

Inter changing rows and columns,

$$A^T = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$\therefore$ $$A^TA = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}^2 = A^2$$

$\Rightarrow$ $|A^TA| = |A^2|$

but, $A^TA = I$ (given)

$\therefore$ $|A^2| = |I| = 1$

$$|A| = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$$

$\Rightarrow$ $R_1 \to R_1 + R_2 + R_3$

$$= (a+b+c)\begin{bmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{bmatrix}$$

$\Rightarrow$ $C_2 \to C_2 - C_1$ $\quad C_3 \to C_3 - C_1$

$$= (a+b+c)\begin{bmatrix} 1 & 0 & 0 \\ b & c-b & a-b \\ c & a-c & b-c \end{bmatrix}$$

$= (a + b + c)\ [(c - b)(b - c) - (a - c)(a - b)]$

$= (a + b + c)\ (-b^2 - c^2 + 2bc - a^2 + ac + ab - bc)$

$= -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

$= -(a^3 + b^3 + c^3 - 3abc)$

$\because$ $abc = 1$ (given)

$\Rightarrow$ $-(a^3 + b^3 + c^3 - 3)$

$\because$ $|A^2| = 1$

$\Rightarrow$ $(a^3 + b^3 + c^3 - 3)^2 = 1$

$\because$ a, b, c are positive

$a^3 + b^3 + c^3 > 3 \times 1$

$\therefore$ $a^3 + b^3 + c^3 - 3 = 1$

$\Rightarrow$ $a^3 + b^3 + c^3 = 4$

12. (c)

We parameterize the curve using $t = y$

$$x = 2 - 3t^2 \qquad -1 \le t \le 1$$

$$y = t$$

Then

$$dx = -6t\, dt$$

$$dy = dt$$

$$\int_c 2y^3 dx + 3x^2 dy = \int_{-1}^{1} \left[2t^3(-6t) + 3\left(2 - 3t^2\right)^2 \right] dt = \int_{-1}^{1} \left(15t^4 - 36t^2 + 12\right) dt$$

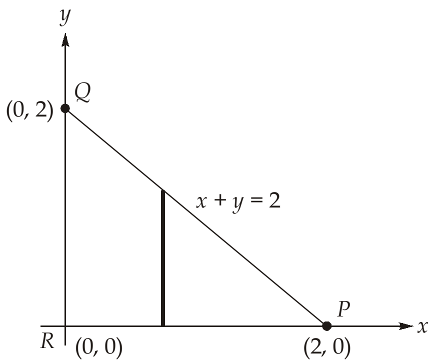
$$= \left[\frac{15t^5}{5} - \frac{36t^3}{3} + 12t \right]_{-1}^{1} = \left[3t^5 - 12t^3 + 12t \right]_{-1}^{1} = 3 + 3 = 6$$

13. (c)

$$I = \int_0^2 \int_0^{2-x} 5y\, dy\, dx = \int_0^2 \left[\frac{5y^2}{2} \right]_0^{2-x} dx$$

$$= \int_0^2 \frac{5}{2}(2-x)^2 dx = -\frac{5}{2} \frac{(2-x)^3}{3} \Bigg|_0^2$$

$$= -\frac{5(2-x)^3}{6} \Bigg|_0^2 = -\frac{5}{6}(-8) = \frac{20}{3}$$



14. (d)

$$x + \frac{1}{x} = 2\cos\theta$$

$$x^2 - 2\cos\theta \,.\, x + 1 = 0$$

$$\Rightarrow \qquad x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$$x = \cos\theta \pm i\sin\theta$$

$$\therefore \qquad x^r = (\cos\theta \pm i\sin\theta)^r = \cos r\theta \pm i\sin r\theta$$

$$x^{-r} = (\cos\theta \pm i\sin\theta)^{-r} = \cos r\theta \mp i\sin r\theta$$

$$x^r + \frac{1}{x^r} = 2\cos r\theta$$

Hence, option (d) is correct.

15. (a)

Given, $\quad u = x \log xy$

and $\quad x^3 + y^3 + 3xy = 1$

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$$

$$\frac{\partial u}{\partial x} = x \cdot \frac{1}{xy} \cdot y + \log xy$$

$$= 1 + \log xy$$

$$\frac{\partial u}{\partial y} = x \cdot \frac{1}{xy} \cdot x = \frac{x}{y}$$

Also, $$3x^2 + 3y^2 \frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y \cdot 1\right) = 0$$

$\Rightarrow$ $$\frac{dy}{dx} = -\left(\frac{x^2 + y}{y^2 + x}\right)$$

$\therefore$ $$\frac{du}{dx} = (1 + \log xy) + \frac{x}{y}\left\{-\left(\frac{x^2 + y}{y^2 + x}\right)\right\}$$

16. **(b)**

We know that, if eigen values of a matrix satisfies an equation that matrix also satisfies that equation

$\therefore$ □ $|A - \lambda I| = 0$

gives $\lambda^3 - 4\lambda^2 - 3\lambda + 11 = 0$

or, $A^3 - 4A^2 - 3A + 11I = 0$

Also, we know,

Sum of eigen values = sum of principal diagonal elements

and product of eigen values = elements determinant of the matrix

for a given cubic equation,

$$ax^3 + bx^2 + cx + d = 0,$$

$$\text{sum of roots} = -\frac{b}{a}$$

$$\text{and product of roots} = -\frac{d}{a}$$

$\therefore$ $$\text{sum of eigen values} = -\frac{b}{a} = \frac{-(-4)}{1} = 4$$

and product of eigen values = $\frac{-d}{a} = -11$

$\because$ for given options,

sum of principal diagonal elements in all the matrices = 4

we check determinant of the matrices,

$\because$ $$\det\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix} = -11$$

$\therefore$ eigen values of this matrix satisfy the given equation and,

$\therefore$ this is the required matrix A.

17. (b)

Probability of a success $= \frac{2}{6} = \frac{1}{3}$

Probability of failures $= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ Prob. of no success = prob. of all 3 failure

$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

$\Rightarrow$ Prob. of one success and 2 failures

$= 3C_1 \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

Probability of two successes and 1 failures

$= 3C_2 \times \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$\Rightarrow$ Probability of three successes

$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

Now,

x_i	0	1	2	3
p_i	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{1}{27}$

$\therefore$ mean, $\mu = \Sigma p_i x_i$

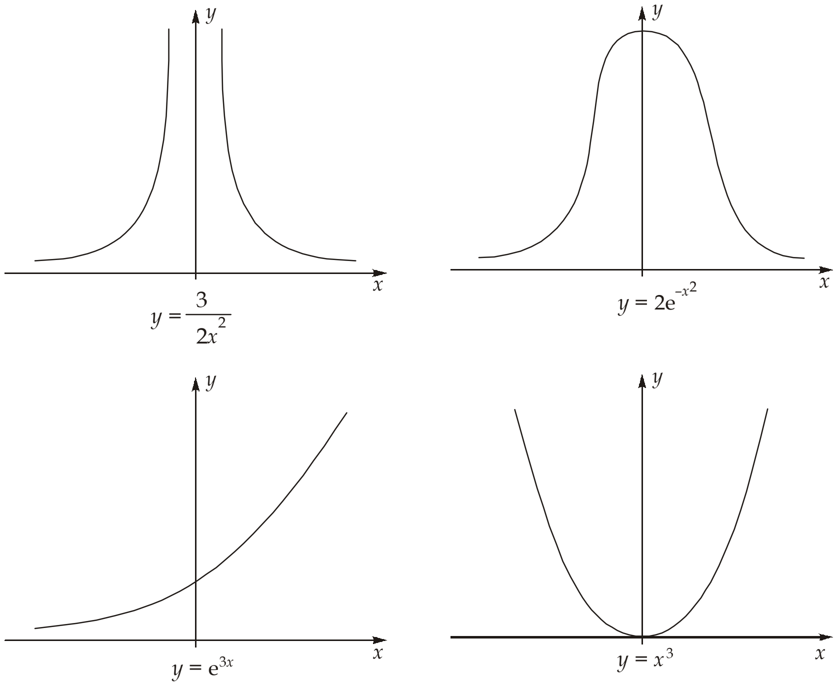
$= 0 + \frac{4}{9} + \frac{4}{9} + \frac{1}{9} = 1$

Also, $\Sigma p_i x_i^2 = 0 + \frac{4}{9} + \frac{8}{9} + \frac{9}{27} = \frac{5}{3}$

$\therefore$ Variance, $\sigma^2 = \Sigma p_i x_i^2 - \mu^2 = \frac{5}{3} - 1 = \frac{2}{3} = 0.667$

18. **(b)**

From the graphs below, we can see that only $2e^{-x^2}$ is strictly bounded.



19. **(a)**

Given that the partial differential equation is parabolic

$$\therefore \quad b^2 - 4ac = 0$$
$$b^2 - 4(4)(4) = 0$$
$$b^2 - 64 = 0$$
$$b^2 = 64$$

20. **(b)**

$$P[X > 1] = \int_1^\infty f(x)\,dx = \int_1^\infty e^{-2x}\,dx = \left.\frac{-e^{-2x}}{2}\right|_1^\infty$$

$$= -\left(\frac{e^{-2\infty}}{2} - \frac{e^{-2}}{2}\right) = \frac{e^{-2}}{2} = 0.067$$

21. **(d)**

$$\frac{d^2y}{dx^2} + y = \cos x$$

$$(D^2 + 1)y = \cos x$$

$$PI = \frac{\cos x}{D^2 + 1}$$

Putting $\quad D^2 = -1$

$$PI = \frac{\cos x}{-1+1} \quad [\text{Makes denominator zero}]$$

$\therefore$ Differentiating numerator and denominator

$$PI = x \cdot \frac{\cos x}{2D}$$

$$= \frac{1}{2} x \int \cos x \, dx = \frac{1}{2} x \sin x$$

22. (b)

$$y = 7x^2 + 12x$$

$$\frac{dy}{dx} = 14x + 12$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 26$$

$$\left.\frac{dy}{dx}\right|_{x=3} = 54$$

$\because$ x is defined in open interval $x = (1, 3)$

$\therefore$ $1 < x < 3$

$\therefore$ $26 < \frac{dy}{dx} < 54$

23. (d)

$$\frac{dx}{dt} = 9x - 11y$$

$$\frac{dy}{dt} = 7x + 13y$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & -11 \\ 7 & 13 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9x & -11y \\ 7x & +13y \end{bmatrix}$$

24. (a)

The volume of a solid generated by revolution about the x-axis, of the area bounded by curve $y = f(x)$, the x-axis and the ordinates $x = a$, $y = b$ is

Volume $\int_a^b \pi y^2 dx$

Here, $a = 2$, $b = 3$ and $y = 2\sqrt{x} \Rightarrow y^2 = 4x$

$\therefore$ Volume $= \int_2^3 \pi 4x dx = 4\pi \left[\frac{x^2}{2}\right]_2^3 = 2\pi[x^2]_2^3$

$= 2\pi\,[9 - 4] = 10\pi$

25. (c)

Given,

$$\text{Trace } A = 9$$
$$|A| = 24$$
$$\lambda_1 = 3$$
$$\lambda_1 + \lambda_2 + \lambda_3 = 9$$
$$\Rightarrow 3 + \lambda_2 + \lambda_3 = 9$$
$$\Rightarrow \lambda_2 + \lambda_3 = 6$$

26. (a)

$$\int_0^{\pi} \frac{dx}{c\left(\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right) + d\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)} = \int_0^{\pi} \frac{dx}{(c+d)\cos^2\frac{x}{2} + (c-d)\sin^2\frac{x}{2}}$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}\,dx}{(c+d) + (c-d)\tan^2\frac{x}{2}} = \frac{1}{(c-d)}\int_0^{\pi} \frac{\sec^2\frac{x}{2}\,dx}{\frac{(c+d)}{(c-d)} + \tan^2\frac{x}{2}}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\left\{\tan\frac{x}{2}\sqrt{\frac{c-d}{c+d}}\right\}\right]_0^{\pi}$$

$$= \frac{2}{c-d}\sqrt{\frac{c-d}{c+d}}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{2}{\sqrt{(c-d)(c+d)}} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{c^2 - d^2}}$$

27. (b)

Let
$$I = \int_0^{\infty} \frac{e^{-x}\sin bx}{x}\,dx$$

$$\frac{dI}{db} = \int_0^{\infty} \frac{\partial}{\partial b}\left(\frac{e^{-x}\sin bx}{x}\right)dx = \int_0^{\infty} \frac{e^{-x}x\cos bx}{x}\,dx$$

$$= \int_0^{\infty} e^{-x}\cos bx\,dx$$

We know that

$$\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int_0^{\infty} e^{-x}\cos bx\,dx = \left[\frac{e^{-x}}{1+b^2}[-\cos bx + b\sin bx]\right]_0^{\infty}$$

$$\frac{dI}{db} = \frac{1}{1+b^2}$$

Integrating both sides, $I = \tan^{-1} b$

28. **(d)**

$$y'' + 4y' + 4y = 0$$
$$(D^2 + 4D + 4)y = 0$$
$$\Rightarrow \quad (D + 2)(D + 2) = 0$$
$$\Rightarrow \quad D = -2, -2$$
$$\therefore \quad y = C_1 e^{-2x} + C_2 x e^{-2x}$$
$$y(0) = 0 \Rightarrow 0 = C_1$$
$$y(1) = 0 \Rightarrow 0 = C_1 + C_2$$
$$\Rightarrow \quad C_2 = 0$$
$$y = 0 \text{ is the solution}$$
$$\therefore \quad y(2) = 0$$

29. **(d)**

$$|A - \lambda I| = 0$$
$$\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$
$$\begin{vmatrix} 3-\lambda & 1 \\ -2 & -\lambda \end{vmatrix} = 0$$
$$-3\lambda + \lambda^2 + 2 = 0$$
$$\lambda^2 - 3\lambda + 2 = 0$$
$$A^2 - 3A + 2 = 0$$
$$A - 3I + 2A^{-1} = 0$$

30. **(c)**

$$AX = \lambda X$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -6 \end{bmatrix} = (-6)\begin{bmatrix} 3 \\ -6 \end{bmatrix}$$
$$3a - 6b = -18 \quad \text{... (i)}$$
$$3c - 6d = 36 \quad \text{... (ii)}$$
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = (-3)\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$
$$3a - 3b = -9 \quad \text{... (iii)}$$
$$3c - 3d = 9 \quad \text{... (iv)}$$

From equation (i) and (ii), $a = 0$ and $b = 3$.
From equation (ii) and (iv), $c = -6$ and $d = -9$.

$$\therefore \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -6 & -9 \end{bmatrix}$$

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